Space Flight Mechanics

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Lecture No. 19
Three Body Problem (Contd.)

In the last lecture, we have been discussing about the restricted three body problem. So, in this lecture, we are going to conclude this.

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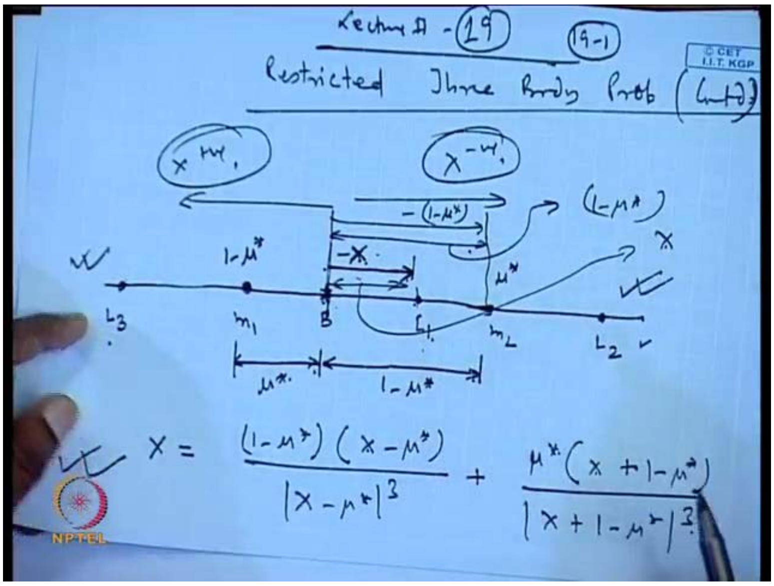

So, we had started with two primary masses, which were the m 1 and m 2. In the normalized situation, we wrote this as, one, this mass as 1 minus mu star and this mass as mu star. The B, the barycentre was shown as B. So, this distance from here to here, this is nothing, but mu star and distance from here to here, this is 1 minus mu star, on the normalized scale. And, we took the Lagrangian points on this side as L 3, and this side as L 2, and here, we took as L 1. So, these are the three Lagrangian points L 1, L 2 and L 3, which are lying in a line; and, we were trying to derive the equation for the X, which is the distance of the Lagrangian point from the barycentre, from the barycentre, at which also, the located is the origin of the synodic reference frame. So, this is basically, in the synodic reference frame, we are looking for this L 1 distance. So, in the synodic reference frame, this distance is X and also, this coincides with the barycentre,so that, this, this distance remains same. So, now, given this distance…

So, we wrote for the Lagrangian point L 3 one equation, for L 2 we wrote another equation, and for L 1 also, we wrote on one equation. So, we took an alternative route to describe all this three equations. And besides, we were progressing with the, our earlier model, in which we wrote X is equal to 1 minus mu star times X minus mu star divided by…So, we had started with this equation and then, by putting X is equal to minus X, we solve for the point L 2 and by taking this as it is, we wrote the equation for L 3. So, we wrote the equation for L 3 by taking X is equal to positive; for L 2, as X is equal to negative.Now, we have to work for the L 3 point, L 1 point. So, for the L 1 point, we have, we need to write this as, obviously,for the L 1 point, the distance here, what we are doing on this side, because X is on this side, positive, and on this side X is negative. So, for this three Lagrangian points, we are writing three separate equations, such that, once you solve those equations,so, the positive value of X will give the location of L 1, L 2 and L 3. This is, this was our objective, ok. So, proceeding this way…So, on this side,X is negative. So, we are going to put here the minus sign, here in this place, and similarly, the coordinate of this one is exactly 1 minus mu star, with a negative sign. What this distance, we can write as and distance from here to here, this distance can be written as,1 minus mu star; similarly, this distance from here to here, this can, this is nothing, but X.

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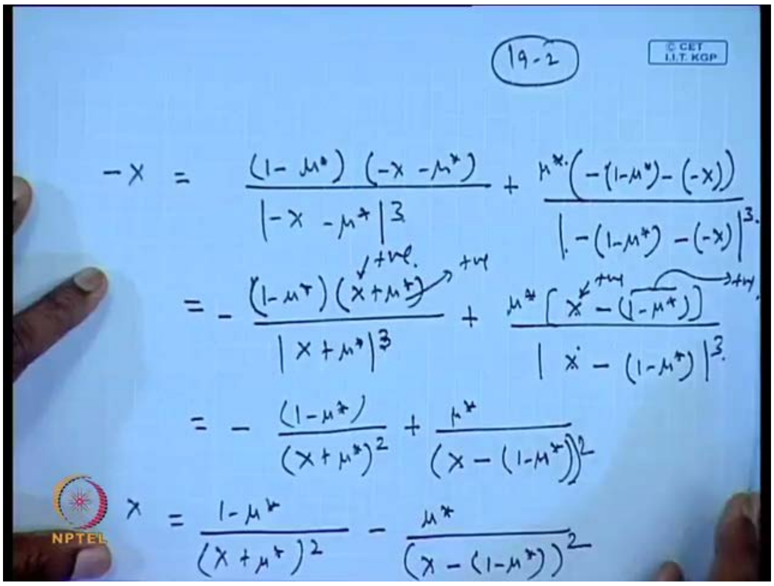

So, to work out for the L 1 point, what we need to do, we put X is equal to minus X, and then, we write, in the same way, 1 minus mu star minus X minus mu star, divided by minus X minus mu star whole cube, plus mu star times…Now, we subtract from, the distance from here to here, this is given to be 1 minus mu star. So, I am taking this with a negative sign; we write it in this way. Now, look into the whole thing, how it, this gets reduced to 1 minus mu star and this minus sign will come outside;X plus mu star; if you look into this quantity, this X is positive here; this is positive; this is positive; mu star is positive;1 minus mu star, this quantity is also positive; therefore, we can cancel the numerator and the denominator, and we can write this as,X plus mu star whole square plus mu star divided by X minus…This facilates our derivation of the equation and if you remember what we did, we cancel the negative sign on both the sides. So, ultimately, the equation gets reduced to 1 minus mu star divided by X plus mu star whole square minus mu star divided by X minus 1 minus mu star whole square. And, you can compare this with the equation that we derived last time. So, let us look into that equation.

