High Speed Aerodynamics
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Lecture No. # 38
Transonic Flow

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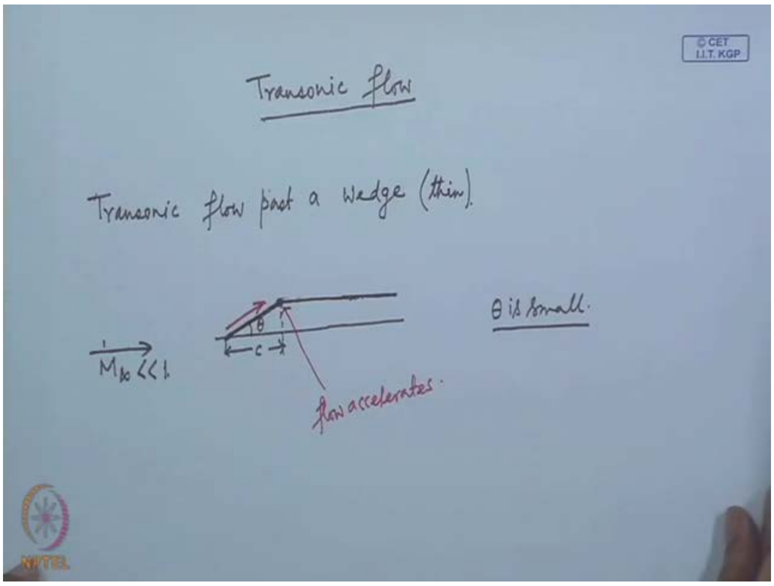

Our next topic of discussion will be transonic flow. Of course, we have already discussed a little bit about transonic flows, in particular we have derived the small perturbation equation for transonic flow. We have also developed the similarity rule for transonic small perturbation equation and we have seen that even for small disturbance the transonic flow equations are non-linear, and close form analytic solutions are not readily available. So, mostly our discussion on transonic flow will be qualitative. We will try to develop the basic physical feature of the flow without going much into the mathematics or developing a solution, trying to develop a solution; most of the solutions are, of course, numerical.

Now, if we look to the similarity rule, that we have derived, that as Mach number increases for a certain geometry, the pressure coefficient goes on increasing and as Mach number approaches 1, it is extremely large and even the pressure coefficient at the point may become infinite, that is what the linearized similarity rule suggests for a given geometry. Of course, the transonic similarity rule, which is actually valid for transonic flow does not predict this infinite pressure coefficient, but predicts a large pressure as a consequence. The drag force also becomes large and again, the linearized theory predicts an infinite drag at Mach number 1 and very large drag coefficient near Mach number unity. Eventually, this is the reason, that in the earlier days of flight, there are the myths of sonic barrier. However, if we use transonic similarity rule, it shows the very large drag, but not of course infinite large drag, which is surmountable, but not impossible to surmount. Now, we have in, within the context of small perturbation flow we have developed a transonic similarity parameter and also mentioned, that if the transonic similarity parameter lies within the range minus 1 to plus 1, the flow becomes, is called transonic.

Now, what are basically these transonic flows? Earlier we have simply said, that when the flow is, when the free stream is close to unity the flow is transonic and also, for transonic similarity parameter case, that if the transonic similarity parameter lies between minus 1 to plus 1, the flow is transonic. But what exactly it means? What are basically the transonic flows? Transonic flows are basically a mixed type of flow where both subsonic and supersonic flow exist simultaneously, that is, transonic flow maybe called, transonic flow is a subsonic region embedded in a supersonic flow or a small supersonic region embedded in otherwise subsonic flow, that is, both subsonic and supersonic flow exist together, it is a mixed flow field. Whenever there is a supersonic range region, the region is usually terminated by a normal shock and which is the region, that of large drag and also many other features associated with transonic flow.

