Space Flight Mechanics

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Lecture No. # 20

Three Body Problem (Contd.)

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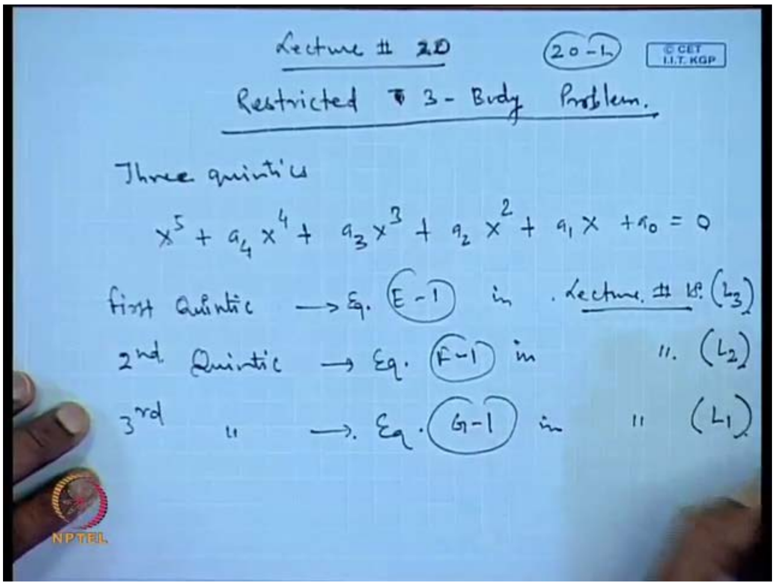

We have been discussing about the restricted 3-Body Problem in the last lecture. So, we continue with this and we will try to conclude it today. So, in the restricted 3-Body Problem, we got three quintics, which we have described by the general form X to the power 5 plus a 4 times X to the power 4 plus a 3 times X cube plus a 2 X square plus a 1 X plus a 0 equal to 0. So, we are the first quintic, this was described by equation number e 1 in lecture number eighteen, and this described basically the point L 3. The second quintic, this was described by equation number F 1, in lecture number eighteen, and this was describing the L 2 point, the second lagranges point. The third quintic, this was described by equation number G 1, in lecture number eighteen and this describe the lagranges point L 1.

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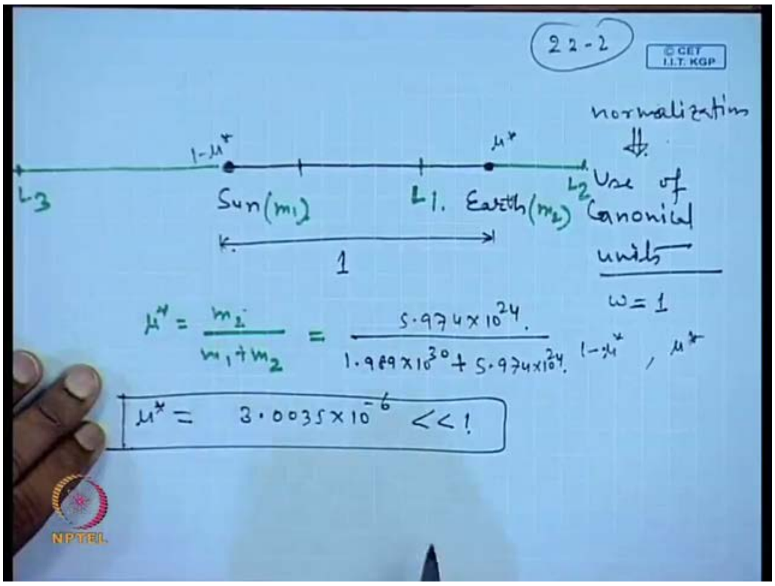

Now for the Sun-Earth System, the distance between the Earth and Sun has already been normalized. So, this normalize normalization what normalization, in technical language, this is called the, whatever the normalization we are representing here, this is basically the use of canonical units. So, we have to use the canonical units, where we represented the quantities in such a way that, this distance got normalized to one. So, this two whole distance, we represented as one unit. Similarly our angular velocity, that got equal to 1.

The mass of these two particles they were represented as, 1 minus mu 1 star and another as mu star. So, this mass was 1 minus mu star and this was mu star. So, the mass also got normalize to 1. Advantage of using this unit is that, it makes the whole representation very simple. And then our L 1 point, it lies somewhere on this side. L 3 point is there and L 2 point lies somewhere on the right side of this. So, L 2 point is here and L 1 point is lying here.

Now, for the Sun Earth system mu star which is m 2, here we are taking for this and m 1 we are writing for this. So, m 2 by m 1 plus m 2 and this is the mass of the Earth. So, this becomes. So, we have used in earlier, I have discussed about how much the mass of the Sun and the Earth is. So, we can use those values. So, Sun mass is around 1.989 into 10 to the power 30 kgs and the Earth mass is around 5.974 into 10 to the power 24 kgs, so 5.974 into 10 to the power 24 kgs. So, this gives you value of 3.0035 into 10 to the power minus 6 and this is much smaller than 1.

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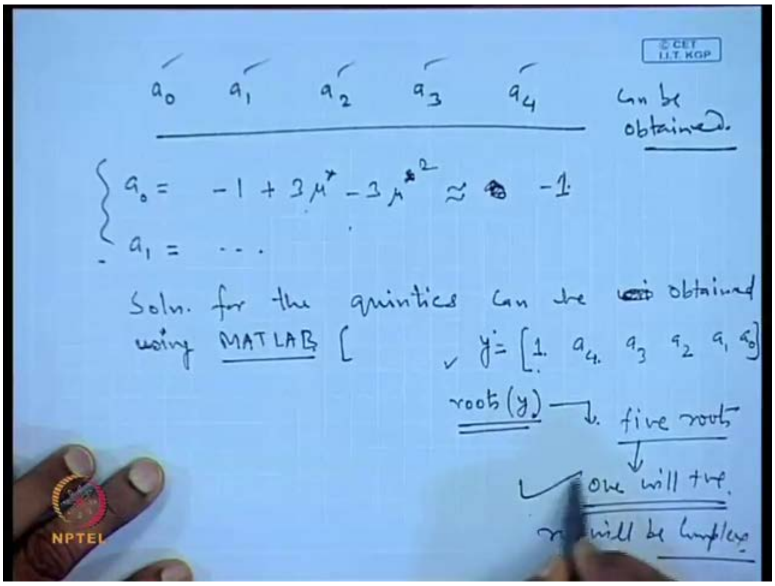

So, once we have got this mu star then the coefficients a 0, a 1, a 2, a 3, a 4 can be obtained. Because the equations for this coefficients a 0, a 1, a 2, a 3, a 4 we have already written. You just need to insert the value of mu star and compute it. This can be done in MAT LAB very easily. But the MAT LAB, it does not give you very accurate calculation unlike if you program in Fortran or C and use double precision there.

Suppose a 0 is written as 1 minus 1 plus 3 minus 1 plus 3 mu star minus 3 mu star whole square. So, with this very small value of mu star, which is around 10 to the 3 times 10 to the power minus 6, so mu square will almost it, this will be 10 to the power minus 11 along with this term along with 3 and this term also 10 to the power minus six. So, this is very small as compared to a 0. So, a 0 can be approximated as, this quantity can be approximated as minus 1.