This is the equation that we worked out for the L 1. X minus 1 minus mu star divided by X plus mu star. If you just look into this, both the equations are exactly same; but this equation, last time, we derived using an alternative route, where we took into consideration, the forces acting on the two masses, forces acting on the mass m, which is placed at Lagrangian point L 1, by mass m 1 and mass m 2; and besides, because it is a moving in this way, or either this way,so, the necessary centripetal acceleration is provided by the subtraction of the two forces. This is pulling it in this direction; another one is pulling into this direction. So, we subtracted this, and this is the, the resultant gravitational force provides this, the necessary centripetal force to move in a circle. So, using this condition, we worked out last time, this equation, and this equation is the same as we have written today. So, this is what we got, the last time. So, both the alternative routes you can take, and you can work it out; you can, there is no difference between this two.

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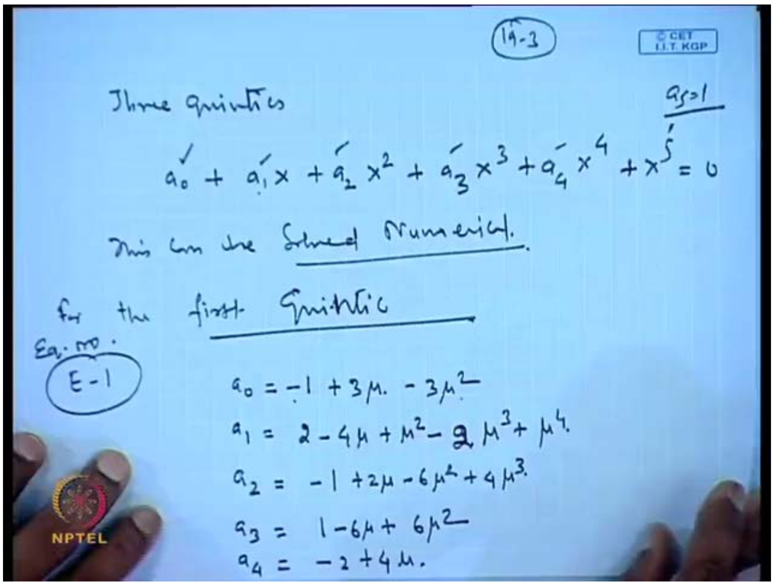

So, ultimately, the last time, what resulted that, we got three quintics, three quintics which were of the form a 0 plus a 1 X plus a 2 X square plus a 3 X cube, and solution of this quintics will give the three Lagrangian points. Now, to get this three Lagrangian points, obviously, it is not easy to work it out. So, this has to be done numerically. So, this can be solved, this can be solved numerically. So, we do not do the numerical exercise here. So, to work out this, the three equations that we developed… So, this was the third equation, the last equation that we wrote as the equation number g 1, or this can be multiplied, the whole thing, denominator can be brought on the left hand side and then, this gets multiplied by, this term gets multiplied by this term and this term gets multiplied by this term. So, we wrote in a, in a polynomial format. So, after writing in a polynomial format, we have got this quintic equation for all the three axioms.

So, for our, for the first quintic, for the first quintic which we wrote as the equation number E1, the solution can be written as, sorry, this coefficients a 0, a 1, a 2, a 3 and a 4, and a 5 here, is nothing, but equal to 1. So, a 0 becomes minus 1 plus 3 mu. Of course, while you are writing this equation in the polynomial format, and after getting the polynomial format, you are getting the X terms with the same degree together and therefore, this coefficients will appear here, in this place, and it has to be done with great patience. So, if you do it, then, you get this coefficients in this format. So, for the first quintic, we get a 0 is equal to minus 1 plus 3 mu minus 3 mu square;a 0 minus 1 plus 3 mu minus 3 mu square; then, we have a 1 is equal to 2 minus 4 mu plus mu square minus

2; minus 1plus2 mu minus 6 mu square plus 4 mu cube, and a 4 is equal to minus 2 plus 4 mu. So, this corresponds to…See, in the last lecture, we have this corresponds to this quintic;X times X minus mu square times X plus 1 minus mu square and the, and this equal to this quantity. So, this was our quintic number E 1.

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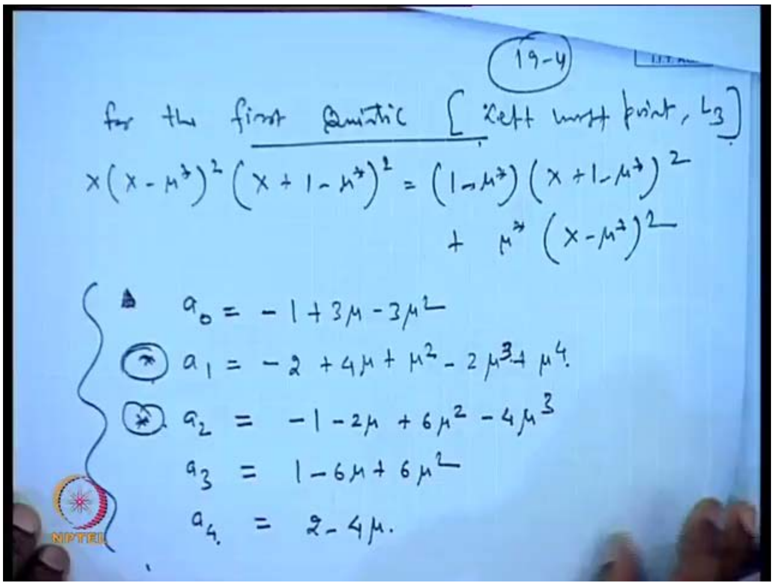

So, for the next quintic which is E 2, for the second quintic, the second quintic, we have written as F1; for the second quintic, which is F 1,so, X is equal to X times X plus mu star square times X minus 1 minus mu star…whole square plus mu star whole square. So, this was our second quintic and for the second quintic, we can write…Little modification is there; so, going back to our earlier thing, this stands for F 1; this is thesecond quintic,in fact; this is second quintic. So, for the second quintic, this is for the right most point, right most point which is L 2, and this one we are writing for E 1here. So, this is the left most point, which in our case, we have written as L 3. So, we need to do the modification here, in this place. So, we replace this byanother term.

So, for the first quintic, now, this is our first quintic. So, for the first quintic,X times X minus mu star square times X plus 1 minus mu star square is equal to 1 minus…This is the first quintic, theleft most point which is L 3. So, for the Lagrangian point L 3 and for this quintic, the solution that we get, we can write as, a 0 remains same. So, a 0, as we have got for the second quintic,so, a 0 will be minus 1 plus 3 mu minus 3 mu square; and a 1 will be, a 1 will be changed minus 2 plus 4 mu mu square minus 2 mu cube plus mu to the power 4; a 2 is also changed. So, we have a 2 is…a 1 is changed, a 2 is changed; a 3 remains same, as for the second quintic; and again, a 4 will change; this is 2 minus 4 mu.

