Space Flight Mechanics
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Module No.# 01
Lecture No.#10
Two Body Problem (Contd.)

Solast time we have been working with the orbit determination using vector methods. So, we continue with that the last few explanation were remaining. So, we will complete this and then move to the scalar form of the orbit determination andwhich is conducive tocomputer implementation.

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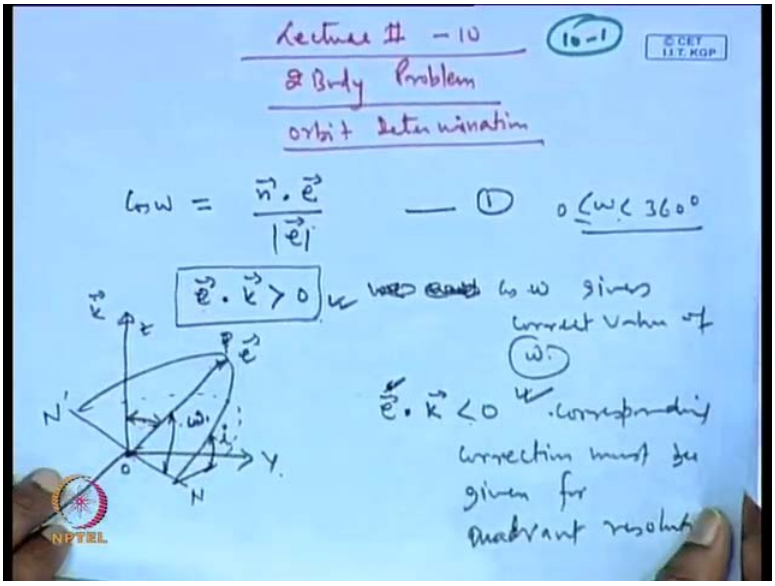

So, last time we had the equation cos omega is equal to n dot ein this equation if the angle between an a is line between 180 degree. So, its fine. So, the whatever the value we get for the omega that' correct, but if it is more than 180 degree then problem arises. So, we need to resolve this. So, here omega lies between 0 and 360 degree. So, we need to resolve this. So, for that resolving what we did we took the dot product in e dot and k dot vector.

So, we had the e vector which is which was lying in the plane of the orbit like this. So, this is the e vector a vector was along the z direction. So, this is the z direction o n and n prime was here and in this direction we had x and here we had yand the angle it was making here the in angle of inclination I. So, if the e vector is lying about the x y plane. So, e dot k this will be less than 180 degree for. So, here what we will do that if this is e dot k is greater than 0 then we write then we have cos omega gives the correct value of omega. So, if e vector is lying over this plane means the jet components of the e vector is positivethen e dot k dot product of this will be greater than 0 if e vector dot product with the k vector if it is less than 0 means e vector is having an negative componentmeans no longer the perigee wildly above the x y plane.

But it will go below the x y plane. So, in that case e dot k will be less than 0. So, from. So, from here to here this angle is positive suppose it goes below this. So, the angle betweenon the right side if it tills to thebelow the x y plane. So, you can see directly that between the angle between the k and the e vector exceeds90 degree and here it, it will vary from till measuring from the angle I is equal to 0. So, if we take the extreme position that the e vector is near about I is equal toline on the x y plane. So, the angle between the e vector and the k vector it is a around less than ninety degree or ninety degree. So, in that case e dot k the dot product will remain positive, but as soon as goes below this plane.

So, this becomes negative and therefore, corresponding correction must be given forquadrant resolution. So, if the angle between here the angle between e and the z vector this is this angle herewhich is nothing. So, y of the x y plane if the e vector is above the x y plane. So, this angle remains less than 90 degree even if e is lying on this side. So, you can see that e will be again less than 90, but as soon as the e goes below the x y plane you will see that this value will exceed the 90 degree and therefore, both this assumption are correct. So, e dot k greater than 0 will indicate that the perigee is line above the this is the perigee here. So, perigee is line above the x y plane and e dot k less than 0 it will show that the perigee is lying below the x y plane. So, accordingly the argument of perigee this can be resolved

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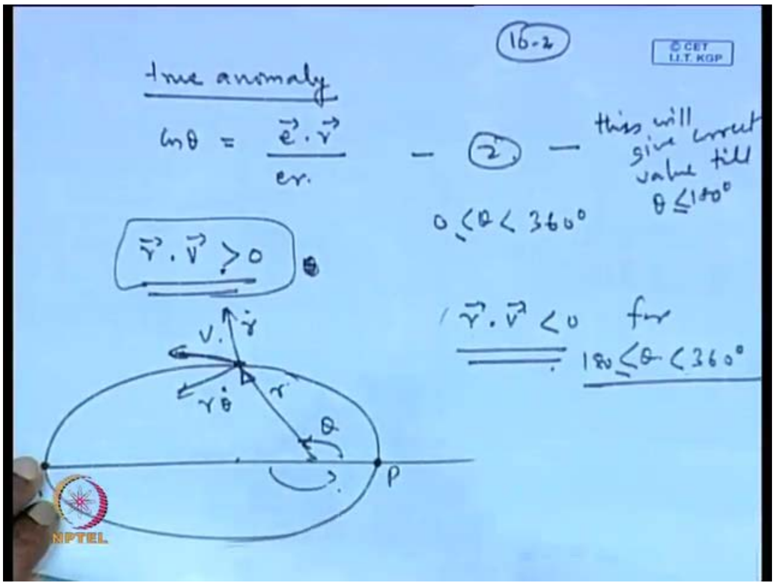

Now the next was true anomaly. So, for the true anomaly again we have the cos theta is equal to e dot rwhere theta wililybetween 0 and360 degree, but the cos theta will only resolve between 0 and360 degree it will give the correct value at if itexceeds the 180 degree value if the angle and r is greater than 180 degree then the quadrant resolution must be done as we are done for a then from dot product of n dot e or the cos omega. So, in the same way we have to fallow here. So, in this case what we see that r times the this will be greater than 0 hence that implies that if we have the orbit herethis is the perigee position and this is the r vector and this is the velocityvector v the components of the r vector this is r vector r dot and this is r times theta dot.

