**High Speed Aerodynamics**

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**Lecture No. # 39**

**Transonic Flow (Contd.)**

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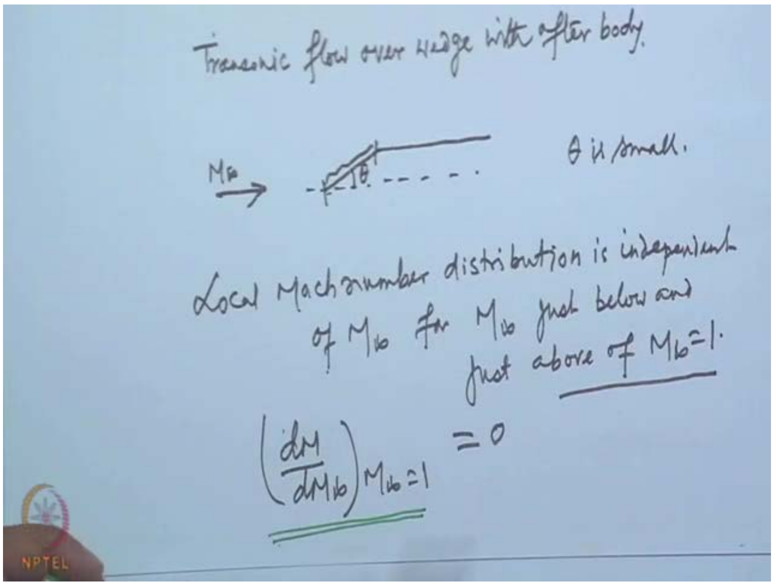


So, we will continue our discussion, on transonic flow over a wedge with upper body, where we are considering a wedge, which is extended and our approximation; so that, we are within the framework of small perturbation theory, that M in theta is small; as theta increases, the accuracy of the results decreases. However in this case, mostly our discussion, the qualitative nature of the flow, so they of course, remain more or less the same. We have seen that, the local Mach number distribution on the wedge; that is, on this part of the wedge, local Mach number distribution Mach number distribution is independent of M infinity.

For M infinity, just below and above just below and just above of M infinity just above of M infinity equal to 1. That is, when the system Mach number is very close to 1? The Mach number distribution on the surface remains the same, which we expressed mathematically as d M d M infinity or M infinity equal to 1 is 0.

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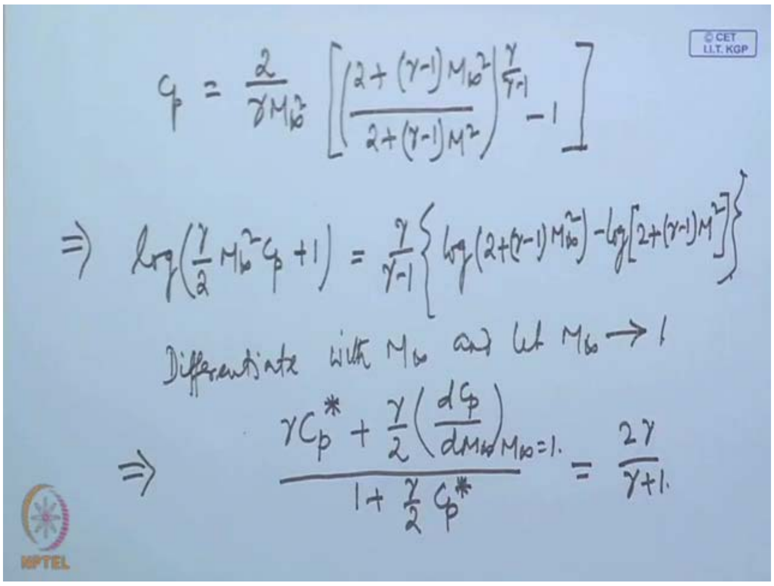


Now let us see, how this affix the pressure coefficient near Mach number unity? The pressure coefficient is given by 2 plus gamma minus 1 M infinity squared by 2 plus gamma minus 1 M squared to the power gamma by gamma minus 1 minus 1. So, this is the pressure coefficient at a point, where the local Mach number is M. Taking algorithm, this gives log of gamma by 2 M infinity squared into C p minus plus 1 equal to gamma by gamma minus 1 log of 2 plus gamma minus 1 M infinity squared 2 plus gamma minus 1 M squared. Differentiate this with respect to M infinity, differentiate with M infinity and let M infinity approaches 1; this gives then, gamma C p star plus gamma by 2 d C p dM infinity M infinity approaching 1 by 1 plus gamma by 2 C p star, that is 2 gamma by gamma plus 1.