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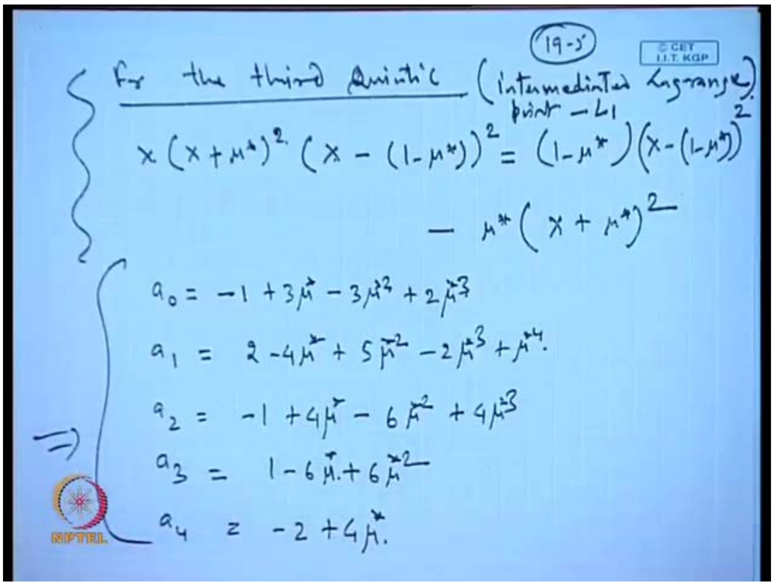

Now, for the third quintic. So, the third quintic, we have written as X times X minus mu star; this is, third quintic is, this is plus; X times X plus mu square times X minus 1minus mu star whole square; this is equal to 1 minus mu star times…minus mu star times…whole square. So, this is our third quintic; this is the intermediate, intermediate Lagrangian point,Lagrange point, which we have written as L 1.So, for L 1, the solution that we get, this is a 0 equal to minus 1 plus 3 mu minus 3 mu square plus 2 mu cube; a 1 equal to 2 minus 4 mu plus 5 mu square minus 2 mu cube plus mu to the power 4; 4 mu cube, and then…Now, in this, coefficients which are written in terms of this mu star, all of them are, this is mu star. So, all of them are written in terms of mu star. So, the earlier equations also, we have written, they are all in terms of mu star. So, all these are mu stars; here too, these are mu stars; these are mu stars.

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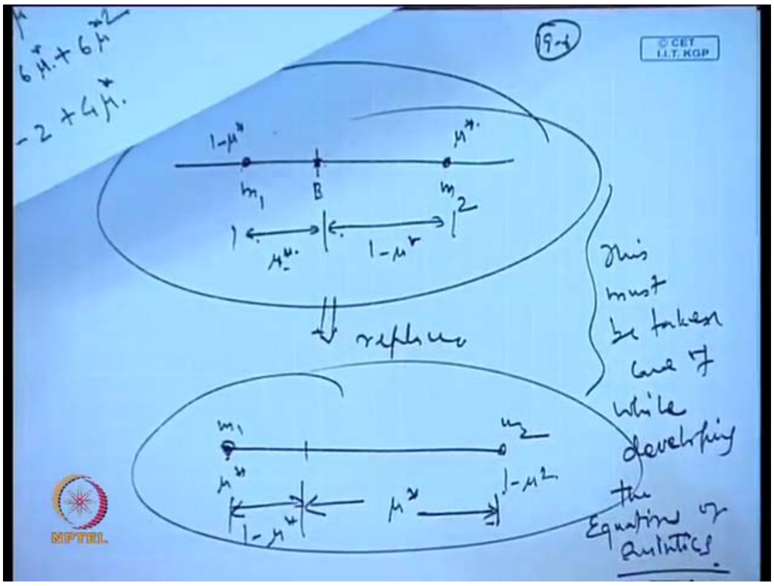

So, one caution must be taken care of. So, if what we have written here, this is our point m 1, and here, the mass m 1 is here, and other mass m 2 is there; and the mass of this, we have written as 1 minus mu star;and the mass of this we wrote as, mu star; and from the origin of the synodic, or the barycentric reference frame, the distance we wrote as, this as mu star and from here to here, the distance we wrote as 1 minus mu star.So, instead of doing this, if you replace the same thing as m 1, m 2 and here, the mass then, to the normalized scale, you write this as mu star and this one you write as 1 minus mu star; so, obviously, this distance, then will become 1 minus mu star and the distance from here to here, this will become mu star. So, if you write, replace it with this. So, the coefficients of this quintics, that we are writing here, so, the sign of them will change. So, this must be taken care of. So, if you do the mistake here, then, so, obviously, it will not match with this.

So, this must be taken care of while developing the equations of quintics. So, a mistake in this, it will totally give you the wrong values of the Lagrangian points. So, once we have done this, now, this quintics can be solved only by some numerical procedure. So, we avoid that, because this is the last lecture on the restricted three body problem. And,the scope of this course is limited, in this respect. So, we can take some example and we can show the, how the L 4 an L 5atleast can be calculated.

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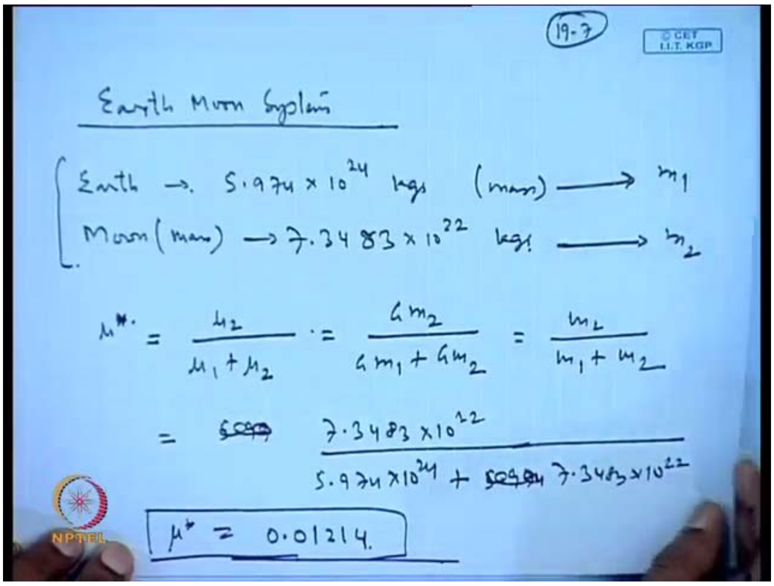

So, say the, we have the earth moon system. For the earth moon system, the earth has a mass of 5.974 into 10 to the power 24 kgs and this is the mass of earth; mass of moon, this is 7.3483…So, mu star can written as, mu star is nothing, but mu 2 by mu 1 plus mu 2; means, we have written this mu 2 is equal to g times m 2 divided by g times m 1 plus g times m 2. So, m 2 here is, stands for moon, and m 1 stands for earth. So, g cancels out and this simply becomes m 2 by m 1 plus m 2; and if you insert this values. So, 5.97, this is 7.3483 into 10 to the power 22 divided by 7.3483 into 10 to the power 22.