Now, when both subsonic and supersonic flow exist together, we know, that the governing equation for subsonic case and supersonic flow case changes its nature, at least for the small perturbation case we have seen, and which of course, is also true for the full governing equation, that is, Euler's equations, that in the subsonic region the nature of the equation is elliptic, that is, the super, subsonic region, the governing equation, governing Euler's equations are elliptic partial differential equation, while in the supersonic range the governing Euler equations are hyperbolic in nature. The requirement of these two types of partial differential equation is completely different.

As we have mentioned earlier, that in case of a subsonic flow when the equations are elliptic, the elliptic partial differential equation needs boundary condition to be satisfied at all the boundaries. There cannot be any discontinuity, neither in the dependent variables nor in their derivatives, while in case of a supersonic flow or when the equations are hyperbolic, there are certain directions known as the characteristic direction, which in case of a supersonic flow we have seen.

The Mach lines are those characteristic direction and across this Mach lines or across these characteristic lines, the dependent flow variable themselves are continuous, but their normal derivatives are usually discontinuous, that is, hyperbolic p d is at least, allows a limited type of discontinuity. There is discontinuity in the normal derivative of the dependent variables across these Mach lines. In addition, of course, earlier we have seen of shock wave when there is jump conditions, again a limited discontinuity in, in the flow variables themselves.

So, this hyperbolic partial differential equation, they allow this type of discontinuity and also, as we have seen in many cases, that boundary conditions are not necessary in all the boundaries. The characteristic line, which originates from the wall, they only supply the information, provide the information and boundary condition will be required on the wall, but the characteristic lines, which originate at the infinity, they do not bring any or convey any perturbation and those boundary conditions are usually not required. So, the requirement for solution of hyperbolic partial differential equation and elliptic partial differential equation are quite different, and since in a transonic flow both these regions are present, that in some region the flow is elliptic or the governing equations are elliptic and in some other region the governing equations are hyperbolic, which a practical problem in mathematical solution of the equations.

Also, in real flow, while the flow is viscous the shock wave boundary layer interaction is also an essential part of all transonic flow and their interaction have considerable effect on the strength and position of the shock, as well as, the growth of the boundary layer, flow separation and so on. So, solving transonic flow is extremely complicated and as we have already mentioned, that the solutions are usually numerical. So, what at best we can do is discuss about the physical features or general nature of transonic flow over some simple boundary.

Now, let us first of all consider transonic flow past a wedge, let us consider transonic flow past a wedge, of course we will consider a very thin wedge or and let us say, that this is followed by a straight section, we will just consider half of the wedge. So, thin wedge means theta is small. Now, what happens, you know, let us consider first a flow, which is much smaller than 1. Consider a free stream m infinity, which is much smaller than 1. We know the flow will accelerate over this wedge, we will reach the highest velocity here let us say, flow accelerates over this part. Now, consequently, as we go on increasing the Mach number, a situation arises when the Mach number reaches here unity and then, there will be a supersonic region behind the shoulder, which will be terminated by a normal shock.

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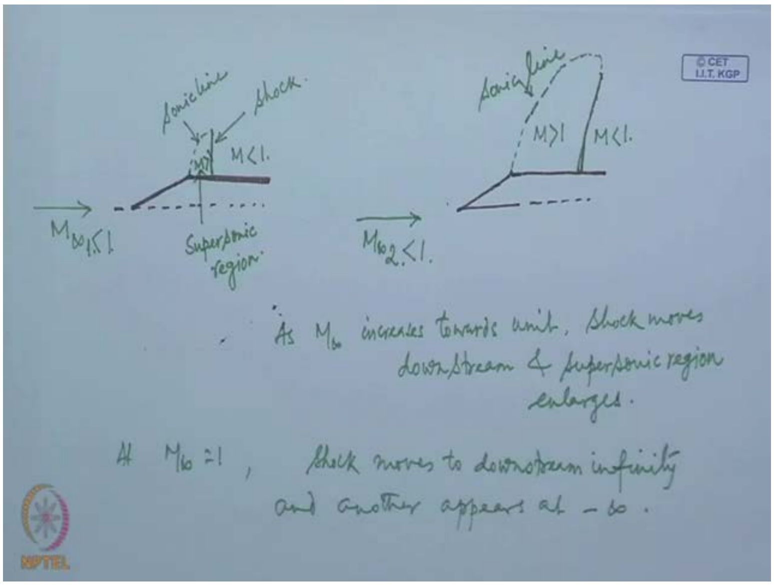