So, r dot v this will be positive while we move from perigee to apogee till this point from here to here this will be positive, but as soon as we go below this. So, r dot v will be less than zero for theta line between 180 to360 degreeand this is valid for theta line between. So, this will give correct value this will give correct valuetill theta less than hundred eighty degree less than equal to hundred degree hundred eighty degree. So, as soon as we are exceeding means we are coming into this domain.

So, then we have to take care of the proper sign and this proper sign can be resolved using this two in equalities r dot be greater than 0 and then r dot be less than zero for theta line between 180 and360 degree. So, if we get this quantity then we ensure that the theta is lying from here to here and if we get this quantity.So, we can ensure that it is a line between this and this range from here to here.

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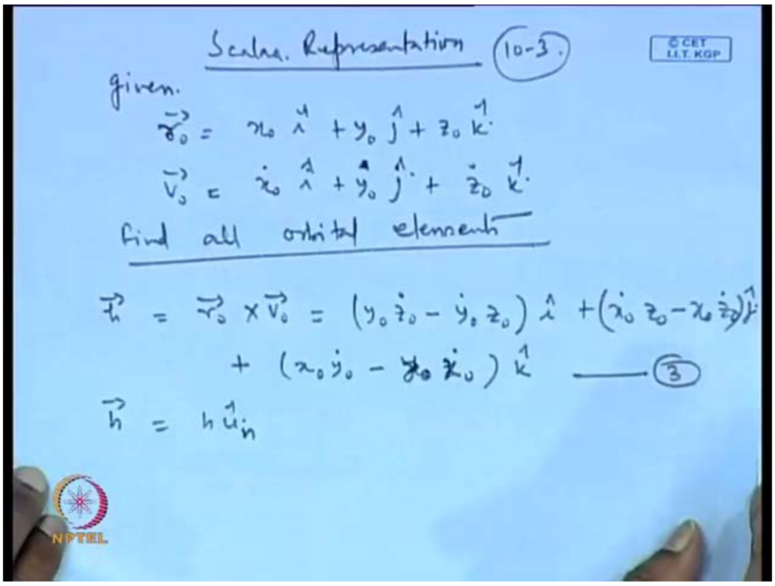

So, this completes ourvector method of orbit determination. So, now we go into thescalar method of say the a method which is a way which is conducive to computer implementation. So, we will work out that. So, this will be not in terms of the vector, but this will be represented in terms of the scalar means the components of the xcomponents of the radius vector and the velocity vector which are x y z and x dot y dot z dot. So, given.So, scalar representation. So, given r 0 is equal to f 0 I plus y 0 j and v 0 is equal to x dot 0 Iplus y dot 0 j plus y dot 0 j plus z dot kpoint all orbital elements. So, here we write h is equal to r 0 plus v 0.

So, if we take the cross product of this this will be y 0 z 0 dot minus y 0 dot z 0 I cap 0 minus x 0 z 0 r cap plus 0 y 0 dotminus y 0 x 0 dot k. So, this is our equation number 3. So, h we can write as h timesu n where u n is the unit vector perpendicular to the orbital plane. So, we need to determine the u n vector.

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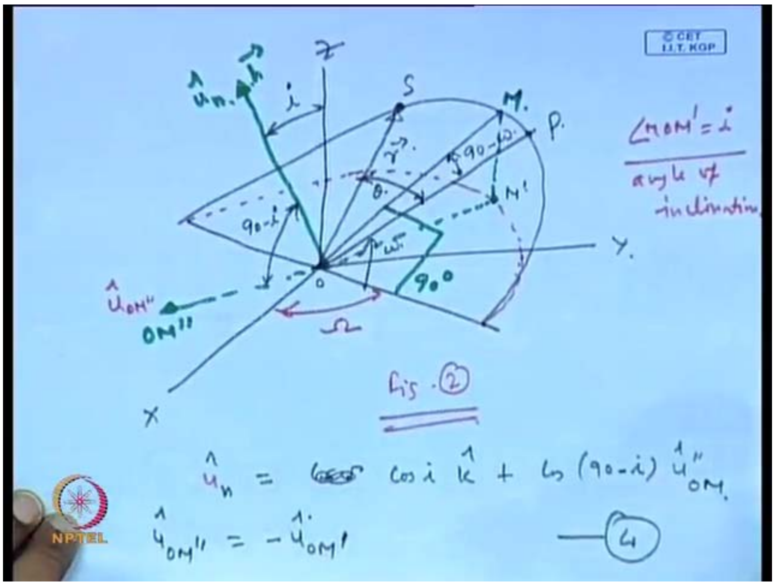

So, for this we draw the figure the X y and z this is o say this is the perigee position. So, angle from here to here iswe take a line herem hose angle is here 90 degreeand this is m the satellite is somewhere here in this position. So, this is the position of satellite s this is the orbit vector. So, the angle from here to here this will be 90 minus small whether and the angle which we are measuring from the perigee this angle from here to here this is total angle is theta. So, this angle is theta anglethis is the z and we have the u n vector which is perpendicular to the orbital plane. So, this is the s vector and u n is also along this directionnow the projection of point m on the x y plane.

We show like this. So, this is our m prime and we can connect them the projection the orbit we can show in this way. So, the angle m o m prime is nothing, but I which is the angle of inclination this angle from here to here is capital omega argument perigee. So, opposite to the o m vector we extend here and we write o m double primeand unit vector in this direction we will show this as u o m double prime cap now using this figure. So, this is our figure two. So, using this figure we will be able to work out all the details. So, from here we can writee 1 cap is equal to cos I a cap times cos ninety minus I u cap double prime o m. So, the angle from here to here this is the angle I. So, thisangle from here to here this is ninety minus Iwhich is pretty clear from figure.So, this is our equation number fourthen we writeu o m cap double is equal to minus u o m cap. So, o m u o m prime o m double prime they are opposite to each other this is valid