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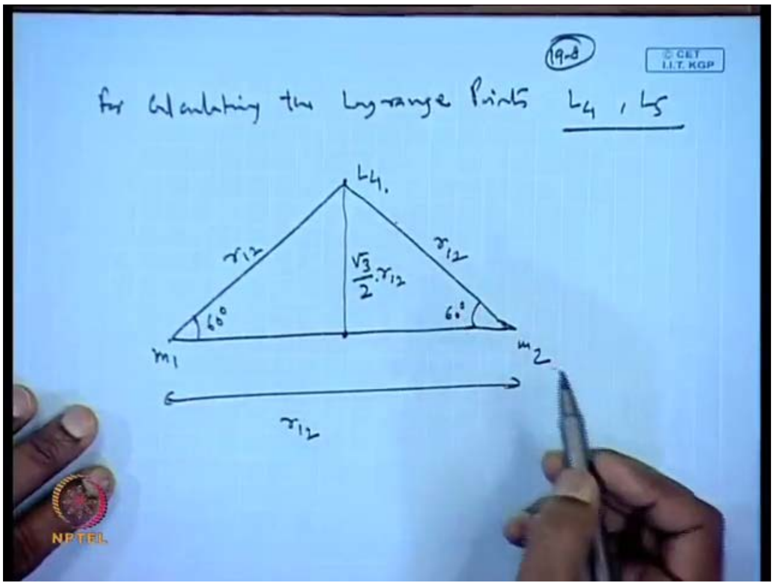

And, this value will approximately turn out be 0.01214. So, this is mu star. Now, for calculating Lagrange points L 4 and L 5, this is mass m 1 here, this is mass m 2 and this is the point L 4, say. So, we know this is root 3 by 2 on the normalized scale, and on the non-normalized scale,so, this is the distance r 1 2;so, this becomes r 1 2. And, this distance, from here to here, this is nothing, but r 1 2; this is also nothing, but r 1 2. These angles are 60 degree. So, it is a very easy to work out, where this L 4 will depend, if the distance r 1 2 is known to us.

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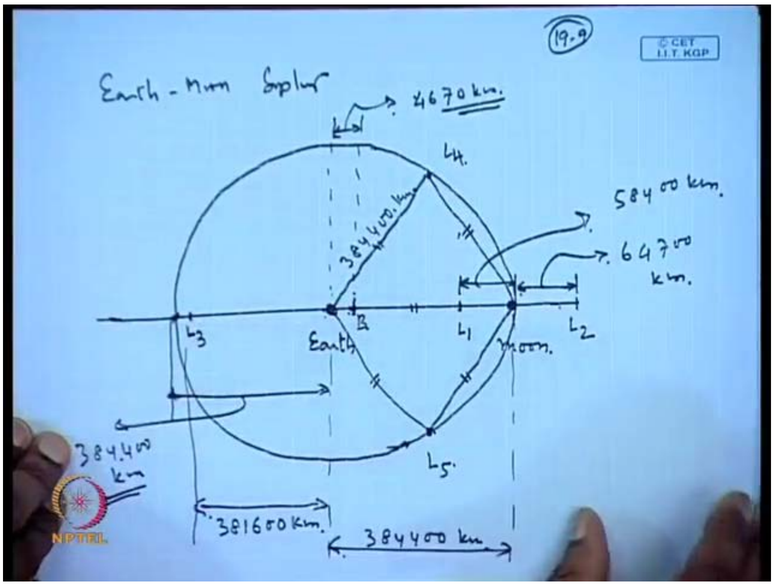

Now, if, if for the earth moon system, if we have earth here, in this place, suppose, the moon is present here, in this place…So, L 4 and L 5,they together make the equilateral triangle, with respect to this earth and moon. So, we have the equilateral triangle which is present like this. So, this is our L 4; this is L 5. And, if you remember, the barycentre is lying very close to the earth. So, if the barycentre is lying here,so, the whole thing, this is moving about this barycentre; and, if you look into the motion of the moon, with respect to the earth,so, with respect to the earth, moon will keep moving like this; say in the circular orbit, it is moving like this. So, if it is moving like this,so, simultaneously, the barycentre, which is ainitial point…So, the earth moon system, the earth is, and the moon, both are moving about this barycentre. So, the moon motion, it appears to be, with respect to earth, is lying on a nothing, but a circle.

So, as the moon moves on the circle,so, you will see that, the Lagrange points L 4 an L 5, they will also keep lying on this circle. Now, what about the point L 1? So, L 1 point, suppose, this lies here; and we have the L 2 point that we have written, which lies here on this right hand side, and L 3, L 3 point, it lies little, sort of this point. So, say, this lies here, in this place. This is too much magnified on this scale, to keep the clarity. So, the distance between the earth and the moon…So, we have this distance, this is 384400kilometers. So, this distance also then becomes 384400 kilometers; and this distance also, this will be the same. So, this distance are equal. This point will lie at a distance of 64700 kilometer and the point L 1…So, the, this distance from here to here, this is 58400 kilometer. And, the distance to the L 3 point from the earth, this is 381600 kilometer; while the distance between this and this point, from earth and this is nothing, but the radius of the orbit of the moon.

So, this is 384400 kilometers and the barycentre, this is lying here. So, it falls at a distance of, this is around 4670 kilometers. So, this gives you a nice picture of all the Lagrangian point, three Lagrangian points are,Lagrange points are lying, with respect to the earth and moon. So, as the moon is moving around the earth, with respect to the earth, actually both are moving about their, their barycentre; but from the earth, we can see that, the moon is moving around the earth. So, if you take this relative motion,so, the Lagrangian,Lagrange points L 4 and L 5, they will appear to move in, on this circle and L 3, L 3 lies little inside this circle.