Similarly, a 1 you can find it out. So, after finding this quantities, the solution for the quintics can be obtained using MAT LAB and this you can do for the checking purpose. It is very easy to do what you need to do here, is that the quintic equation you need to write in a format, where you will indicate a vector, which is y. So, y we will write as 1 and then a 4, a 3, a 2, a 1 and a 0. So, this is the way of describing a polynomial in MAT LAB and thereafter you use a command roots y. So, this will give you the roots of this polynomial which describes this y. So, the polynomial of the quintics now once you get this solution, the roots. So, you will obtain 5 five roots, out of this one will be positive and rest other will be rest will be complex. So, we reject the complex root and accept only the positive root.

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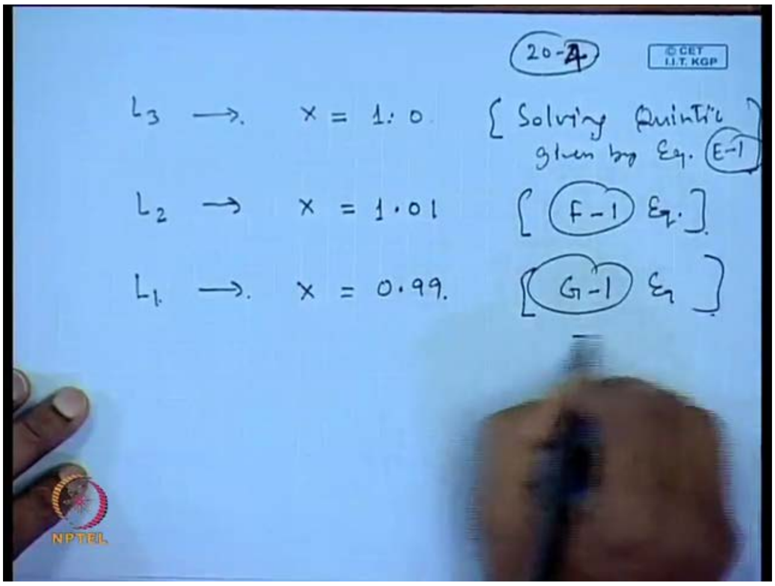

So for L 3 point once you solve. So, you get X is equal to 1.0 and what is the point X, we have indicated earlier. Point X is the distance of this is, suppose this is the barycenter. So, X indicates the distance of the lagranges point from the barycenter. Now in this case, mu star is very small and the distance also we have noted earlier that, this distance is nothing, but from here to here this is mu star and the distance from this place to place this is, 1 minus mu star. So, this is your mu star.

So this is the distance which is X, which gives you the position of the point L 3. So, L 3 can be obtained by solving this quintic and that gives you X is equal to 1.0, so similarly solving quintic given by equation e 1. Similarly for L 2 you get X is equal to 1.01 and this you obtain from equation F 1. For L 1 you get X is equal to 0.99 and this you get from equation G 1.

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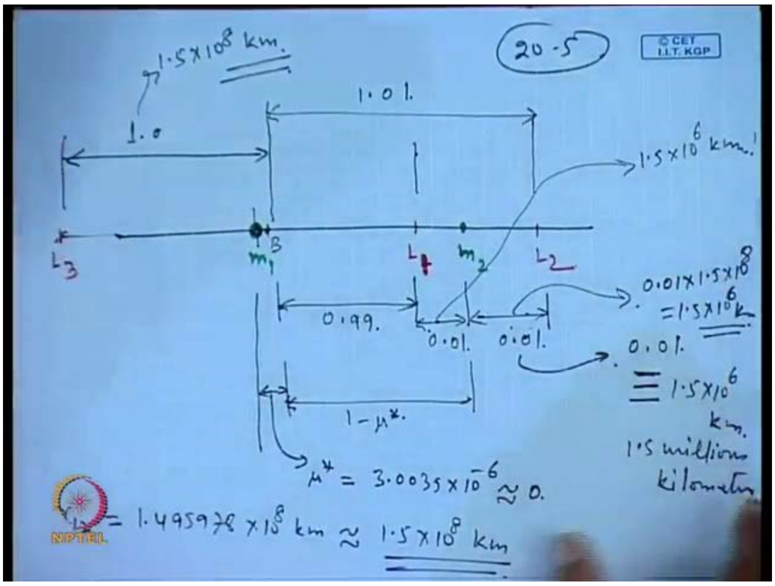

Now, the barycenter which we have shown earlier in the figure, here this is the barycenter which is mu star, is very small. So, this almost coincides the point b almost coincides with the center of the Sun. So, we can neglect it and show all the distances from the Sun itself. So, the distance is. So, we can bring b close to this. So, here this is your b which is very close to the Sun or either you can, because the b, it lies inside the Sun itself. It is so heavy with respect to the Earth showing b here in this place, so all the distances can be written here now. This is 1.0 and distance to the L 2 point, this is 1.01. Distance to the L 1 point, this is 0.99 and therefore, this distance becomes 0.01. And the distance from here to here also, this becomes 0.01. So, this quantity, this is nothing but 1 minus mu star. This is your mu star, which we have taken as 3.0035 into 10 to the power minus 6 and this is nearly equal to 0.

Now this are the distances in canonical units are the on the normalized scale and this you can expand it to the original scale. So, if you expand it to the original scale, so this will become 0.01. This will be equivalent to 1.5 times 10 to the power 6 kilometers. Because the distance this is, for the original distance from here to here. This is distance r 1 2. This is 1.495978 into 10 to the power 8 kilometers which is nearly equal to 1.5 times 10 to the power 8 kilometers.

So, to expand this distance to the original scale, we need to multiply 0.01 into 1.5 into 10 to the power 8 and this gives you 1.5 into 10 to the power 6 kilometers, that is 1.5

million miles million kilometers. So, this we have already discussed many times that last time we have discussed this that, this distance of the L 1 and L 2. This is around 1.5 million on the left and right of the Earth. So, this distance also gets to 1.5 into 10 to the power 6 kilometers and this distance output, this is 1.5 times 10 to the power 8 kilometers. So, this is how your lagranges points are distributed about the Sun Earth on a single line.

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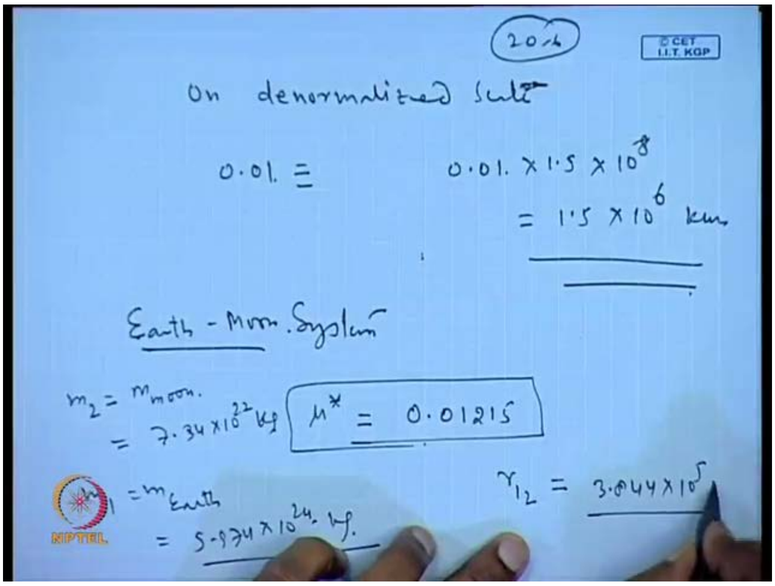

So, on denormalized scale, we have 0.01. This is equivalent to 0.01 times 1.5 times 10 to the power 8 equal to 1.5 into 10 to the power 6 kilometers. Once we have done this. So, if the same kind of treatment can be given to the Earth-Moon System Earth-Moon System. For the Earth-Moon System we know the mu star this is around 0.01215. Because of, in this case, the m 2 will be the mass of the Moon. So, m 2 is nothing but m Moon and which is equal to 7.34 into 10 to the power 22 kgs and m 1 is m earth which is 5.974 into 10 to the power 24 kg and distance between the Earth and the Moon. So, r 1 2 this is around 3.844 into 10 to the power 5 kilometers. So, using this you can do the same kind of computation and you can get the results.