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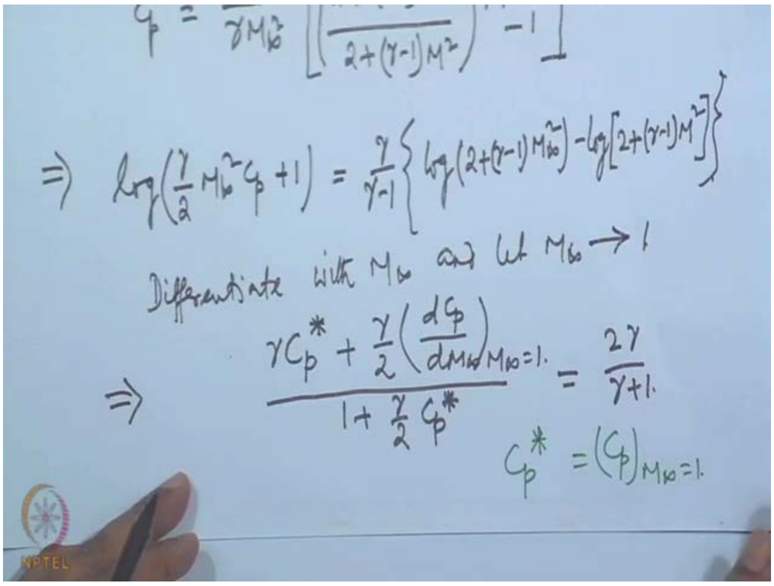


In this, the C p star is C p at M equal to 1. In which, this C p star is C p at M infinity is equal to 1 with this, then d C p dM infinity; at M infinity equal to 1 4 by gamma plus 1 to 1 minus half.

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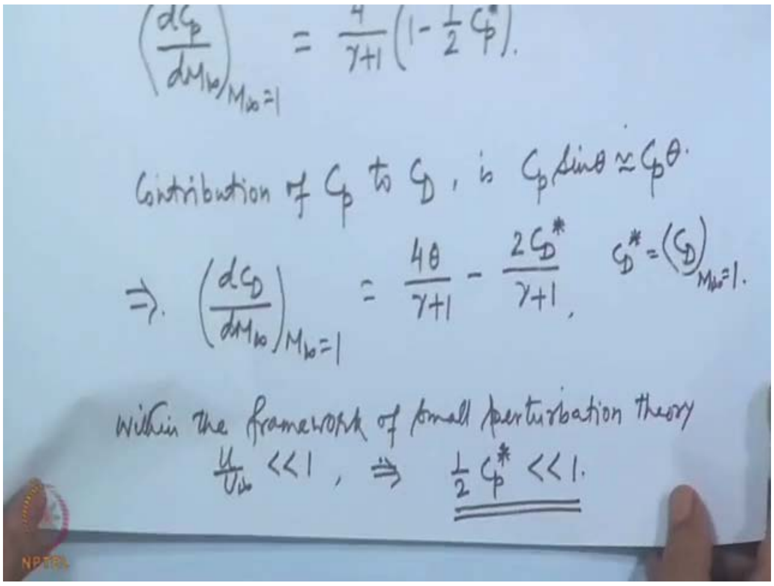


Now at any point, where the pressure coefficient is C p, that contributes to the drag as C p theta. So, contribution of C p to C D is C p sin theta; where theta is the semi vertex angle of the wedge, which for small theta is C p into theta. So, we can now get that, d C D d M infinity four theta by gamma plus 1 minus 2 C D star by gamma plus 1 and again C D star is C D at M infinity equal to 1. So, this very simple analysis gives us, some indication about the drag curve flow, near M infinity.

Now, within the small perturbation theory, where; within the framework of small perturbation theory, that is for u by v infinity is much less than 1 and in this case, this half C p star is much less than 1. Of course, this result holds for wedge with very small semi vertex angle; as the vertex angle increases, the accuracy of this approximation decreases.