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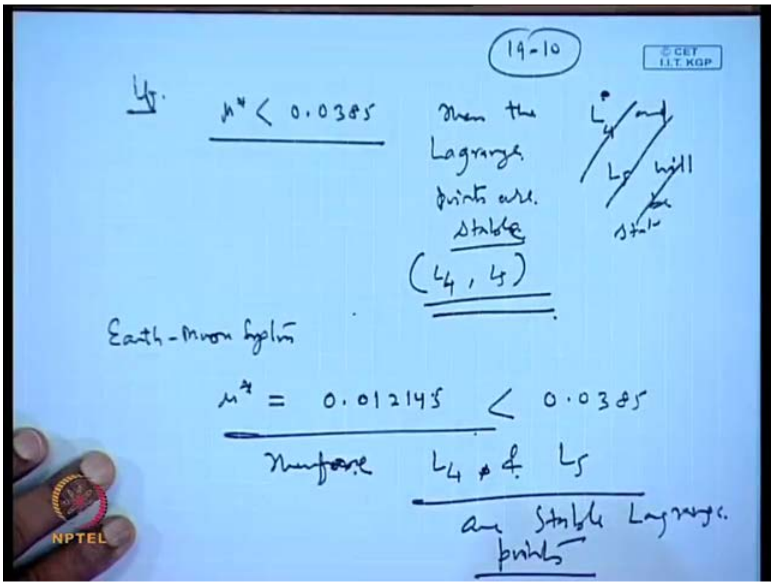

Now, the, as already we have discussed that, the L 4 and L 5, these are the stable points. So, proving the stability of this system, is again beyond the scope of this course, but rigorous analysis can show that, for the, once we are working for this Lagrange points... So, if mu star, this less than 0.0385, then, the L 4 and L 5 will be, or say, the Lagrange points will be stable, rather than writing L 4 and L 5; then, the Lagranges points are stable. So, in this case, only L 4 and L 5.

So, for the earth moon system, for the earth moon system, just now, we have calculated that, mu star is nothing, but 0.012145. So, this is less than 0.0385; and therefore, L 4 and L 5, they are stable Lagrange points. So, this implies that, if you put any object at L 4 and L 5,so, it will remain, it will appear to be a stationary, with respect to, to the earth and the moon. So, means, this will maintain a equilateral triangle; but in reality, what happens, because of the perturbation of the sun…So, really, this point needs a station keeping; means, you have to provide, from time to time some thrust; say, if, if your are putting some satellite,so, it will try to deviate from this point. So, you need to do the correction by giving appropriate amount of thrust. So, without thrust, you cannot keep the satellite in this two points; even for the earth moon system, because, the naturally, the perturbation of the sun is present. While the L 1, L 2 and L 3, they are the unstable points and obviously, if you, if you want to keep the object in this place, then, you definitely need the station keeping and besides, the sun perturbation is already there.

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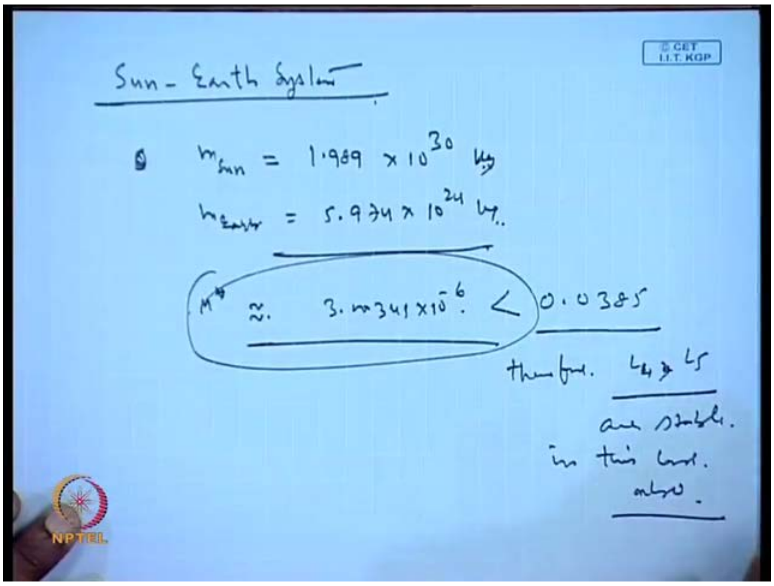

So, for the sun earth system, we have sun mass as m sun, we can write as 1.989 into 10 to the power 30 kg, and m earth, we already know that, this is5.974 into 10 to the power 24 kgs. Therefore, mu star in this case, this will be approximately 3.00…10 to the power minus 6; and obviously, this quantity is much less than 0.0385 and therefore, L 4 and L 5 are stable, in this case also. So, earlier NASA has sent some satellites and it has tried to utilize this Lagrangian points,Lagrange points for doing the astronomical observation.

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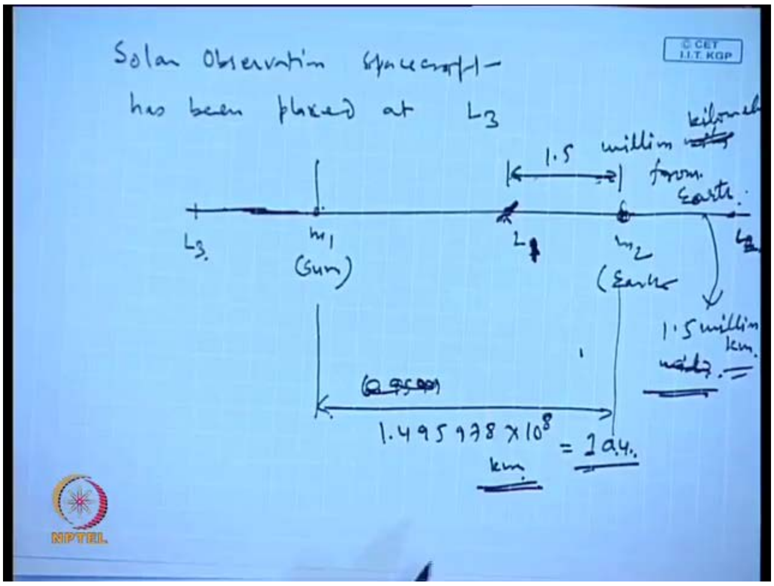

So, solar observation spacecraft has been placed at L 3. So, L 3 is present between these two masses, m 1, this is sun and here, this is m 2. So, this is earth. So, L 3 is present here, in this place. Now, L 3, we have taken, in this case, we have taken here, as L 1. So, to observe the sun,so, satellite has been put here, in this place, but obviously, this requires station keeping and this distance is around 1.5 million miles from earth. And similarly, L 2 also, it lies around at the same distance, little bit difference is there. So, if this also lies around at 1.5 million miles. This is million kilometers; this is kilometers; million kilometers, 1.5 million kilometers from this side and 1.5 million kilometers from this side, and L 3 is lying here. So, for calculating allthis distances, if we solve the, solve this equations numerically…So, all this distances can be found out. Now, in the case of the earth sun system, the distance between the earth and the sun… So, this distance from here to here, this is around 6.9599; this is the mean radius of the earth; this is 11.4595978 into 10 to the power 8 kilometers; and this is nothing, but 1 astronomical unit, the distance between earth and the sun, this is taken as 1 astronomical unit. So, if you take this distance, and you know this is 1.5 million miles, million kilometers,so, you can see, how large this distance is. So, it is almost near the earth, while this point is far more away.