So, last time we have shown it on the figure also how the L 1, L 2 points are distributed and L 3 and L 4 they lie on the circle on which the Moon moves. So, now going into the next topic now let us go to the Jacobi Integral.

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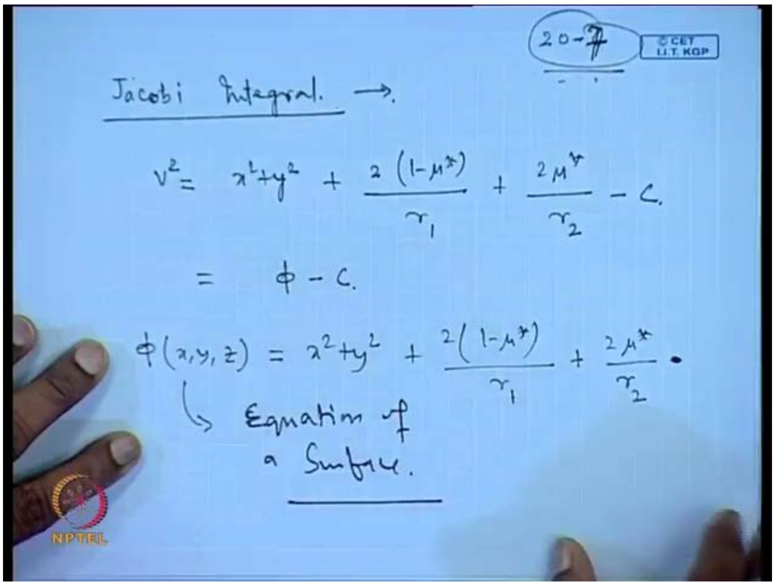

So, we have already described the four points L 4, L 5, L 1, L 2 and L 3. So, how to find out the lagrangian points that is over. Now we get back to the Jacobi Integral, which we obtain for the relative motion by integrating the equation. So, Jacobi Integral basically it indicates the energy in some relative sense not in a absolute sense. So, this integral was written as v square equal to x square plus y square plus 2 times 1 minus mu star plus 2 mu star divided by r 2 minus C. We wrote it in this way, where phi x y z, now this describes is the equation of a surface. So, basically this equation describes the equation describes the surface.

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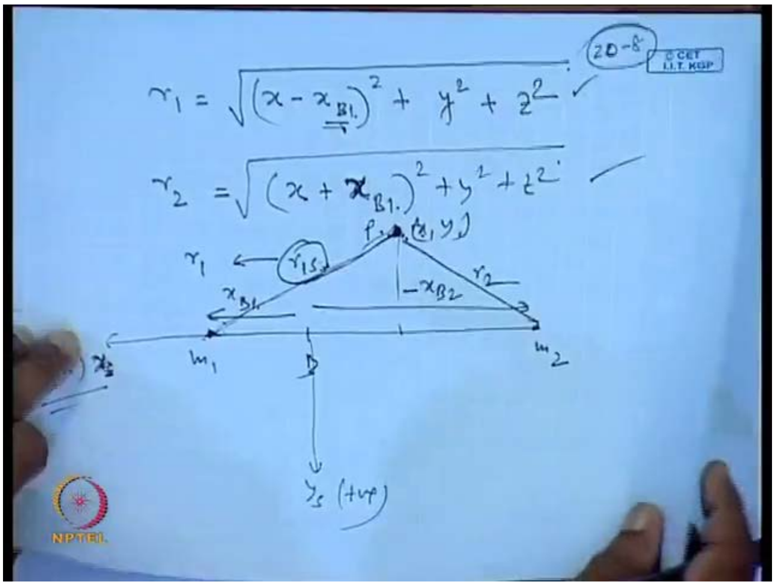

Now, if you look into r 1. So, r 1 is nothing but x minus x b 1 which is the distance of the mass one from the barycenter. So, x b 1 square plus y square plus z square under root and similarly r 2 is written as x plus x b 1 x b 1 or r b 1, whatever the notation we have used. So, y square plus z square. So, here is the mass m 1 and here is the mass m 2 and this was the barycenter. So, this distance we wrote as x b 1 and this distance as x b 2 and if we put the proper sign here and in this direction, our the synodic positive, synodic reference frame is there. And here the y s is positive.

So, therefore, the any point the x, y we choose. So, the distance radius vector from here to the radius from here, to the point p which is at which the mass m 3 is lying. So, this we wrote as r 1 s and later on, we dropped the notation s and just we wrote it as r 1. Similarly from here to here this is r 2. So, this r 1 and r 2 are described by these two equations.

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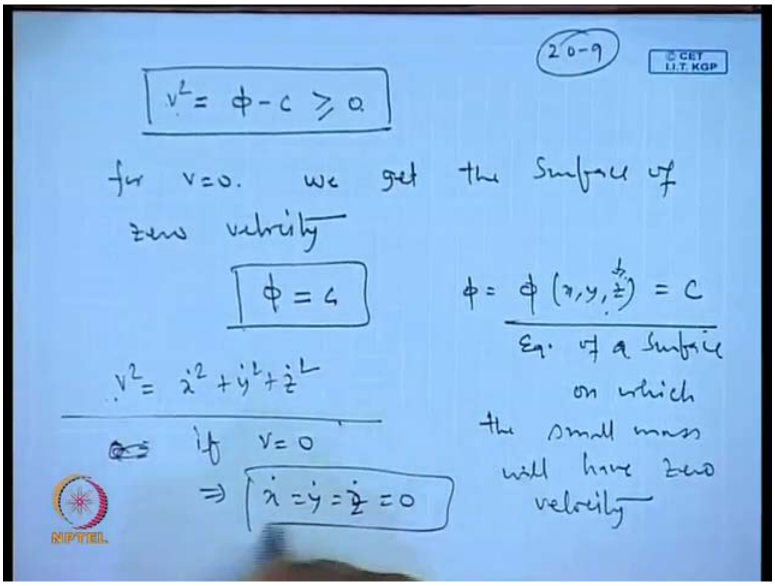

Now we know that v square, previously we have written v square is equal to phi minus c and v square is a positive quantity. So, it cannot be negative. So, it is inequality must be satisfied. So, for v equal to 0 we get the surface of zero velocity and this surface is described by phi equal to c. So, the phi equal to phi x y z equal to c. So, z is coming in picture, because of this r 1 and r 2 the X square y square, there is such in this here whatever we have written, so z is coming from this place and if you look into this equation. So, here there is no existence of z here in this place.

So, the x square y square, if we write this as the distance r. So, r square becomes X square plus y square. So, x and y are the distances being measured from the barycenter b. So, this gives you the equation of a surface equation of a surface, on which the small mass will have zero velocity. Because the v square we are writing as x dot square y dot square plus z dot square, if you refer to this equation Jacobi Integral that we developed earlier. So, if v equal to 0. So, this simply implies if v equal to 0. So, this implies x dot equal to y dot equal to z dot equal to 0. Now, this equation that we have written, let us write this equation as equation number 1, phi x y z is equal to x square plus y square and plus this quantity.