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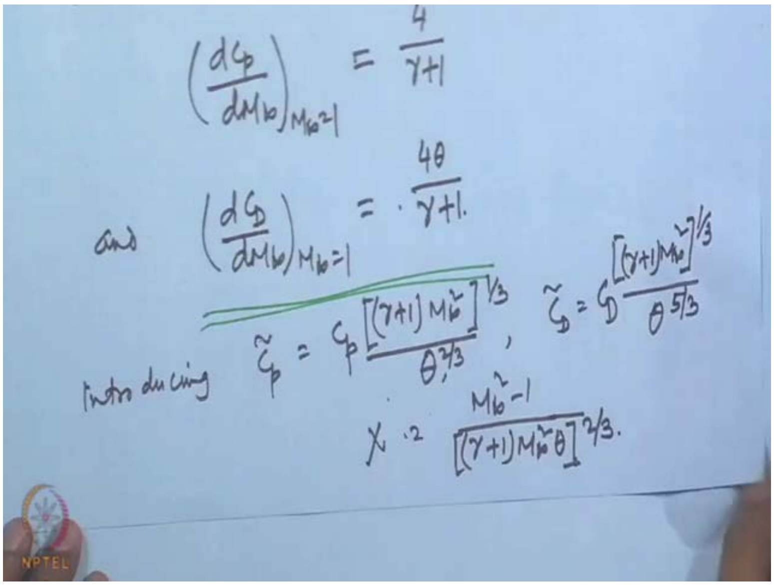


So with this approximation, with this approximation we have and 4 theta by gamma plus 1. So, this is where is the slope of the drag versus Mach number curve near M infinity equal to 1? Now, if you introduce the transonic similarity parameter, that modifier pressure coefficient corresponding to transonic flow gamma plus 1 M infinity squared to the power 1 by 3 by theta, C D star is C D into gamma plus 1 M infinity squared to the power 1 by 3 theta to be power 2 by 3 and this is theta to the power 5 by 3 and the transonic similarity parameter that is M infinity squared minus 1 divided by gamma plus

1 into M infinity squared into theta to the power 2 by 3. So, if these definitions are introduced here; if these are introduced in these two relations.

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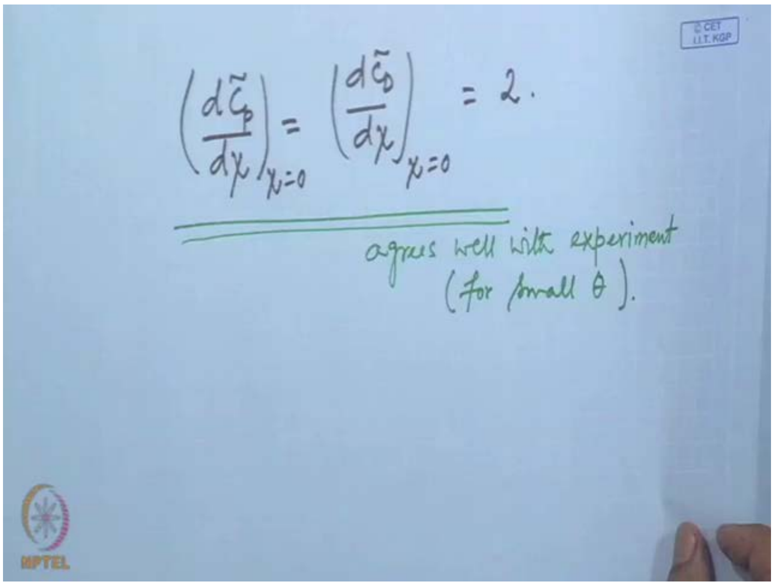


We get 2 so that, the modified drag coefficient in terms of the transonic similarity parameter, has a slope of 2 at the sonic point; free stream sonic point of course. And it found that, this result agrees well with experiment for small theta. Say of, theta is of the order of 10 to 15 degree, this result agrees quite well.

Meaning of for a wedge of 20 to 30 degree vertex angle, this result compares very well with experimental observation. And of course, that validates the transonic similarity theory and as well as, transonic small perturbation theory. Now once, after this discussion; we will now, have a brief discussion of transonic flow, about airfoils which is of course, a practical interest and perhaps, the most important as far as the aerodynamics are concerned.