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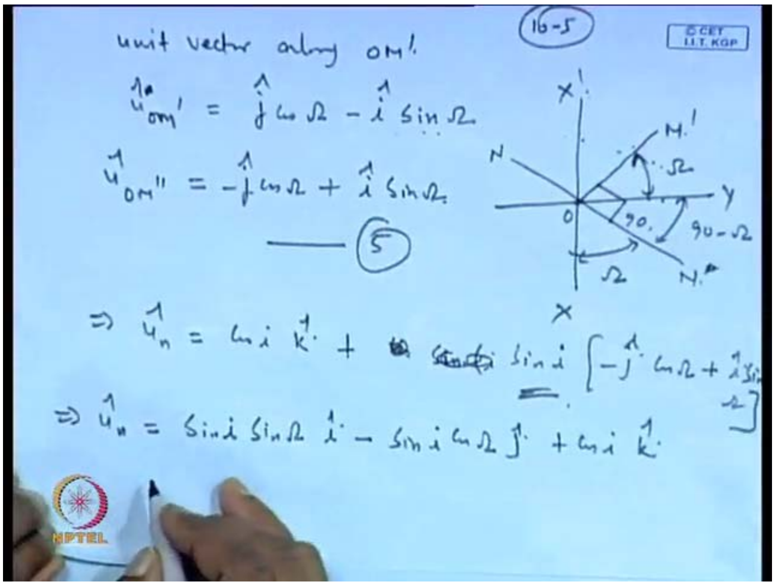

Now unit vectorlong o m prime we can write it as you cap a cos capital omega minus I sine capital omega this is n n prime o and m prime is perpendicular to this line this ninety degree angle from here to here is capital omega this is ninety minus capital omega this angle is then capital omega. So, we can get theunit vector along this direction j cos capital omega and here this is X and Y and his is X prime. So, this is minus I cos ninety minus omega which gives us the minus I sign capital omega. So, we get this value. So, therefore, u cap prime double prime this will be minus j cos capital omega.

And this is cap plus I capital cap and capital omega. So, this is equation number five. So, this implies a 1 cap will be cos i of k plus sine we need to replace u n double prime. So, this becomes sign I times minus j cap cos capital omega plus Icapital omega sign capital omega. So, here this is nothing, but sign I. So, we are using the same thing here. So, expanding it we can write a 1 cap is equal to sine I and capital omega I cap minus sign I cos capital omega j cap plus cos i k

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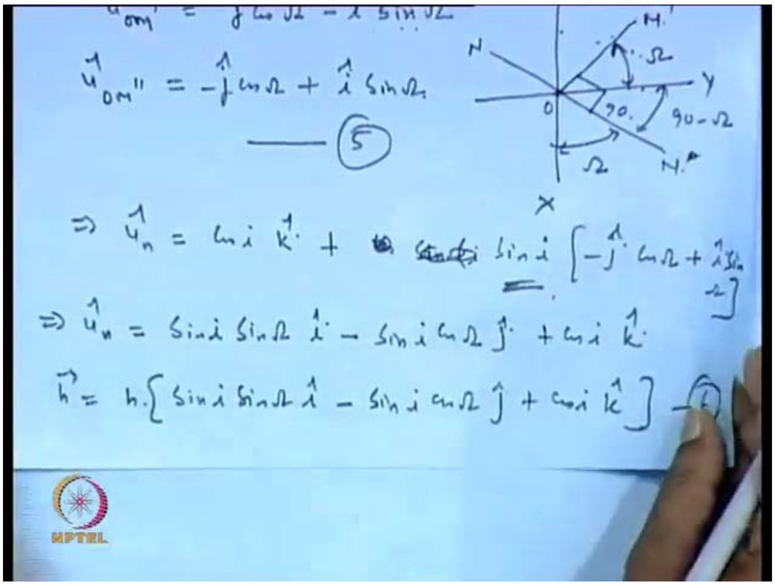

And h we have therefore, we can write as h times sign I sine capital omega I cap minus sine I cos capital omega j cap plus cos i j cap this is our equation number 6

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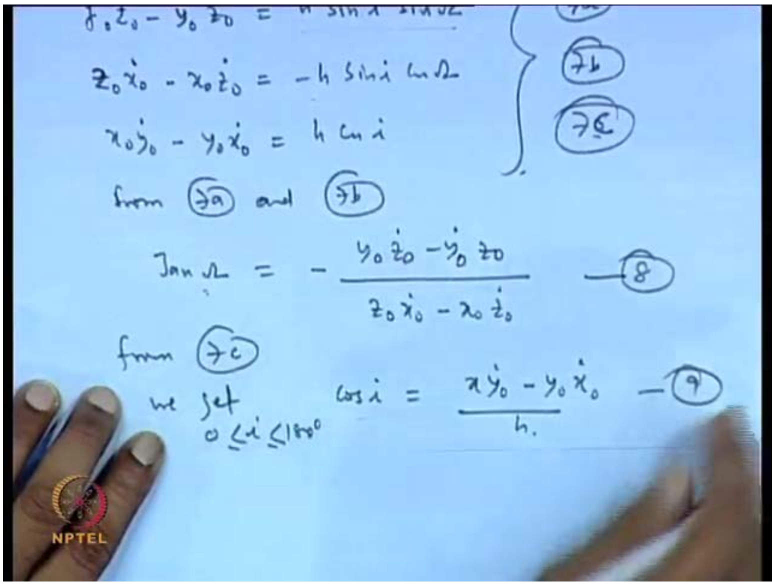

Now comparing equation 3 and equation 6 we can write x 0x 0 dot minus y 0 dot z 0 is equal to h sine I sine capital omega this is 7 a then x 0 x 0 dot minus x 0 z 0 dot is equal to minus h sine I this is 7 c. So, from 7a and7 b7 a and 7 b.

We can write tan capital omega dividing them minus y 0 z 0 dot minus I 0 dot z 0 divided by x 0 z 0 dot minus x 0 z 0 dot and there is no ambiguity of sign because the separately we need thewe know this term and this term. So, we are dividing this depending on the sign the in which quadrant the capital omega line this can be resolved this is our equation number 8 now 7 c from 7 c we getcos i equal to x y 0 dot minus y 0 x 0 dot by h this is our equation number nine and there is ambiguity in this casebecause the I wilily between 0 and 180 degree. So, forI less than ninety degree it is a called the protrude orbit and I greater than 90 degree it is a called the retrograde orbit and for I is equal to 90 degree its exactly it is a called the polar orbit. So, herethere will be no ambiguity of the sign of I because the I will vary from 0 to 180 degrees. So, it lies between 0 and 180 degrees. So, there is no ambiguity in this case