So let us say, at…

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Now, consider these cases where here, let us say we have free stream Mach number, M infinity 1 and (no audio 14:22 to 14:39) this is the sonic line, that is, at all the points, which lie on this line, the flow velocity is sonic. This is a supersonic region, this is a supersonic region and again, here it is subsonic. This supersonic pocket is terminated by a shock, which is almost a normal shock.

Now, let us consider another free stream Mach number, which is higher than this first free stream Mach number. Let us again call it M infinity 2. Again, in this region the flow will become…

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In other way, that as the free stream Mach number now goes on increasing, but they are still less than 1. Subsonic, the free stream Mach number is still subsonic and however, they are increasing and we have seen, that as the free stream Mach number increases, the size of the supersonic region increases. And similarly, as we go on increasing this Mach number, the free stream Mach number, this shock goes downstream, shock moves downstream and the supersonic region increases. So, as increases towards unity shock moves downstream and supersonic region become larger and larger.

Now, when Mach number, the free stream Mach number reaches unity, this shock moves to downstream infinity. However, simultaneously, another shock appears at minus infinity, so as shock moves to downstream infinity and another appears at minus infinity. Now, let us say further increase of m infinity, that is, the free stream is now supersonic and is going away from 1. This bow shock, that has first appeared at minus infinity will now move towards the body.

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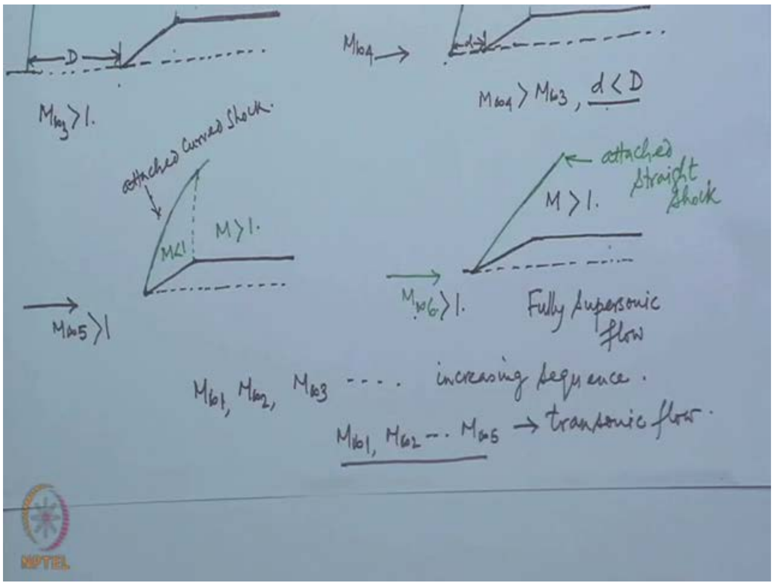

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So, we will call it M infinity 3, the detached bow shock there and the sonic line they are behind the detached shock. The flow here is subsonic and here, the flow is supersonic. This is once again the sonic line, detached bow shock, if the free stream Mach number is increased further

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We have here M infinity 4 greater than M infinity 3 and d is smaller than D, that is, the shock approaches the body, however still detached, and there is a subsonic region here. If the free stream Mach number is further increased one stage, it will come where the shock will, shock will attach the nose of the wedge, however we still remain curved. Let us call that M infinity 5. The shock has now attached to the nose, that is, this detachment distance has become 0, but it is still little curved. This is the region where M is less than 1 and in this region M is greater than 1. So, this is now an attached curve shock and then if we increase it further…

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So, this is now fully supersonic flow attached state shock; all these are greater than 1.