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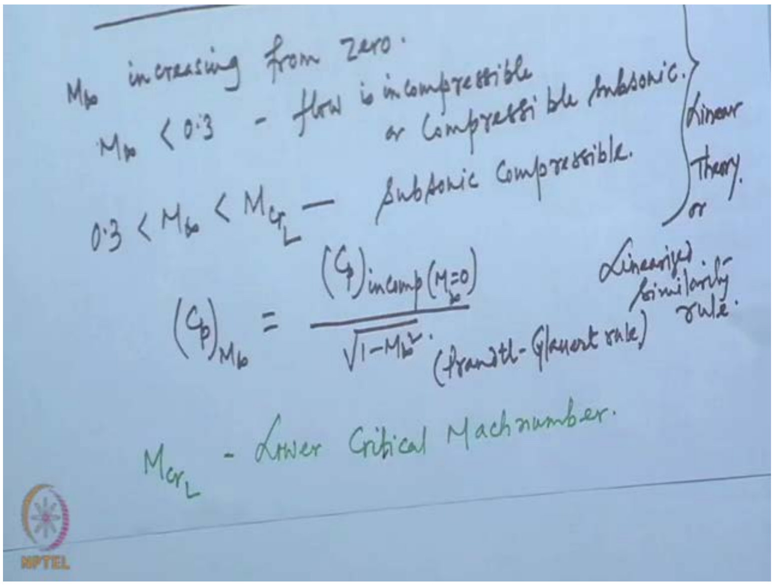


So, let us now, consider transonic flow about airfoils. Again, let us consider that, M infinity increasing from zero, initially up to certain Mach number; let say, as a thumb rule, which we have taken as that M infinity less than point three, this flow is incompressible and we know, what is the flow? Of course, they can be treated as compressible subsonic, then say; we will call this M critical subsonic compressible and both can be used by both can analyzed by linearized theory linear theory or even, linear similarity rule; meaning if, we think in terms of prandtl-glauert rule.

Then once we know, the pressure distribution on the airfoil at an incompressible case, which correspondence really to Mach number 0; free stream Mach number 0. Then of course, at any other Mach number within this range, we can get the pressure coefficient simply by the prandtl-glauert rule, which is the C p incompressible divided by square root of, so C p at M infinity is C p incompressible divided by root over 1 minus M infinity squared. When the free stream Mach number reaches M c r L, which is lower critical Mach number; which we have already defined in our last lecture, that M c r L is lower critical Mach number.

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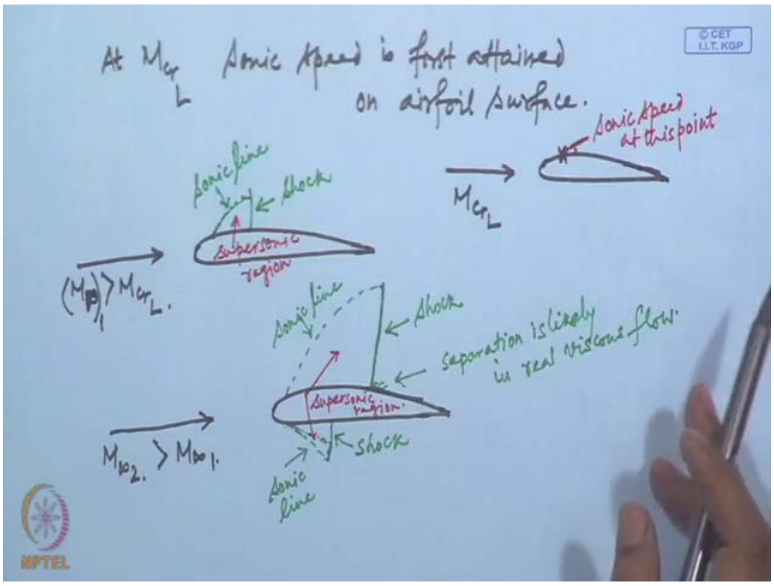


Where, sonic speed is first attained on the airfoil surface at one point. At M c r L, sonic speed is first attended on airfoil surface. This of course, inherently means that, we are considering inviscid flow; where the flow slips over the body and as we have seen earlier, the maximum velocity is reached on the surface. So if now, free stream Mach number increases beyond this. Then of course, there will be a subsonic reason on the surface of the airfoil over certain reason and as this Mach number increases, the size of the supersonic region increases and this supersonic region terminates with a shock

So let us consider, one such a free stream; remember in this discussion, we are also considering that the angle of attack is remaining constant, because this lower category Mach number depends also on the angle of attack. As this directly, effects the configuration. Let say, now this is M infinity and once again, let us denote it M infinity 1. So, there will be; so this is shock; this is a sonic line, just to show for that M sonic speed at this point. If the Mach number increase still slightly, now will have a much larger sonic region and in a viscous flow, they are might be even a separation is likely in real viscous flow.