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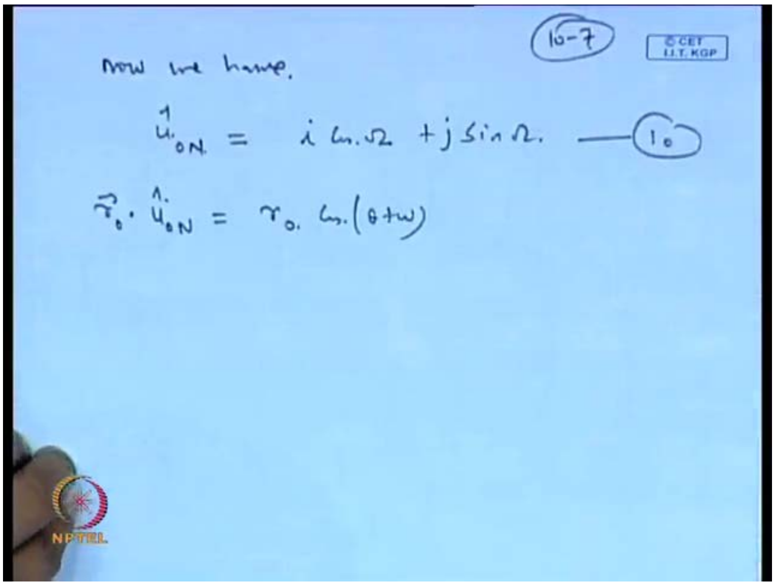

Now we have unit vector along with the point direction this can be written as I cos capital omega plus j sine capital omegaand this is very obvious from this figure we can take the unit vector along the 1 direction here. So, this is our equation number ten now r 0 dot a 1 cap this will be r 0cos theta plus omega is here what we are trying to do we are trying to work out theargument of perigee theta is already known to us theta.

We have determined in our earlier lectures. So, if we get the value of theta plus omega which is the angle measured from the line this is n and the n prime. So, if we measureif we have the total angle level theta plus omega then it will be very easy to find out provided that theta is known. So, omega can be derived after subtracting from theta plus omega if we subtract theta then we get the value of omega. So, here we are tempting to find out the value for cos theta and omega. So, this angle we know the angle

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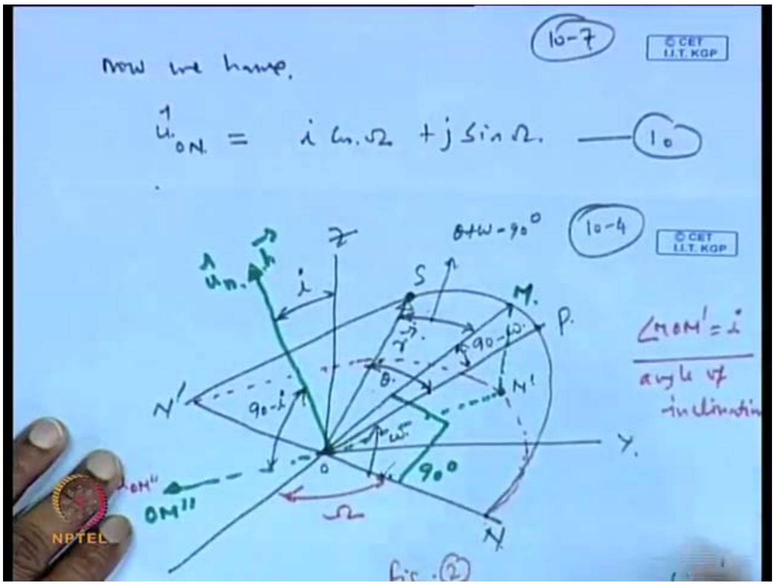

Let us look into what this the quantity of what this value is between s and o m. So, from here to here we see that this is the angle theta and from here to here this is the angle omega. So, this total angle from here to here is theta plus omega and if we subtract from this the ninety degree.

So, this angle will be nothing, but theta plus omega minus 90 degree. So, we can utilize this to find out theta plus omega. So, exactly going through this process. So, r 0 dot u o nthis is r 0cos theta plus omega.

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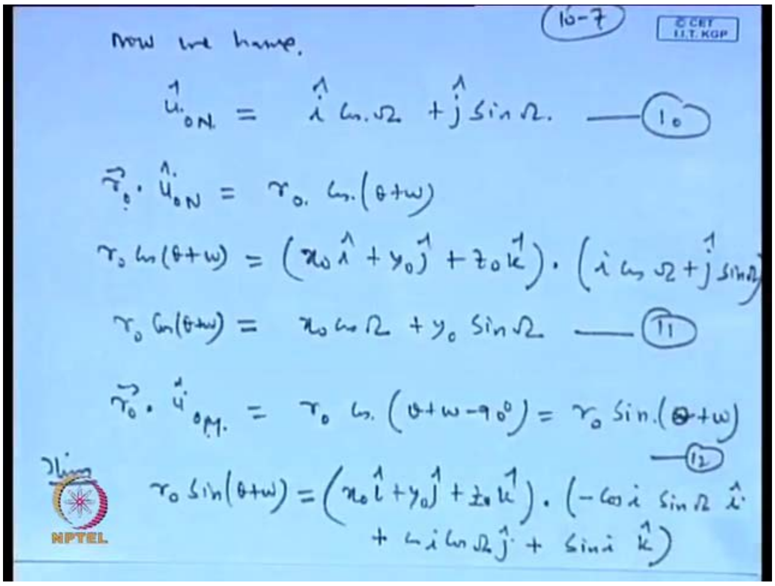

So, we have r 0cos theta plus omega now break this the left hand left hand side we are writing here. So, this is f 0 I y 0 j plus z 0 k and you know vector we know I cos this is I cap j cap I cos capital omega plus j cap sine capital omega. So, this gives us x 0cos capital omega plus y 0 sine capital omega. So, r 0cos theta plus omega this is quantity this is our equation number elevennow we have taken alongdot product of r 0 and u o n now we can take the dot product of r dot and u cap o m. So, this will be r 0 times cos theta plus omega minus ninety degree. So, this will be r 0sine theta plus omega.

So, thus this is our equation number twelve this r 0 sine theta plus omega we can write this as f 0 I cap plus y 0 j cap plus z 0 a cap and dot product u o m vector. So, u o m vector we already know. So, multiply this here and this is u o m we have written as cos i times sine capital omega I cap plus cos i cos capital omega j cap plus sine I k cap this is the u ovector we have written earlier.