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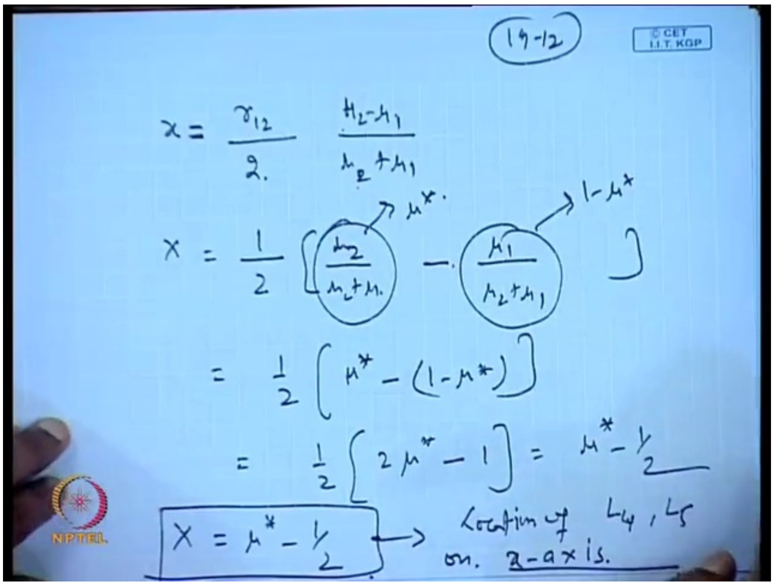

So, next, we can look little bit more into the equation for the X, for the X, for the L 4 and L 5, we wrote this as r 1 by 2 times mu 2 minus mu 1by mu 2 plus mu 1. And, on the normalized scale, the same thing will appear as, r 1 2 will be written as 1 and this is 2and the whole thing, then, you can mu 2 by mu 2 plus mu 1, you can divide it and this quantity is nothing, but your mu star. And then, you have mu 1 by mu 2 plus mu 1; this quantity is nothing, but 1 minus mu star. So, what you get here, 1 by 2mu star minus 1 minus mu star. So, this becomes 1 by 2 2 mu star minus 1. So, this is mu star minus 1 by 2. So, X is given by this value. So, this is the location of the, location of L 4, L 5 on x axis. And obviously, the y on the normalized scale, you can write y as, this capital Y and this is nothing, but plus minus root 3 by 2.

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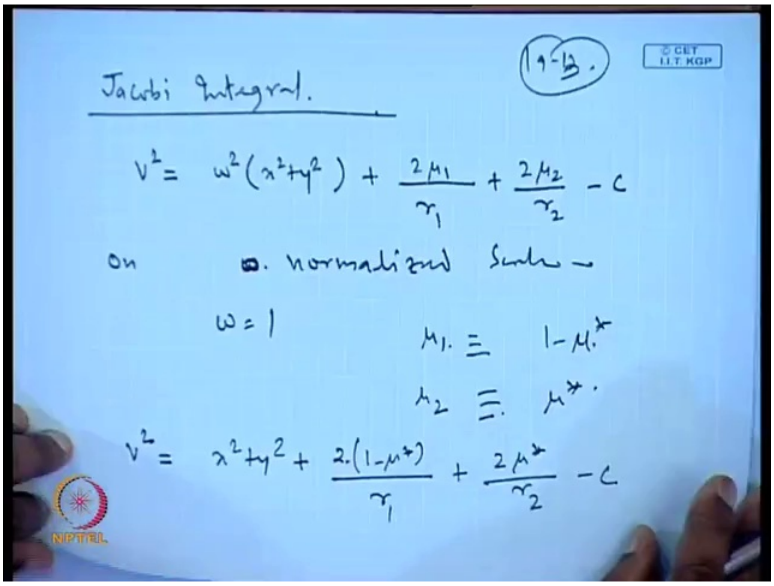

Now, we can proceed to our Jacobi integral. So, we got the Jacobi integral in this format. So, again, on normalized scale, which we, on normalized scale…So, we wrote omega is equal to 1 and mu 1, obviously, then, we are writing as 1 minus, this is equivalent to 1 minus mu star and mu 2 then becomes equivalent to mu star. So, then, this equation gets reduced to x square plus y square plus 2…

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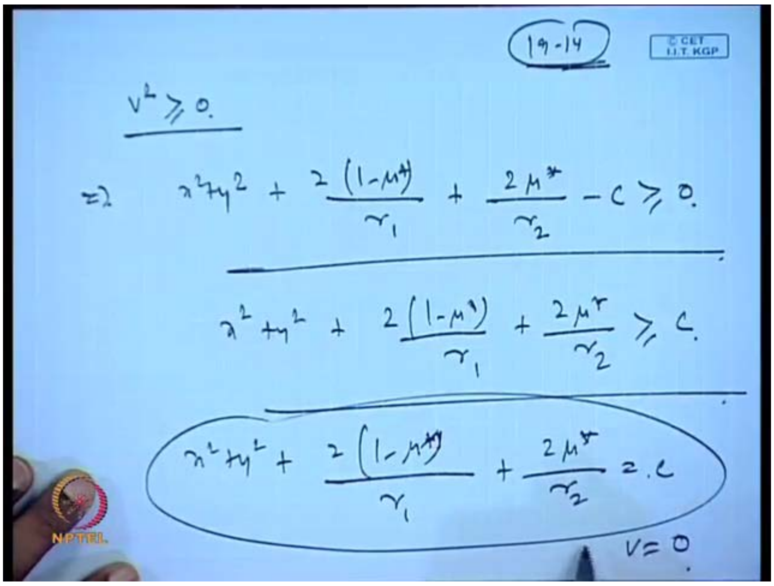

Now, we know that, v square, this is a positive quantity, or atmost it can be 0; and therefore, this implies that,x square y square plus 2 times 1 minus mu star by r 1 plus 2 mu star by r 2 minus C, this will be greater than equal to 0, or x square plus y square… So, the equality sign, when x square plus y square 2 1 minus mu star, when this equality signs holds means, v equal to 0; this implies v equal to 0. So, already, we have discussed in the last few lectures that, this represents a surface.