So, this is actually the solution that corresponds to the earlier shock wave angle, that we have discussed in case of oblique shock, that corresponding to a particular theta there is a minimum Mach number at which an attached oblique shock is possible. So, this corresponds to that attached oblique shock. So, this is a fully supersonic flow and all the other sequence, that is, M infinity 1, 2, 3, M infinity 1, M infinity 2, M infinity 3, M infinity 4 and M infinity 5, they represent all transonic flow. So, we will mention here, that M infinity 1, M infinity 2, M infinity 3 is an increasing sequence, that is, M infinity 2 is higher than M infinity 1, M infinity 3 is higher than M infinity 2, M infinity 4 is higher than M infinity 3, and so on. And this flow is transonic for all M infinity 1, M infinity 2, M infinity 3.

The flow corresponding to these free stream Mach number are transonic, so the transonic flow is possible for free stream Mach number less than unity, as well as, free stream Mach number higher than unity, as long as, if we consider it is a slender, very slender cone. Then, as long as, that transonic similarity parameter lies between minus 1 to plus 1 or in other general case, as we have mentioned, that the transonic flow is a mixed flow containing both subsonic and supersonic flow together. And also, as we have seen in this qualitative description of flow over a wedge with a state after body, that there is a minimum Mach number subsonic at which the transonic region starts and again maximum supersonic Mach number at which the transonic regime, say ends.

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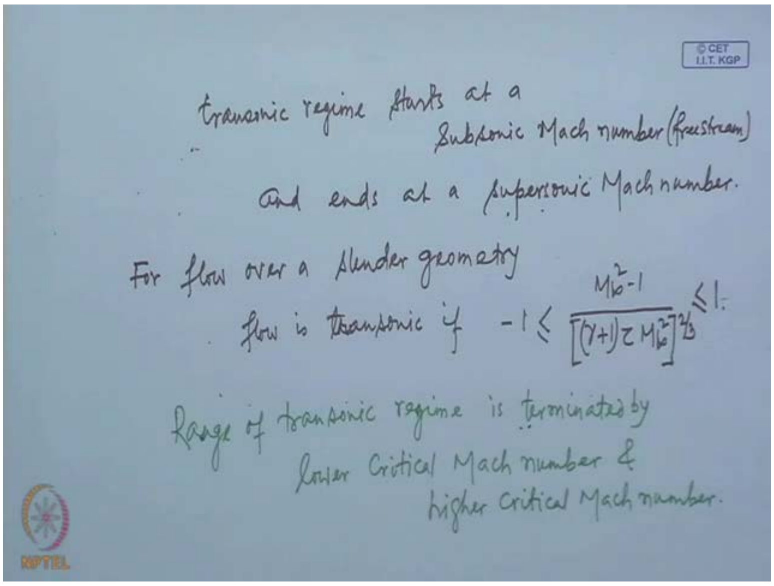

So, we have, so transonic region starts at, at a subsonic Mach number free stream, subsonic free stream Mach number and ends at a supersonic free stream Mach number. For small perturbation this is, for flow over a slender geometry, for flow over a slender geometry, flow is transonic if minus 1 less than equal to M infinity square minus 1 by gamma plus 1 into tau M infinity square to the power 2 by 3 less than equal to 1.

So, see, that two, two ends of this regime is characterized by the value of minus 1 and plus 1 for this transonic parameter and these are, these Mach numbers are called the lower critical Mach number and the higher critical Mach number. Range of transonic regime is terminated by lower critical Mach number and higher critical Mach number, that is, the transonic regime starts at the lower critical Mach number and ends at the higher critical Mach number. Based on this definition, the lower critical Mach number corresponds to minus 1 and the higher critical Mach number corresponds to plus 1. And considering the general case, the lower critical Mach number is, Mach, free system Mach number at which the highest local Mach number over the body is unity.