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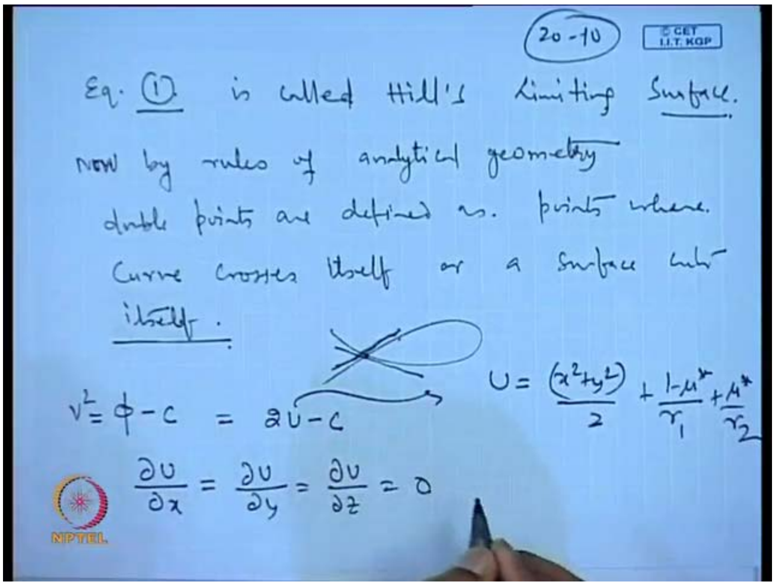

So, this equation 1 is called Hills Limiting Surface. Now by rules of analytical geometry, double points are defined as points, where curve crosses itself or a surface cuts itself. So, double point will exist here in this place. So, you will have two tangents here, one like this and another like this. So, double points.

So, phi we have written v square, we have written as phi minus C. And this can also we written as 2 U minus C, where u is the quantity x square plus y square divided by 2 plus 1 minus mu star by r 1 plus mu star by r 2. So, this double points will be given by dou u by dou x equal to dou u by dou y equal to dou u by dou z equal to 0. So, this is a standard result from analytical geometry.

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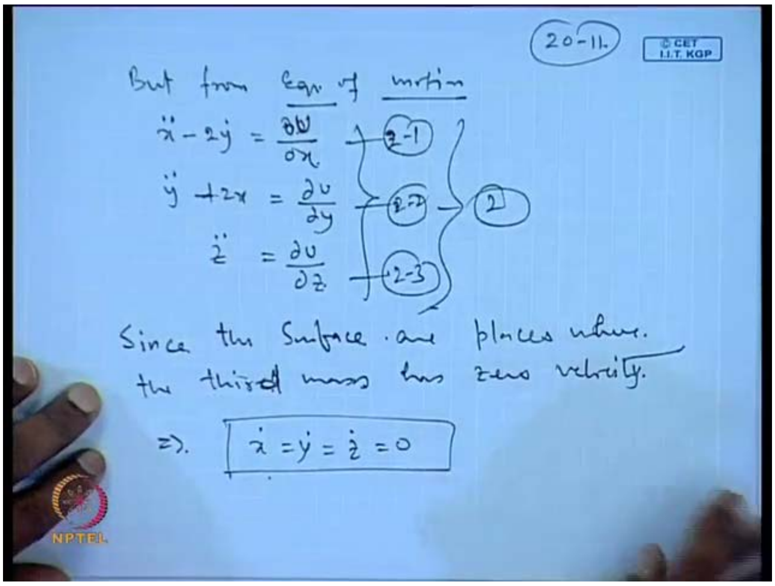

But from the equation of the motion, that we developed earlier in the synodic reference frame, we know that x double dot minus 2 y dot is equal to dou u by dou x, y double dot plus 2 x, this is equal to dou u by dou y and z double dot is equal to dou u by dou z. So, this we can write as equation number 2-1, 2-2 and this as 2-3.

Since the surface that we are defining, so the surface are places, where the third mass has zero velocity. So, this implies x dot equal to y dot equal to z dot equal to 0, that we have already stated. So, from this equation, if we put already a double points we see that dou u by dou x dou u by dou y dou u equal to dou z equal to 0 and from here we get that, this is 2 x dot. So, and from here, if we insert the value for y dot and x dot equal to 0. So, if we are inserting this. So, similarly we get here x double dot y double dot is equal to z double dot equal to 0. So, this implies, this is from equation 2.

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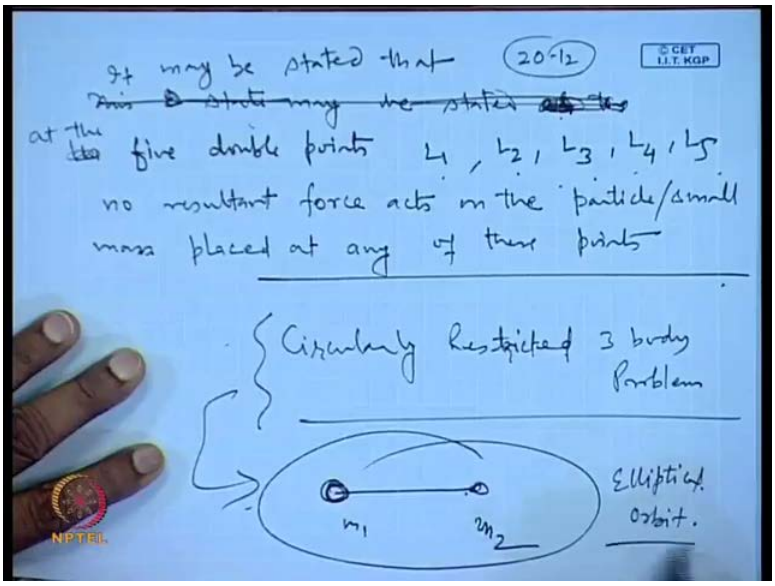

So, this statement may be stated as the 5 double points L 1, L 2, L 3, L 4, L 5 we stated as we have stated here we write, it may be stated it may be stated that, at the 5 double points at the 5 double points L 1, L 2, L 3, L 4 and L 5 no resultant force acts on the particle or the small mass. So, this conclusion from where, we have got first, we got the zero velocity surface and thereby we put here, x dot y dot equal to 0 and at the double points we put the quantities, where the occurs in subset or touch each other. So, at that point, we kept dou u by dou x dou u by dou y and dou u by dou z equal to 0.

So, this quantity becomes zero and here all this the first derivative are becoming zero. Therefore, this is implying that there is no resultant force acting on this and while deriving the quintics what we did. So, at that time we assumed that, in this equation we assume that there is so far deriving the quintics means, for finding out the equilibrium points. So, at the equilibrium points, we know that there will not be any acceleration. So, we started with that assumption, that there is no acceleration. So, we put x double dot y double dot z double dot equal to 0 and also we put x dot y dot z dot equal to 0.

So, now, we can see that the way we get from the Jacobi Integral or from this equation and plotting in terms of the Jacobi Integral, because we are taking this Jacobi Integral here. And this way are writing in terms of u where, if u phi equal to 2 u. So, and dou u by dou x dou u by dou y dou u by dou z equal to 0. So, from this integral we are getting back to getting the results that there are points where there are no acceleration. We do not have to assume that the points where the acceleration will be zero. So, this is a very good result and both way it confirms that even, if we proceed from the Jacobi Integral, we get the same result or either we start from the equation of motion assuming that, if quantity x double dot and y dot equal to zero. So, we still we get the same kind of result.

So, if therefore, now it becomes very easy to discuss about the lagranges points and how to develop them now, what we have got here the result, one very important thing to state, this is for the circularly restricted. All this results are for circularly restricted 3-Body Problem, if the two primary masses m 1 and m 2 they are moving in an elliptical orbit, then no Jacobi Integral exists. This you should remember.