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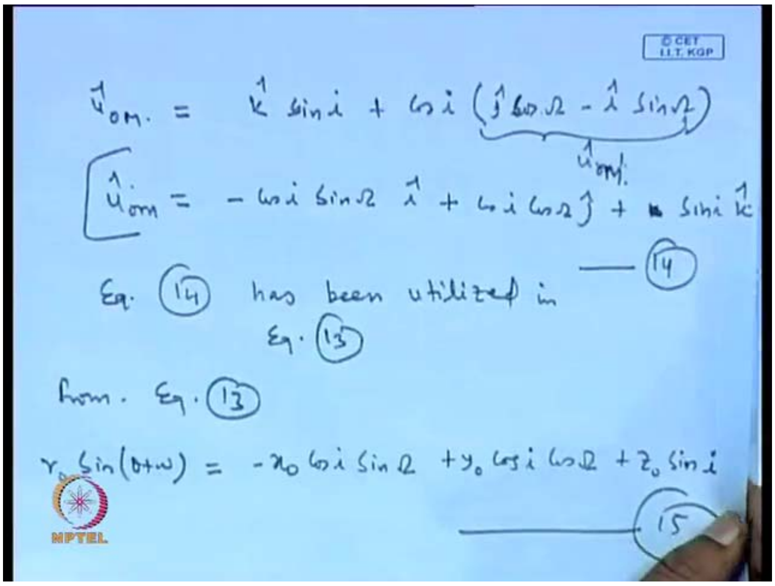

So, let us work out he u m vector. So, u o m vectorwe can write as k cap sine Iu o m the unit vector along the o m direction. So, we take the component of the k vector along this. So, this is k sine I this angle is ninetyminus I from here to here. So, k sine I pluscos i times the unit vector along the o m prime direction. So, we already know the unit vector along theo m prime direction this is the unit vector along this quantity is nothing, but u o m prime.

So, now we resolve this. So, if we resolve it this will result incos i sine capital omega I cap pluscos icos capital omega j cap plus k plus sine I k cap. So, this is u o m and this is what we have utilized in the previous equations. So,this is our equation number thirteenand this is our equation number fourteen. So, equation fourteenhas beenutilizedequation thirteen. So, from equation 12 we can say it from equation thirteen we can see that the expanding this. So, from equation 13 you can write r 0 sin theta plus omega this will be minus x 0cos i sin sin capital omega plus y 0cos i sin cos capital omega plus z 0 sin i this is our equation number 15 So, once we have got r 0 sin theta plus omega and r 0cos theta plus omega. So, we can get the value ofsin sin theta plus omega and this is 10 8.

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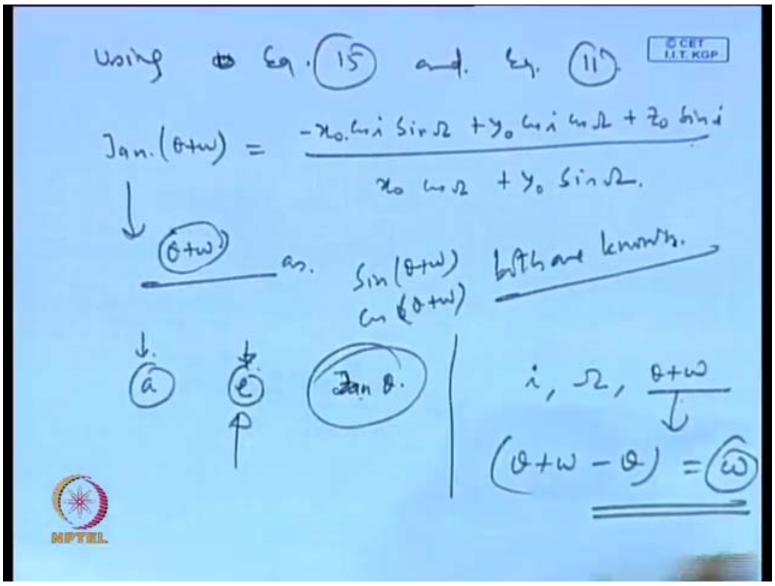

So, usingequation fifteenand equationelevenwe can write tan theta plus omega minus x 0cos ix 0 sin i divided by x 0cos capital omega plus y 0 sin capital omega.

So, thus we have this will give theta plus omega and there will be no problem in determining the quadrant as sin theta plus omega and cos theta plus omega both are known. So, the any ambiguity can be resolved. So, in the previous lectures we have determine awe determine eand for ethe got number of equation thereafter we also determine tan theta we determine tan theta and then the the also worked outa e tan theta 3 parameters we have done here. So, for the rest 3 parameter we have worked out here which are high capital omega and theta plus omega. So, if this suppress from this theta plus omega minus theta. So, this will give us the omega value. So, this complete the process of the orbit determination. So, we have a number of ways of finding the value for e. So, a you can go in the previous lecture can look for all this things I did not repeat all these againand this is can

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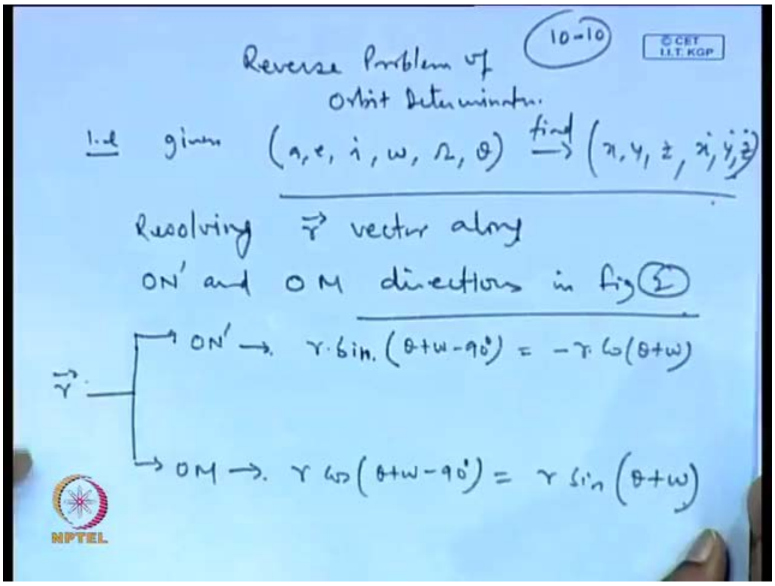

Now, the reverse problem orbit determination with it the reverse problem of orbit determinationthat is given a e i small omega capital omega and theta point x y z x dot y dot z dot. So, for this we need thethe help of p got two. So, very easy to work out here what exactlywe do we have this vector or this is the r vector available to work. So, break this r vector along the o n prime directionalong two perpendicular direction o n prime and o mthereafter break the o m along the z direction and the o m prime directionwhich is laying in the x y frame then break the o m prime along the o x direction and o y direction similarly break the o n frame along the o x and o y direction.