This M infinity is of 2 is of course, larger than M infinity 1 and it is also possible to have, ((no audio 27:21 to 27:51)) this is; super these are all supersonic region. ((no audio 27:56

to 28:35)) Of course, the actual location where this shock will occur and at with Mach number, the shock will be relatively week and sonic region will be little smaller. They of course, change from airfoil to airfoil at and also, changes with angle of attacks, but qualitatively that as Mach number increases, the flow behavior becomes like this.

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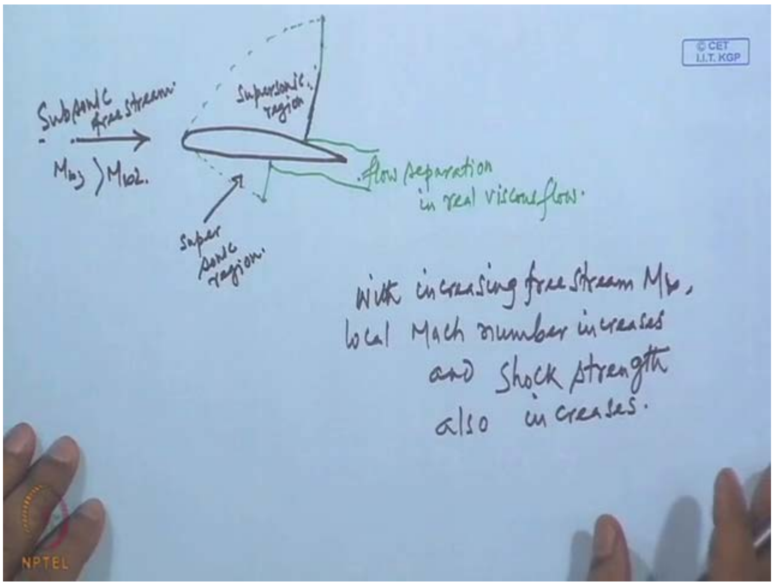


Let us now consider, free stream Mach number which is little less than one; say of the order of 0.9 or little higher then of course, the and flow separation in real viscous flow. So, in real viscous flow, there will be a strong interaction between the shock and the boundary layer and which will effects the location and strength of the shock, as well as the growth of the boundary layer. So here we see that, there is an interdependence between the shock wave and the boundary layer of course, in real viscous flow. In inviscid flow of course, that flow separation and shock boundary layer interaction will not be present.

However, as we can see that, as Mach number increases; the supersonic region on the upper surface, as well as on the lower surface increases. In all these case, subsonic free stream as we mentioned that, this corresponds to a case where Mach number may be of the order of 0.9 or so, and so you see that, as the Mach number goes on increasing; free stream Mach number goes on increasing, the supersonic region also increases and consequently, the local Mach number also increases and the shocks become stronger. So, with increasing free stream Mach number and shock strength also increases, because if, we recall that shock strength for normal shock is M 1 squared minus 1. So since, the upstream Mach ahead of the shock, the Mach number is increase.

Now, compared to the earlier cases, the shock is much strong here. So as this subsonic free stream Mach number increases towards unity, the shock strength goes on increasing. Now, let us consider two cases, which are just marginally less than so if, we go on increasing this Mach number, when the Mach number comes very close to unity? Then, the shock moves downstream, to say near about the trailing edge. Let us say two cases, if, we consider two such cases, very close to unity one subsonic and other supersonic.

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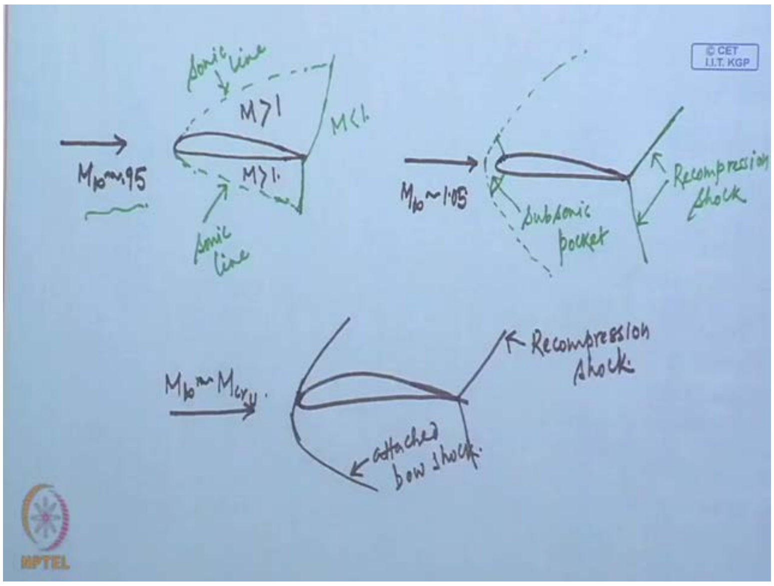