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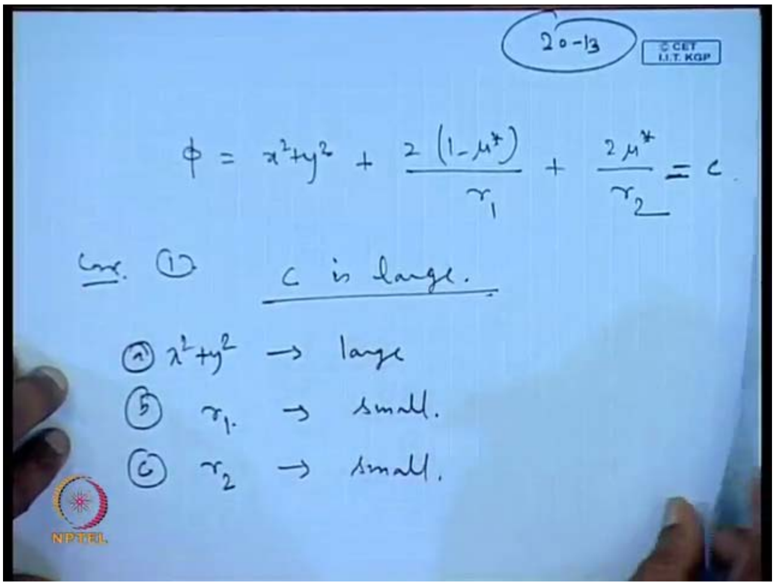

Now we can discuss about the Jacobi Integral and expand it. So, we have phi equal to x square plus y square plus 2 times 1 minus mu star. So, we can discuss the case 1 and this equal to quantity C on the zero velocity surface. So case 1, C is large. So, this can happen by x square plus y square, this becomes large or r 1 becomes small, r 2 becomes a small.

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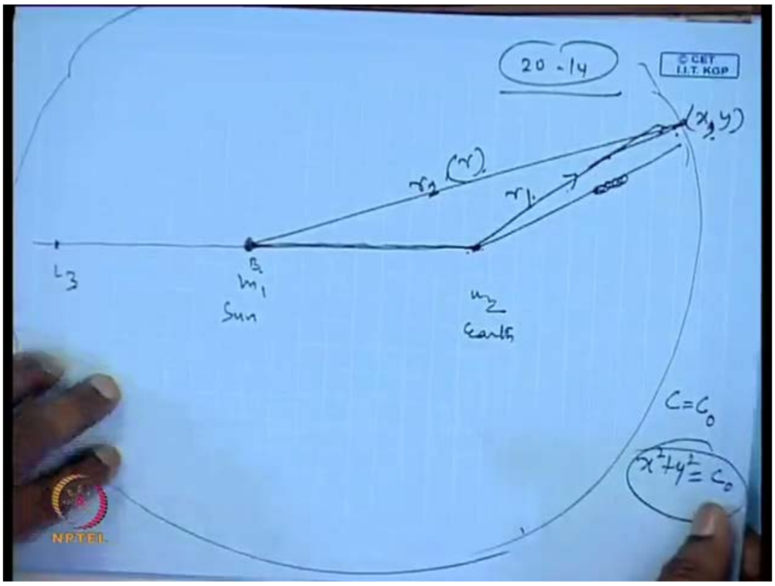

In the case of Sun and Earth on the normalized scale, we know this distance is unity and the barycenter will lie inside the Sun itself. So, we can assume that barycenter is lying here itself, and then look into the Jacobi Integral. So, if x square plus if C is large. So, that can happen say the L 1 point it is almost L 3 point, is lying here at the same distance. This distance and this distance is equal. So, we can make a very big circle here in this place, like this and if we take any point on this circle, whose coordinate is x and y.

So, you can you see that, this is r 1 and this r 2 and barycenter is also lying here. So, r is also lying here, in this place itself. So, r 1 and r 2 they become large here, if this distance is unity this distance is unity. So, we are taking a circle of radius greater than unity. So, the quantity is quite large. So, if the C is large, then this quantity is nearly equal to 1 and this quantity is almost zero. So, this quantity anyhow, this is very a small and r 1 is large. So, this quantity also becomes a small. So, mainly the contribution comes x square plus y square, this becomes equal to C. So, this describes the equation of a circle. So, C equal to we can write C 0 and this becomes x square plus y square is equal to C 0.

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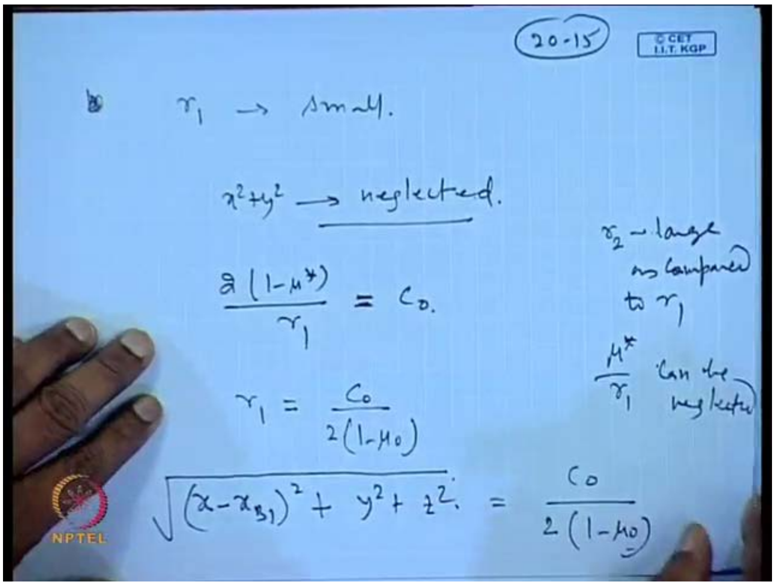

Next we can take the b case, where the r 1 becomes a small. So, if r 1 is becoming a small means, we are in the never rude of the point m 1. So, we can say that now, if we are never rude of this point. So, now, this becomes your r 1 and this becomes your r 2. Here this is your r 2, this is not r 1 and this is r 1, we have taken. So, do the correction here. So, this is your r 2 and this quantity is your r 1. So, you can see that r 2 is large quite large as compared to r 1. And x square y square, this is almost equal to r 1 and this distance is unity. So, therefore, this implies that r 1 will be much less than the unity value.

So, if r 1 is much less than the unity value means, x square plus y square, this we can neglect. Because here, now this distance is also x square plus y square under root. So, this can be neglected and therefore, what we get this, 2 times 1 minus mu star divided by r 1 and also r 2 is large now it is around the value of, say one or little less than one. So, r 2 being large as compared to r 1. And therefore, mu star by r 1, this can be neglected. So, if you neglect this, so this quantity becomes equal to C 0.

And we can write this r 1 equal to C 0 by 2 times 1 minus mu 1 and r 1, you know what it is. This is x minus x b 1 square plus y square plus z square under root is equal to C 0 by 2 times 1 minus mu 0 square and if you are working in the x y plane so; obviously, we will set z equal to 0.