So, the corresponding component which you were you get the x component of the radius vector similarly you get the y component of radius vector and z component is available here and it take the derivative of this then we get the corresponding velocity component. So, we write here resolvingr vector along o n prime and o m direction in figure two. So, r vector we are resolving alongo n prime and o m direction. So, this will give us r sintheta plus omega minus ninety degree is equal to minus r cos theta plus omega similarly this will give r cos theta plus omega minus ninety degree. So, this gives us r sin theta plus omeganow o m prime the component o m can be broken along the z direction and the o m prime direction.

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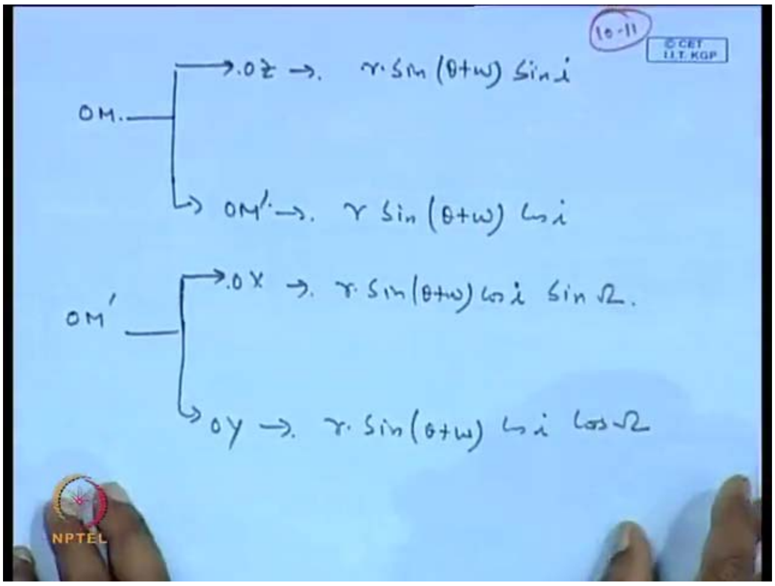

So, next we break the component o m along the o z direction and along the o m prime direction. So, along the o z direction.

This will giver sin theta plus omega sin I and this will give r sin theta plus omegacos i. So, next we need to break o m prime along the o x and the o y direction o x and the o y direction. So, breaking this r sin theta plus omegacos i and sin capital omega similarly this will give us r sin theta plus omegacos i sin cos capital omega and this you can very early of of z come this place because this angle are known. So, if we are breaking up o m prime. So, angle from here to here this is capital omega and this angle is also known the angle from here to here this is90 degree which is laying in the x y frame. So, angle from here to here this is also known . So, this angle is capital omega this angle is capital omega hereten eleven.

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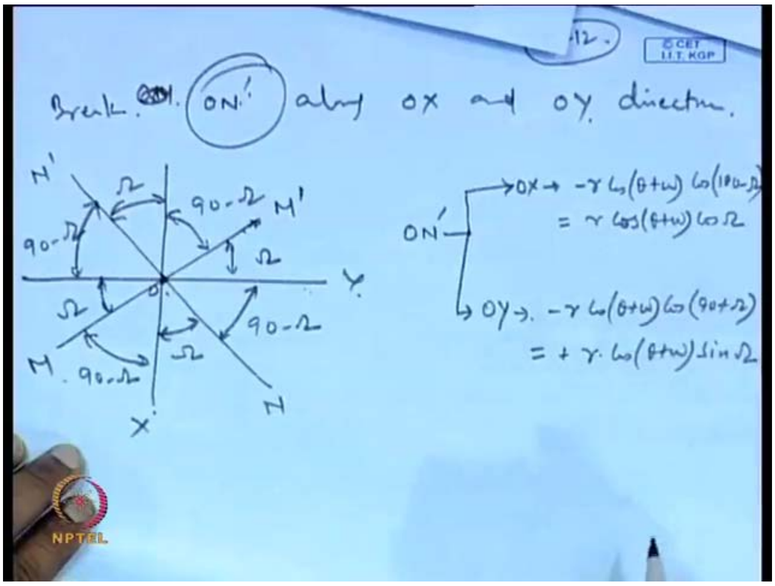

Once we have broken theo m next we need to break o m prime. So, o m prime we have already done. So, next we go for breaking up o n prime. So, breaking break o n prime along o x and o y direction. So, we can take help of the figure here this is x y this is n n prime here this is o this is our m prime m this angle is capital omega this is 90 minus capital omega here this is capital omega90 minus capital omega90 minus capital omega. So, what we need to break o n prime along the ox and the o y direction. So, o n primebreaking this along the o x direction this will give uso n prime minus r cos theta plus omega and the angle from here to here this is nothing, but 180 minus capital omega. So, this become cos180 minus capital omega.

So, this become equal to rcos theta plus omega cos omega similarly we breako y. So, this will give us r cos theta plus omega and along the o y direction the angle is ninety plus capital omega. So, this become cos ninety plus capital omega. So, this become equal tohere this minus plus r cos theta plus omega and this becomes sin capital omega .