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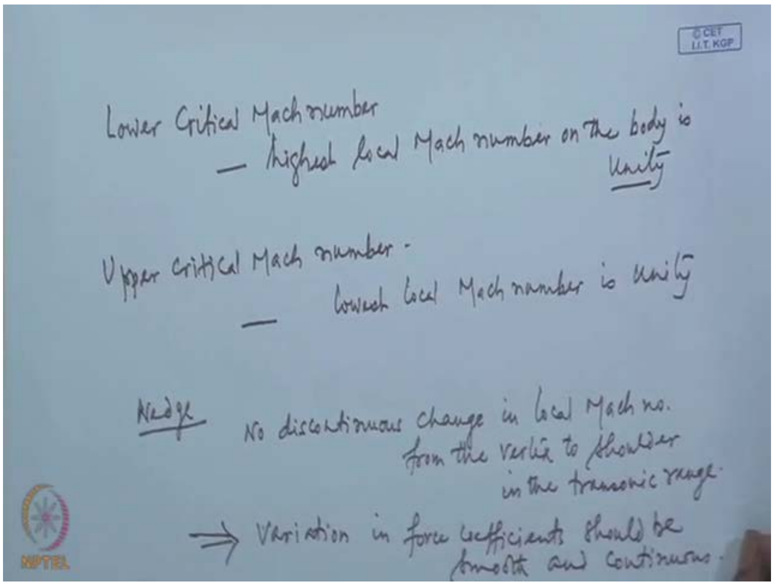

So, we can lower critical Mach number at which the highest local Mach number on a body, on the body is unity, highest local Mach number on the body. Similarly, the upper critical Mach number is, at which lowest local Mach number is unity, that is, in case of the lower critical Mach number, there is only one point in the flow field, which is sonic and in the upper critical Mach number, there is only one point, again in the free stream, which is for the lowest velocity is sonic. And since the transonic flow is, regime is very, very important, particularly in case of aviation industry where most of the, or all the commercial airliners, they fly in the transonic range, so estimation of lower critical Mach number is very, very important for design of airfoil and wings.

Now, again going back to our discussion on the flow over the wedge, what we see that going back to the wedge flow, that there is no discontinuity in the local Mach number distribution from the vertex to the shoulder. So, let us go back to that wedge, what we have observed that there is no discontinuous change in the local Mach number distribution; no discontinuous change in local Mach number from the vertex to shoulder over the entire transonic range. And the changes from subsonic to supersonic, flow is smooth and continuous where we have seen, that up to M infinity 5 the flow over this wedge remains subsonic and at, where Mach number for M infinity 6, the flow over this wedge has become supersonic in this part. So, this change from subsonic to supersonic over this wedge surface is smooth and continuous, there is no discontinuity over this entire Mach number range, say from M infinity 1 to M infinity 5.

Now, if the drag is plotted in this Mach number, this of course, shows a very rapid increase towards infinity. Now, from the basic nature of the flow, since this change over from subsonic to supersonic is smooth, so the variation of, in the drag should also be smooth. This implies variation in forces or force coefficient should be smooth and continuous.

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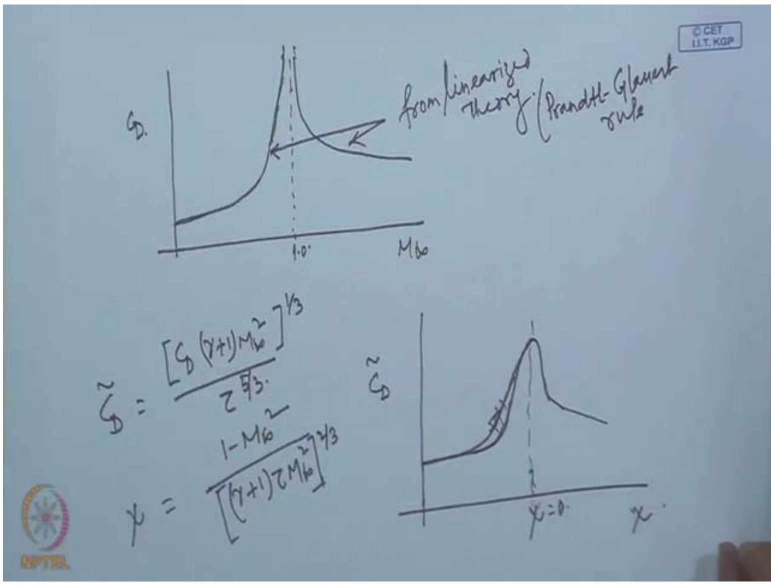