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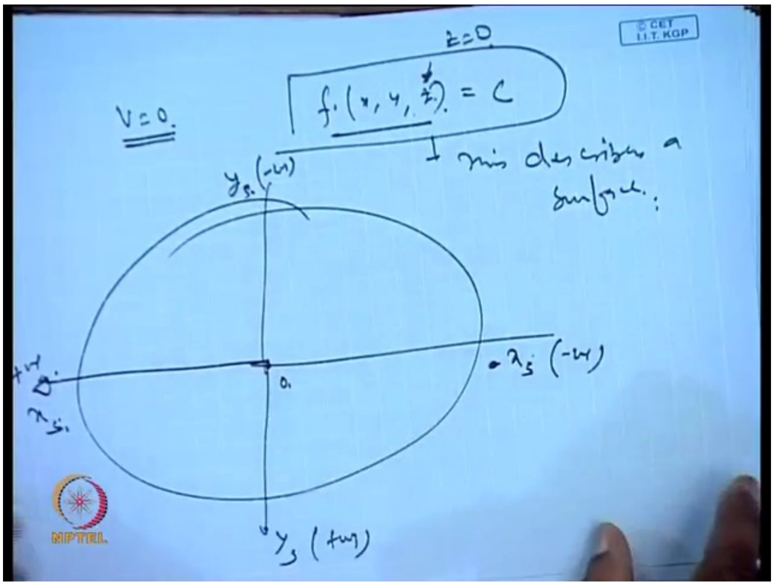

So, when v equal to 0…So, we have f x, y, zis equal to C, this describes a, this describes a surface. Now, it is a very interesting to see, here, z is not present,of course, in this case, but we have put z equal to 0. So, if we have z equal to 0, so, we are looking in the x y plane. So, we have x here and y here, or either, in our case, we have plotted x on this xaxis and y is on this axis; this is the negative side of this. So, xis negative here, on the side; this is positive; and this side, we have written this as positive.

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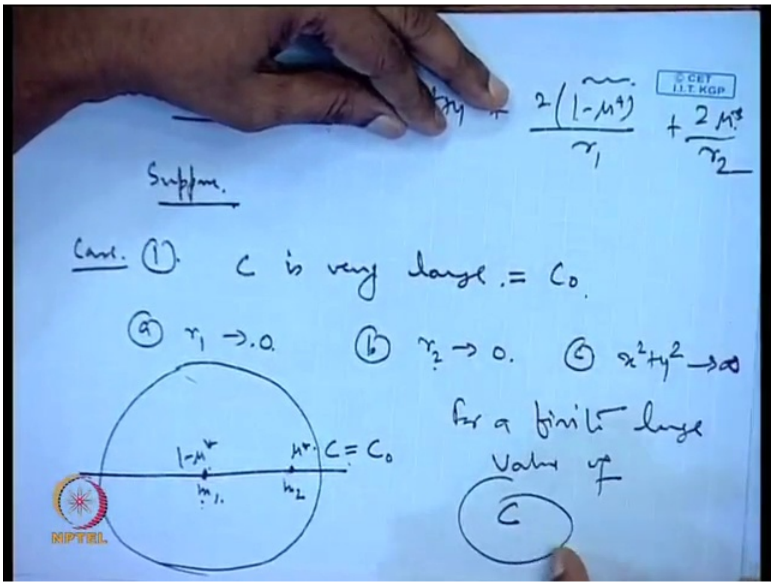

So, this f x y equal to C, which is describing a surface. So, now, look into the equation for the f x y; this is x square plus y square 2 times 1 minus mu star by r 1 plus 2 mu star by r 2 and this is equal to C. So, we have different cases, we can consider here. We can consider a case, let us say, one by one, discuss all of them. So, case 1,C is very large. So, this can happen in the following ways; either r 1 tends to 0, r 1 tends to 0; therefore, this tends to 0, this whole quantity will become very large; or either r 2 tends to 0; here, this quantity r 2 tends to 0, therefore, this quantity will become very large; and, the c is x square plus y square itself tends to infinity; if this happens…

So, what you get, this is the surface, let us say C equal to C 0,and this quantity equal to C 0. So, this is very large quantity, and you have, you have the points here, say m 1 and m 2, which is present here. So, if r 1 tends to 0 means, the point is lying near to this m 1 and r 2 tends to 0,so, point is lying close to m 2. So…So, for a finite large value of C, for a finite large value of C, you can see from this place that, if we are not taking, the point is either close to this, or either close to this…So, depending on the value of C, value of r 1 will depend. Now,1 minus mu 1, this quantity is larger than mu star, because the left hand side, this mass m 1, we are taking as the larger mass; this is our 1 minus mu star and this is mu star. So, this is larger mass. So, this implies that, to, for the C, we have to now equalize the left hand and right hand side; and, if we are close to this, so, obviously, we can equate them, and if we equate…So, r 2 is from here to here, this distance is quite large. So, if you look, the, if your mass is close in the vicinity of this, lying somewhere

here,so, this distance will be quite large, as compared to this. So, this will be…So, if we equate for a finite value of C...

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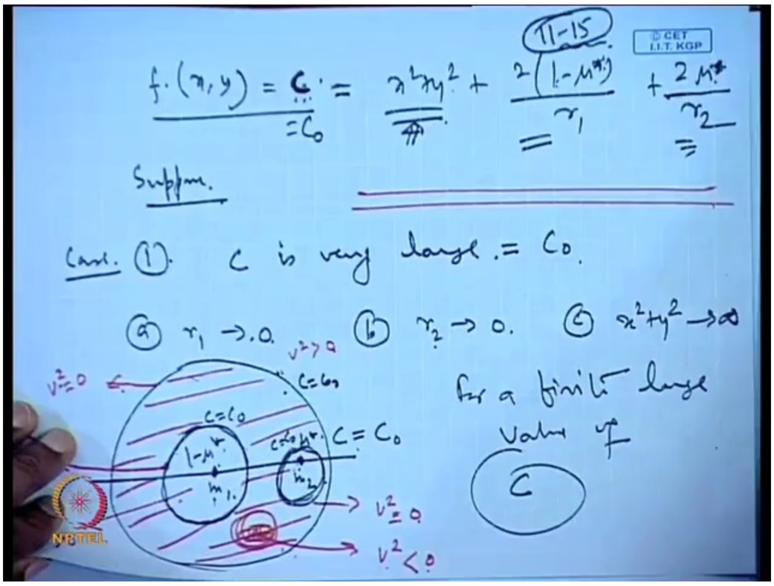