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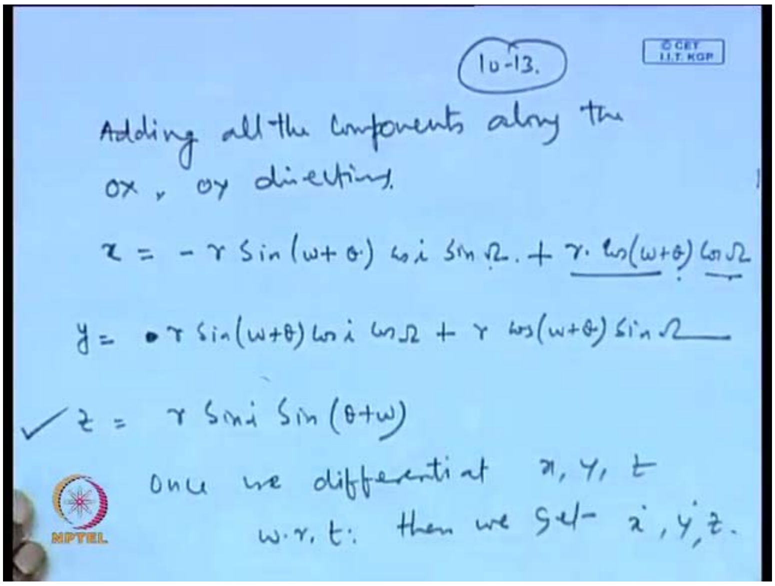

So, now, add all the component along the o x y and o z direction this is ten thirteenadding all the component along the o x o y direction. So, if they add we get the corresponding x y and z values. So, x will be given by minus r sin omega plus theta cos i and sin capital omega you can check 1 by 1. So, this is the component here o x we have taken r sin theta plus omega cos i from sin capital omega this is appearing here next another component of o xanother component of the o x is r cos omega plus theta sin capital omega.

Generally the y can be written as r sin omega plus thetacos icos capital omega r cos omega plus theta and sin capital omega and z can be written as r sin I sin sin theta plus omegayou can verify this. So, o z component you can see here o z component we wrote like this r sin theta plus omega sin i. So, this is a same thingo x component this partthis broke o m prime. So, where this is . So, x iscoso x is mu coso n prime o m primeo m prime we have broken along the o x direction and the o y direction. So, o m sin we addhere o m prime is given here itself. So, this o m prime we are breaking along the o x direction. So, along the o x direction this is r sin theta plus omega cos i which is copied from this place and then the along the o x direction. So, o m prime once we have are breaking. So, this angle from o m prime from here to here the angle between these.

And this is ninety plus capital omega. So, cos90 plus capital omega. So, this here this would be a minus sin. So, here this is a minus sinon the page number 10 and 11 we have a correction here this is .

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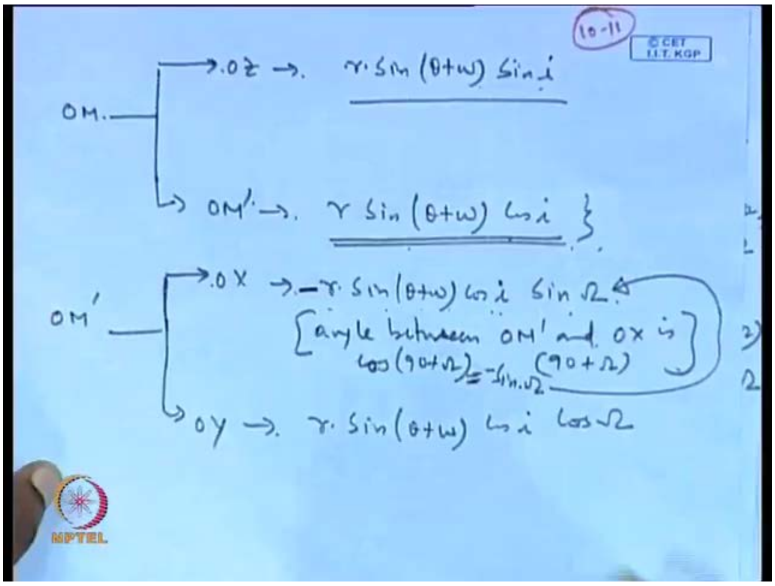

So, this we have got fromthe angle betweeno m prime ando x is ninety plus capital omega. So, once we take cos ninety plus capital omega. So, this is equal to sin capital omega which a minus sin. So, this is what as appeared here in this place. So, the minus sin here should be introduced. So, this is o x is equals to minus r sin theta plus omega cos i sin omega

And the last component we add to this this was o x which was written here r rcos theta plus omega sin cos capital omega. So, this component we have put here in this placesimilarly we can complete forthe y and znow once we have got this values.

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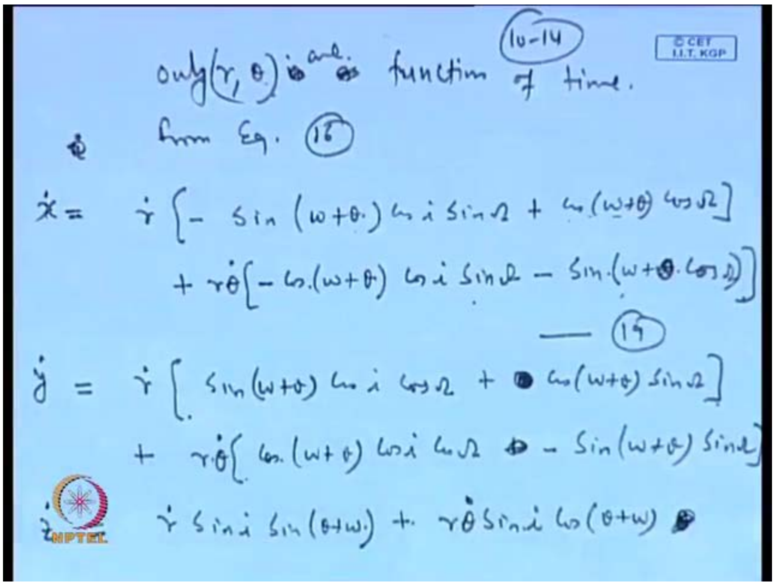

So, if it differentiatex y and z with respect to tthen we will get theonce we differentiatex y z with respect to t then we get x dot y dot z dot we have to take here that only theta is function of time only theta is aonly theta is a function ofof time therefore, only the derivative of this will we taken. So, now, taking the derivative of x dot. So, x dotwill be given by. So, will go back to this slide and number thisequationsthis we can write this aslet us say write this as sixteen this as seventeen and this as the equation number eighteen

So, from equations sixteen x dot becomes equal tohere only theta is a function of time and theta and r only theta r and theta are function of time. So, if we differentiate from equation sixteen we can write r dot time is minussin omega plus theta cos i sin capital omega plus cos omega plus theta cos capital omegaplus r times minushere theta is coming inside. So, we have to could differentiate this if we differentiate this. So, sin will become cos omega plus theta and theta dot will appearcos isin capital omega and here again the if we differentiate cos omega plus theta. So, here we have to write sin omega plus theta this minus sin and cos capital omegaand theta dot will appear. So, theta dot appear in both this terms. So, we write theta dot outside here.