Let us say, this is M infinity; say near about 0.95 then the flow, where the airfoil is all most entirely supersonic? Of course, this number has no real significant; it simply implies that, Mach number is very close to unity. It of course, depends on configuration to configuration that is, the particular geometry of the airfoil and the angle of attack

Similarly, let us say another case, where again Mach number is very close to unity? In this case, as we know that, there will be a bow shock wave with a subsonic pocket. These are the sonic line ((no audio 37:49 to 38:26)) and these are recompression shock. Further, higher Mach number see if, we have further higher Mach number then and of course, a recompression shock. This is M infinity, still M critical upper. So, we have briefly describe, the flow behavior or flow pattern in the transonic regime and talked about different type of shock appearance at different Mach numbers and particularly, this have a tremendous effect on the drag coefficient.

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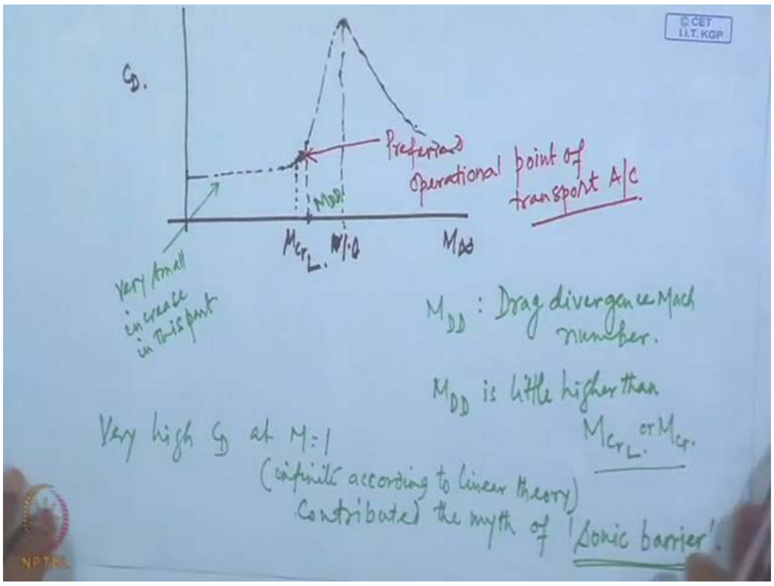


Let us say, if we plot drag coefficient that is; it is Mach number say, nearly up to critical Mach number, very hardly any effect of increasing Mach number the drag coefficient increases marginally. In this part however, after the Mach number has cross the lower critical Mach number then, shock wave appears on the airfoil and drag started increasing very rapidly and ((no audio 43:45 to 44:16)) this is called M D D. M D D is drag divergence Mach number, which of course is, little higher than critical Mach number; little higher than.

As this quite obvious, that the amongst the two critical Mach number, the lower critical Mach number is perhaps, more important as in terms of design or for aviation and hence, usually the critical Mach number is refer to lower critical Mach number. As we mentioned that, at M lower critical Mach number, the sonic point is first reach and after that as Mach number increases, the there is a supersonic sonic pocket on the airfoil, terminated by a normal shock. So, when the Mach number is increased little bit from lower critical Mach number or critical Mach number then, the shock appears and once, the shock appears that contributes the rapid increase in drag, this and also, we have seen that, as now Mach number increases the strength of the shock increases considerably and of course, the drag rise is quite first and this high value of drag so of course, which we earlier mentioned that according to linearized theory, this would have been infinite.

So, very high C D at infinite according to linear theory. This contributed to the myth of sonic barrier this contributed the myth of sonic barrier. Now of course, aircrafts which wants to operate at high speed, but without going supersonic; that is without paying the heavy price of drag rise and fuel price; that is for the transport aircraft. The most preferable point of operation is here; this is the preferred operational point of transport aircrafts that is getting the benefit of high speed, but without paying the penalty of heavy drag or high drag.