So, let us go on the next page;11 16; let us say, this is C 0. So, if the mass is in the close vicinity of, the small mass is close vicinity of m 1,so, the distance from here to this, will be quite large. So, the contribution of this quantity to this C 0 will be very small; and obviously, in this case, f 0,x square y square also, that will contribute less. And therefore, this will be totally decided by the quantity which is present here. So, depending on the value of C 0, this r 1radius will be decided. Similarly, if the mass is in the close vicinity of the m 2, so, this will not contribute much; this will not contribute much.In the close vicinity, this is going to, in the close vicinity this is contributing, this quantity is contributing, contributing in close vicinity.But once the particle is on this boundary, where C equal to C 0, so, in this place, x square y square is not contributing; while once it is here, so, then, it will tend to contribute; but if the mass is, say the, this mass is very large, as compared to this mass. So, again, this quantity x square plus y square…So, if here, the barycentre is lying quite close to this one. So, this quantity will add a small, while this quantity, because this r 1 is asmall. So, this quantity is going to add less.This quantity will become large, and this quantity, this quantity will become large, and this quantity will become a small.And, this is also not very small, but this contribution is still listed here.

So, from here, now, we can draw the (( )), a circle around this; let us say, this is one circle drawing; another circle, I am drawing like this. Now, we can see that, the radius of this two circles are, this one is a larger and this one is a smaller. This will be very clear from the, this equation that, we are considering here. Now, this mass is a small. So, this mu star is a small and therefore, to match it with the C, the r 2 also must be a small. This mass is 1 minus mu star; this is quite large, as compared to this. In the case of earth moon system, you can see that, m 1 is around 81 times greater than m 2. So, that simply implies that, this, if this is large and if you have to match with C equal to C 0, so, our r 1 also must be large, and therefore, this is the reason that, this radius is shown to be large than this radius. In fact, this will, I have exaggerated this figure. In fact, this will be quite a small. As you can look into this figure, here, this radius is quite large and this radius is quite a small. So, here, on this also,C equal to C 0, and here, on this also, C equal to C 0. So, we have not done it to the scale. Now, the region we have drawn here…So, the region inside, this region is not permitted; outside this v square is greater than 0; on this v square equal to 0.

Similarly, on this, v square is equal to 0; on this also, v square equal to 0. Inside this, v square is greater than 0; in this region, the region which is shown here,so, this region, this is v square is less than 0. So, obviously, becausev square cannot be less than 0. So, the shaded region,shaded by the red ink, it is not permissible. And, it is a very easy to see using this equation, if you look into this,so, you will find that, only the shaded region,its not possible. So, the particle can be either inside this, or either inside this, or either it can be outside this; it cannot penetrate this boundary.So, now, as we decrease the value of the C...So, after decreasing the value of the C, the two circles, the, which are appearing here as a oblique, because of this particular term, this makes it not a exactly circle, but rather than a oblique. So, if this two terms together, this is the equation of a circle about the, about the barycentre, or the origin of the synodic reference frame, and these are the quantities visible, not make it the whole circle, but rather it will look in this fashion.

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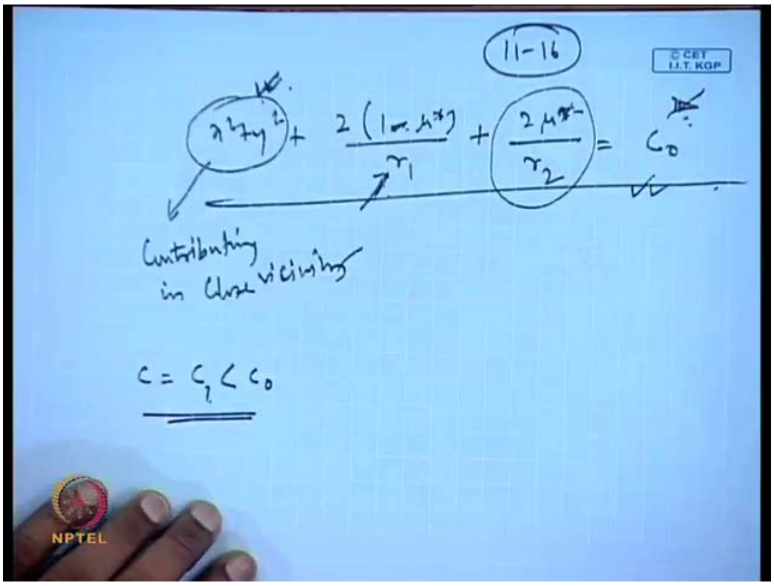

So, as you expand this, as you increase the value of the, decrease the value of the C, means here, we are considering now, a value of C, which is less than C 0; let us say C equal to C 1, which is less than C 0. So, in that case, again, you will see that, this circle this around the, this is the circle. So, this will expand, and this will also expand. And, they will merge somewhere, touch each other in some point, which we now call this as the point L 2; this is the Lagrange point. So, in this, this is the first development which takes place. This is the Lagrange point L 2. Next, as we further decrease the value of the C,so, which is indicated here, by C 3,this two obliques, they merge together, and then, open up. So, now, the particle can be inside anywhere this white space; but we are not sure where it is a line, but it cannot cross the boundary. So, this does not tell you, where the particle is lying exactly, but it is somewhere inside this boundary, because this, by using this equation, we are basically defining the boundaries, not the orbit of the particle. So, if this merge, then, then the particle can move from this space to this space also.

So, it is a permissible. Now, as you further expand it,so, you can see that, it touches the outer boundary. So, the L 3, the, here, in this case, it is written as the L 3; the notation is different here. This is taken from the R C Roy book. So, you can refer to this, for this figure. So...So, Lagrange point L 3 develops. If you further do that, further decrease the value of the C...So, after decreasing the value of the C, the L 1 points then develop. If you further develop,so, this annular shaded region which is here, this becomes short and, and it becomes like this. So, if you keep decreasing it,so, L 4 and the L 5, this Lagrange points they appear.So, this implies that, now, the particle can occupy all these space. Here, the particle can occupy all this white space. Here, the particle can occupy all this white space. So, as the value of the C decreases further and further,so, this L 4 and L 5, they will be accessible. Thank you very much. We continue in the next lecture. So, this is almost cleared and whatever is remaining, we will brief in the next lecture, and then, we move on to thefinally, the trajectory transform. Thank you very much.