So, this gives us the it this is equation number nineteen this gives us velocity in the x direction x velocity component in x direction generally y dot we can writefrom the equation the number seventeen. So, y dot will be equal to r dotsin omega plus thetacos icos capital omega plus r cos omega plus theta sin omega plus r time sin derivative will be cos omega plus theta and theta dot. So, theta dot we will check it outside later on thecos icos capital omega and here this term here r is not ther we have taken outside here. So, this becomes minus minus here sin omega plus theta sin theta dot sin capital omega and theta dot we check it outside and write here in this placein generally z dot we can write as wed taking the derivative of the equation number 18. So, z dot will be r dot and i and sin theta plus omega plus r time sin i and sin theta derivative will be cos theta plus omega sin theta dot.

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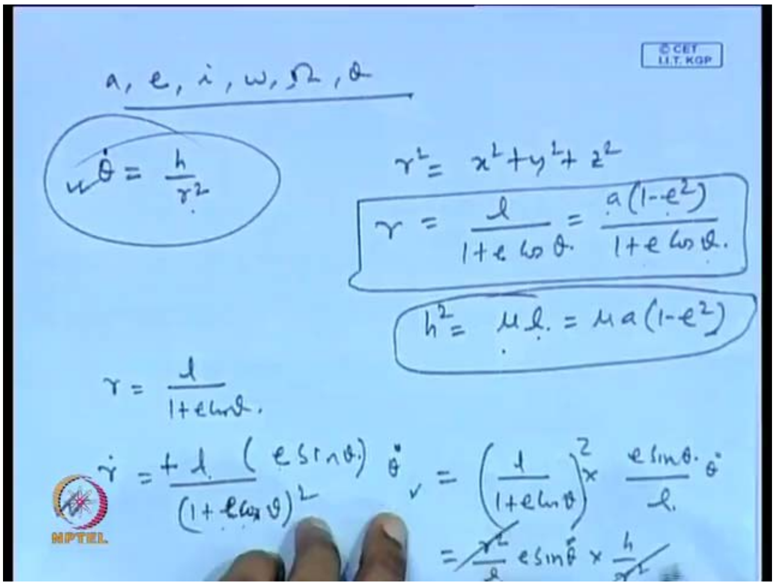

So, theta dot we write here in this placethis is ourequation number twenty and this equation number 21 now we the we are started witha e we were given a e i a small omega capital omega and theta value. So, from here we need to the we are finding this x dot y dot and z dot. So, we can use few substitution and. So, forwhat we need to replace here r dot and the theta dot. So, theta dot we know that theta dot is equal to h by r squareand r square is nothing, but x square plus y square plus z square which also we have written as l by 1 plus e cos thetaand l is nothing, but a times 1 minus e squareby 1 plus e cos theta. So, therefore, e is known a is knowntheta is known e is known here. So, therefore, r can we calculated here r is equals to this quantity.

So, r is known similarly h can be calculated h is nothing, but h square is equals to mu times l mu is known l is known. So, this is mu times a times 1 minus a square therefore, from h is also known. So, theta can be determine. So, theta can be inserted in all these theta dot can be inserted in all this equationsand r dot also similarly can be determine because r is equals to l by 1 plus e cos theta. So, we can take the derivative of this 1 plus e cos theta earlier we have done this exercisee cos theta square minus. So, this becomes e cos theta becomes sin theta and then times theta dot and this will become plus

So, again the all the quantities here are known one is known e theta and here all these quantities is known theta dot already we have determined from this equation. So, using this information will be able to write the equation for r dot also. So, this completes the process reverse process of finding out x y z and x dot y dot z dot. So, we canwrite in a proper way this is 1 by l plus e cos theta times or we can write l y 1 plus e cos theta square divided by e sin theta times l sin theta dot. So, this becomes nothing, but r square divided by l times a sin theta a sin theta sin theta dot we can replace by h by r square. So, this this cancel out this equals to h times e sin theta divided by l

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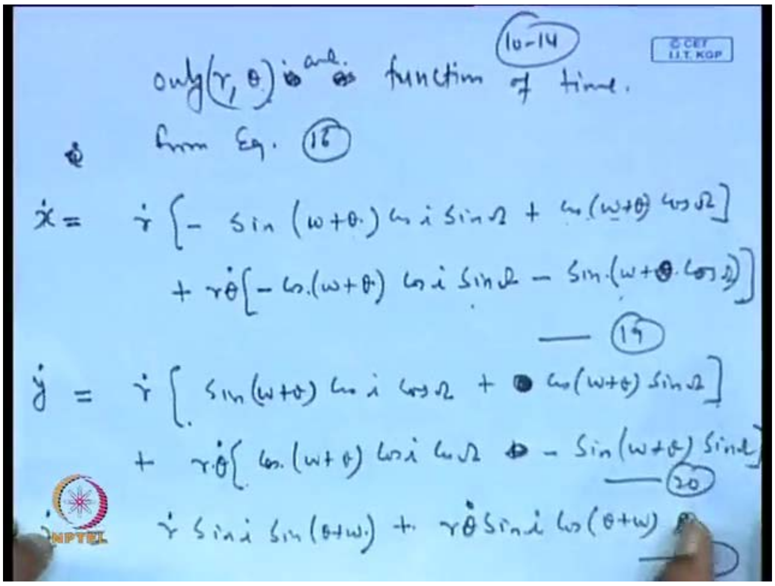

So, this is the final we are getting r dot becomes equal toh times e sin theta by. So, we have got here the value for r dot we have got from herethe value for theta dot. So, our processof the orbit determination is complete. So, next lecture we will look into the kepler's equations of motion and the kelper's problem. So, kepler. So, kepler equation kepler problem it is a kepler equation of motion they are related together, but they are definitely different ofthey look into that problem thank you very much.