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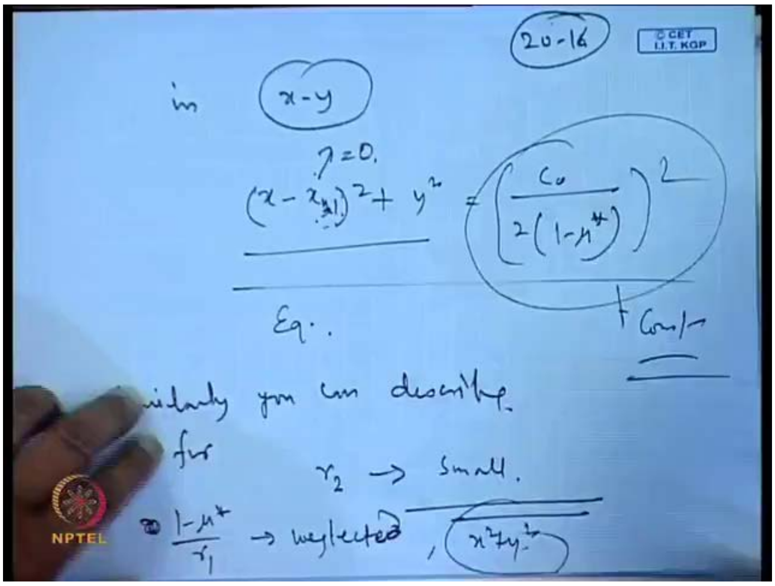

So, in x-y plane, if we are looking for the surface, where it cuts the x-y plane, so obviously, there the z will be equal to zero. So, what you get is x minus x b 1. So, x b 1 also, in this case becomes equal to 0. Because the barycenter almost it is coinciding with the primary mass. So, this quantity square plus y square, this becomes C 0 by 2 times 1 minus mu star whole square. So, this again the right hand side, this is a constant. Therefore, this becomes a equation of a circle, so for a small values of, near the small value, small radius. So, that is around the point b, you will find that this C equal to C 0. This will be almost, this will be describing a circle and for also very large values of r 1, it will be almost a circle, what for intermediate value, it will turn out to be an oblique.

So, similarly you can describe for C case, where r 2 becomes small, r 2 is small. So, if r 2 is a small, then in the same way r 1 and so, 1 minus mu star by r 1. This can be neglected. And also x square plus y square, now we are working in this range . So, now x square plus y square, is the distance from this point to this point. So, here say from this point to this point, this is your r 1 and also this is the distance x square plus y square . So, here x square plus y square, this will be around the unity value or less than that. Because this is a quantity, if you go on the right hand side, this will exceed the value of one.

So, if the square value will be around the one value, but the quantity once you take the mu star by r 1 and r 1being mu star by r 2 and r 2 being a small. So, this is your radius in this case r 2. So, if r 2 is a small, then you will see that this quantity will be quite large than unity. So, the other quantities can be neglected. And we work only with mu star r 2. So, if you work with mu star r 2.

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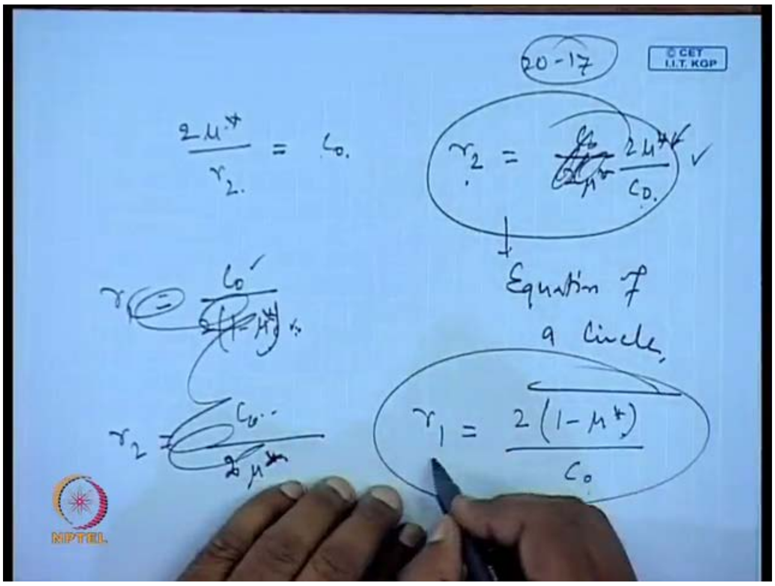

So, mu star by r 2 this will be equal to C 0. 2 mu star times r 2 will be equal to C 0 and this implies r 2 equal to C 0 by 2 mu star and again this is the equation of a circle, more ever one point should be noted that, this circle will be of larger radius and this circle will be of a smaller radius. Reason is very obvious. Because in the two cases, that you are comparing r 1 is equal to C 0 by 2 times 1 minus mu star and r 2 equal to C 0 by 2 mu star. So, C 0 in both the cases it is same. This is large, this is the equation that we have taken, we have written r 1 is equal to 2 times mu star by C 0. So, we have written as the reverse thing. This r 1 is equal to 2 times. Here we have done the mistake, r 1 equal to 2 times 1 minus mu star divided by C 0. So, this quantity becomes 2 times 1 minus mu star divided by C 0. So, here also we need to give the correction.

So, this is 2 times 1 minus mu star divided by C 0. So, here 2 times mu star by r 2 becomes equal to C 2. And therefore, r 2 is equal to C 0 by 2 times 1 minus mu star r 2 is equal to 2 times mu star by C 0. And r 1 we have written as 2 time 1 minus mu star by C 0. Now compare this two equations. Here C 0 is both, in same in the both this cases, but mu star is quite as small as compared to 1 minus mu star. In the case of the Sun Earth System mu star is almost 10 to the power minus 6. It is almost zero. So, virtually r 2 will be very as small as compared to r 1. So, we have not shown on the scale, here in this figure. But this is circle of a larger radius, then the circle of this one will be very, very small.

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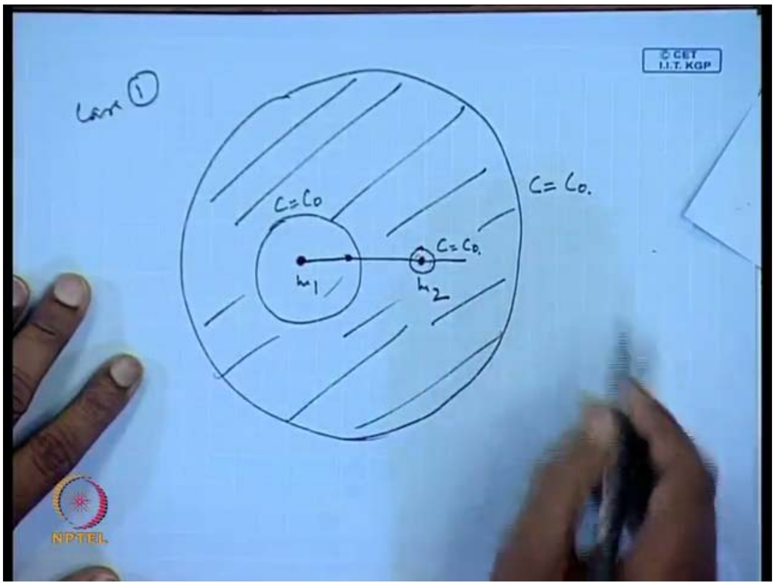

So, the case 1 can be now shown as, we have two masses here m 1 and m 2. And this is C equal to C 0. And here also you have C equal to C 0. And then there is a circle of a small radius C equal to C 0 and you will see that inside this, the b square becomes negative. So, this region is not permitted. So, either the particle can exist here, in this space either in this space or either outside this.