So; this critical; so we have discussed about the flow over airfoil in the transonic range and we have seen, how the supersonic region appears first in the upper surface? Then consequently on the Then subsequently on the lower surface, as well and then, this shock moves downstream towards the trailing edge point and the Mach number reaches very close to unity and downstream of these trailing edge shock, the flow again become subsonic. Even in this case also, this region here is a subsonic region in these cases as well.

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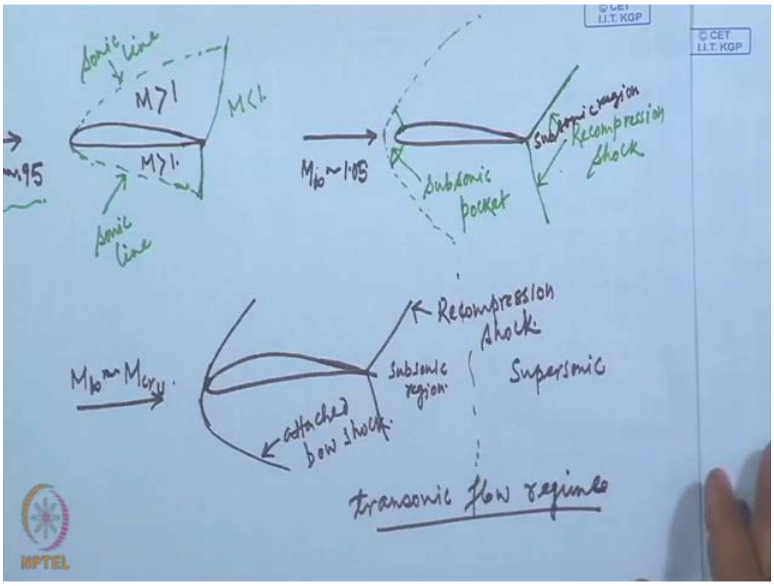


A subsonic region is present here and here also and then of course, a little downstream; the flow become as the free streaming supersonic, that little downstream in these cases also, the flow becomes supersonic somewhere here and so these also belongs to that transonic flow regime. We also have indicated a possible shock boundary layer interaction and separation and rapid growth in boundary layer with thickness, for a real viscous flow and with this, we have discussed how the rapid drag rise.

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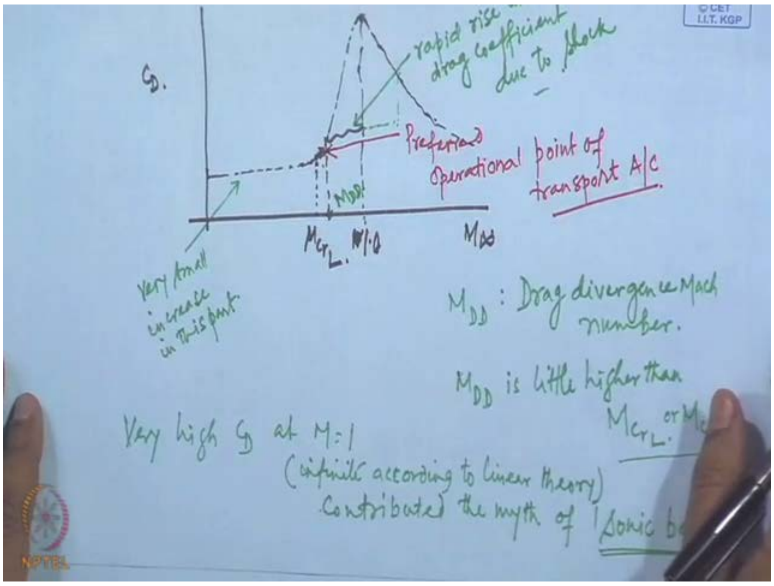


So in this case, rapid rise in drag coefficient in to shock and we also have defined, what is the divergence drag? Divergence Mach number, where the shock wave becomes started contributing largely to the drag coefficient, that becomes the drag divergence Mach number. Now, it a very important to particularly, estimate this lower critical Mach number, because it is that more or less fixes the preferred operational point or cruising speed of a particular aircraft.

Of course, as we have mentioned earlier, that the lower critical Mach number changes with the airfoil shape and airfoil angle of attack. So, we will briefly describe, how we can have a preliminary approximate estimate of lower critical Mach number? And anyway we will consider that, in our next lecture the estimation of lower critical Mach number. So with this, we conclude; so in our next lecture, we will discuss about lower estimation of lower critical Mach number approximately.