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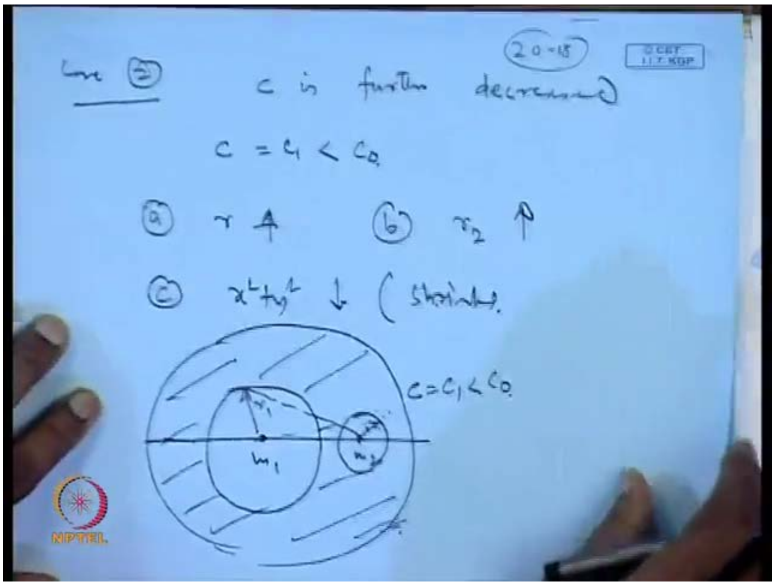

So if we decrease further case 2, C is further decreased. So, C equal to, let us say C 1 is less than C 0. So, this can happen by r 1 going up, b r 2 also by going up and c x square plus y square going down. So, that is this shrinks. So, now, we can depict this goes up, this goes up, m 1 is here; m 2 is here. And this is C equal to C 1, is less than C 0 and this is your r 1. And from here also, you can show this as, r 2 or either, if the particle is inside this. So, this is the boundary value r 1 and r 2 can be shown r 1, will start from here and r 2 will be this value. So, again this region is the prohibited region.

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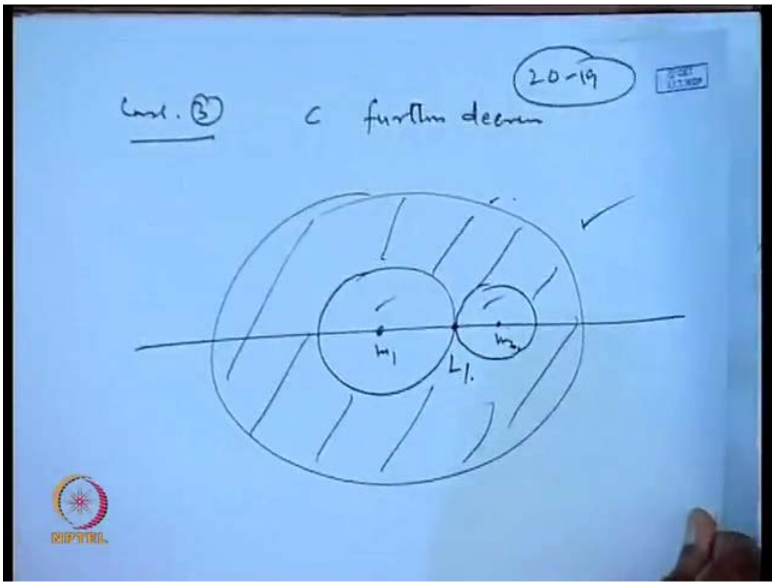

Case 3 C further decreased, if you decrease C, further then there will be a stage, when this two circles will touch each other. So, this is the double points interaction, are developing and this is your point L 1, the first lagranges point has developed now. And again this remains the prohibited region. So, particle can be either inside this or either inside this or on this surface, so if this region opens up then particle can move from this space to this space, and so the permissible region is here here and here.

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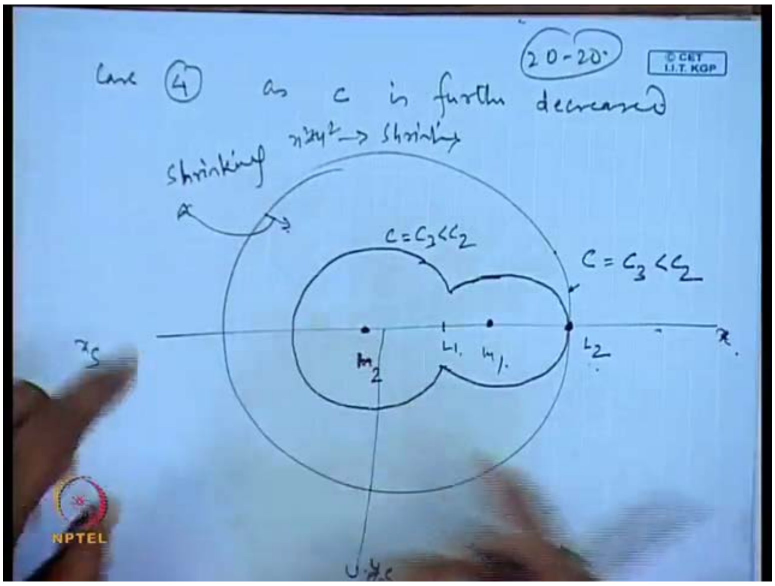

So, as C is further decreased, so C is equal to here. In this case also, we can write here, C is equal to C 2, which will be less than C 1. C equal to C 2 is less than C 1. Here also C equal to C 2 which is less than C 1. So, here C equal to C 3 which is less than C 2 and inside then, you have two curves now, this remember that we are getting by putting z equal to 0, the surface the zero velocity surface, it cuts the x-y plane. So, this is the x s we have plotted here and y s we plotted downward here.

So, this is whole thing we are showing it in the x-y plane. So, now, this 2 circles they merge together to produce this kind of result. So, L 1 point was here, now the point L 2 develops. So, here again you have C equal to C 3 which is less than C 2. So, now, this C is equal to C 3 curve, this meets with this curve and then; obviously, the double point L 2, then develops.

Similarly, in the next step this will become larger than this and this radius will shrink. So, this is shrinking this is shrinking as we have told that this will go down. So, x square plus y square, this is shrinking. So, as it shrinks. So, this and this expands they will come and merge together here in this place and this space this will open up here in this place in the next stage.

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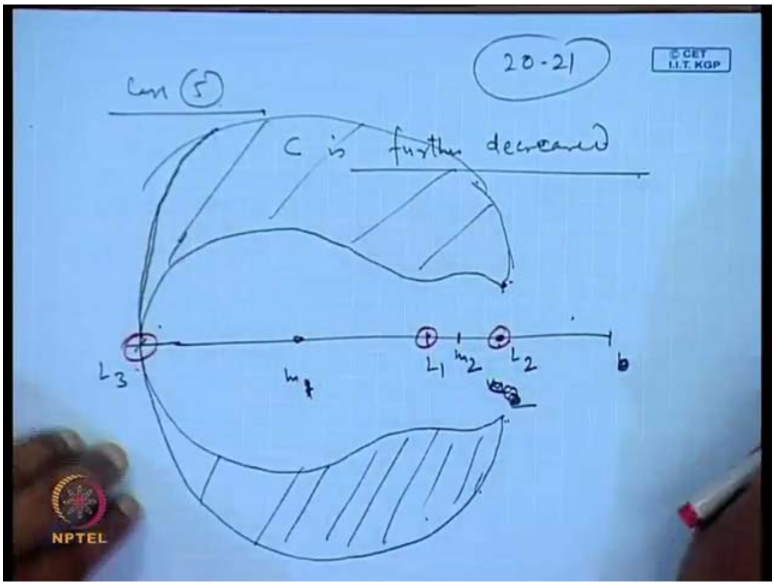

So, case 5 C is further decreased. So, if you decrease further, then we have m 1 and m 2 present here. So, the outer circle now it merges with this. This merges with this. So, this area remains prohibited and this is the lagranges point L 3, which develops here. Here is your L 1 and this side, here L 2 has already developed. So, L 2 is lying here in this place, mass is we can replace the mass inside little bit. So, mass is m 2 here and L 2 has already developed.

So, L 2 is opening. So, L 2 is here, in this place. This has opened up and this is the prohibited region. The shaded area is the prohibited region. So, the particle can move around this. It can, but it still, it cannot go outside because this region is not opened, but now it is possible that particle can go outside even. So this is the bonding region. Figures are not very good, but still it depicts the idea and here the L 1 point is there, here L 1 point, L 2 point is here and the L 1, L 3 point is the recent, now it has developed.

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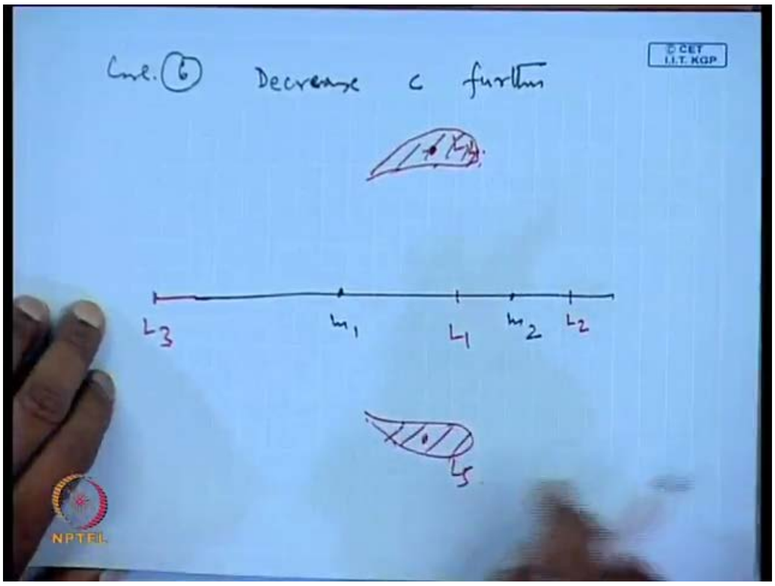

So finally, if you decrease the value of case 6, decrease C further and so, by decreasing C further, what you will see that m 1, m 2 and L 2 is here. L 1 is here and L 3 is lying here in this place. And L 4 and L 5, they are lying somewhere here, which is still they have not developed. So, what happens, this region it shrinks. And it shrinks into this kind of pattern. So, this region still remains prohibited, where the L 4 and the L 5 points are lying inside.

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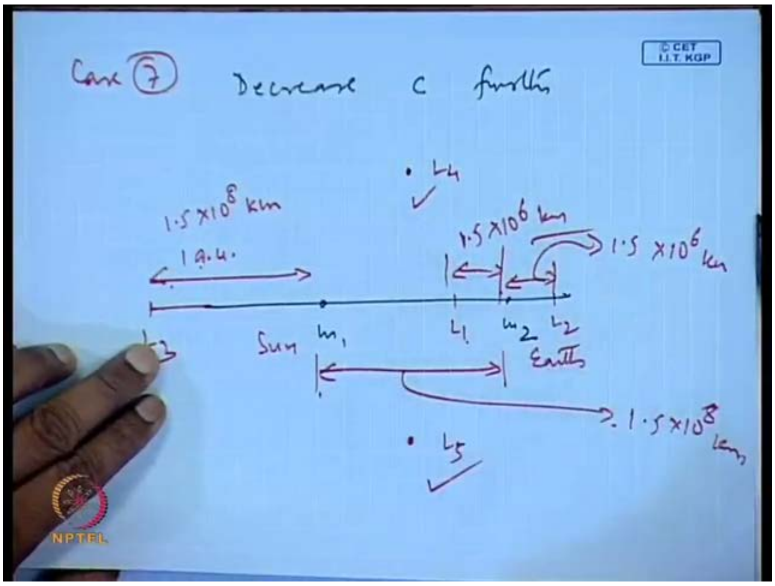

So, as you decrease the value of the C further, so case 7, decrease C further and then you will see that the m 1, m 2. This is the point L 2, L 1 and on this side we have L 3, so L 4 and L 5 that those develop. So, the shaded region now disappears into this point L 4 and L 5. So, the lagranges points L 4 and lagranges point L 5 finally, developed. So, in the case of the Sun-Earth System, we have seen that, this quantity is one astronomical in it, which is the distance between the Sun and Earth 10 to the power 8 kilometer. This distance turns out to be 1.5 times 10 to the power 6 kilometers and this distance turns out to be 1.5 times 10 to the power 6 kilometers .

And for the Moon, earlier we have already written. And this distance from here to here this is nothing, but here 1.5 into 10 to the power 8 kilometers which is the distance between the Sun and the Earth. This is your Sun and this is Earth.

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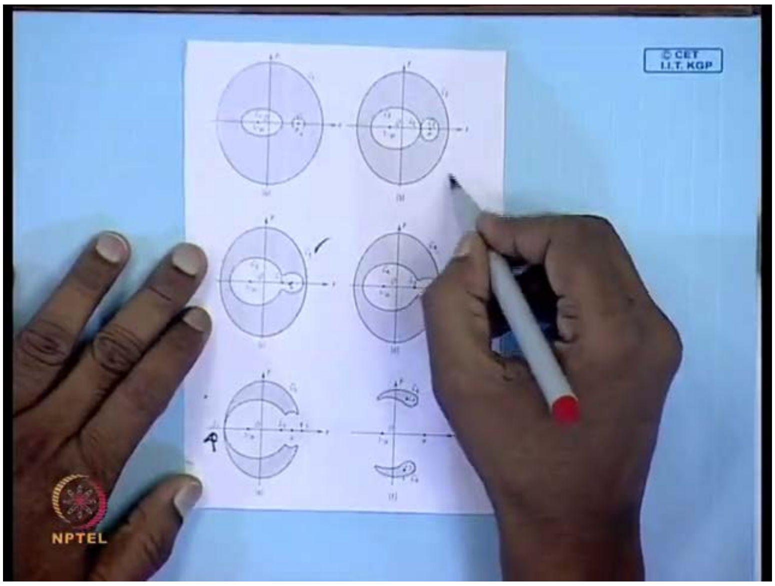

So, this concludes our discussion. But one final thing is remaining. So, whatever I have discussed now, so this is showing you on the x-y plane. So, this is the surface of zero velocity, where it cuts the x-y plane. So, it gives you this kind of picture. And the white portion is the permissible region. And the shaded portion is not the permissible region. So, surface of zero velocity,if you look for that in the x-z plane. Because x is coming. So, on the x plane itself, the L 1 and L 2 points are lying. So, the L 1, L 2 those are visibles here. You can see here, in this place. This is the point m 1. This is the point m 2 and therefore, this point is your L 1. This point is your L 2 and this point is showing here, L 3 point. And this is the contours in the x-z plane, so the surface where it cuts the x-z plane.

So, if you plot, it will look like this and it can be developed in the same way as we have done. And similarly in the y-z plane, you can see this. Because x is not lying here and the lagranges points, they lie in the x. On the x line and therefore, you see here, there is as such no lagranges points like present here. Because the lagranges points, are the double points, where the two surfaces they touch each other. But here, there is no touching point available. And therefore, no lagranges point in this case. So, this is in the y-z plane.

So, thank you very much. We conclude our discussion here and we continue next time with the trajectory transfer. Thank you.