However, if either C D or C L is plotted against Mach number, then we find a discontinuous change, that is, let us say, consider C D.

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This is according to a linearized theory, that is, say as an example, as an example say Prandtl-Glauert rule. However, if the transonic similarity rule is used, if transonic similarity rule is used, then and, and if you remember, that this is what we defined as C D into gamma plus 1 M infinity square to the power 1 by 3 by tau to the power 5 by 3 and the parameter chi is 1 minus M infinity square by…

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That there is no such discontinuity or anything when it is plotted. So, these are basically the appropriate variables that have to be used for the transonic range. And now, let us talk about certain features that of wedge flow.

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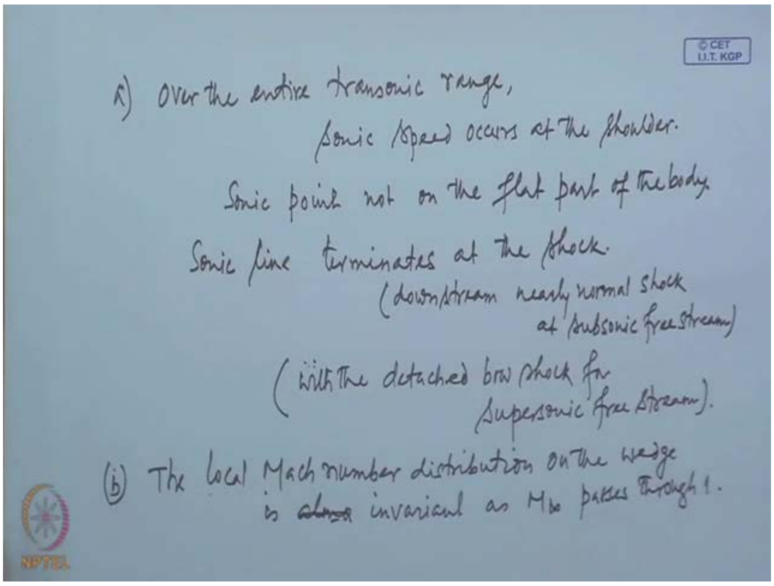

What we have seen, that first of all, that over the entire transonic range, the sonic velocity is, occurs at the shoulder. So, what we have seen that let us say a, over the transonic range, over the entire transonic range sonic speed occurs at the shoulder, that is, sonic speed is not reached on any flat part of the body and this, in case of a subsonic free stream, the sonic line is attached to the nearly normal shock downstream. And in case of a supersonic free stream, this sonic line is attached to the bow shock in front or even in the end of the super transonic range; it is attached to the curved leading edge shock.

So, sonic line, sonic point, not on the flat plate, not on the flat part, flat part of the body, body, sonic line terminates at the shock, which is downstream, nearly normal shock at subsonic free stream and with the detached bow shock for supersonic free stream. Another very important observation, that the local Mach number distribution is almost invariant as the flow passes through unity, that is, the local Mach number distribution, local Mach number distribution on the wedge, on the wedge, is almost invariant, is invariant as M infinity passes through 1. That is, when M infinity is very close to 1, either subsonic or supersonic, the Mach number distribution on the wedge part will remain same or the flow velocity is almost constant when the Mach number is very close, very, very close to unity. So, this can, this can of course, which shows very easily using a little bit of mathematics.

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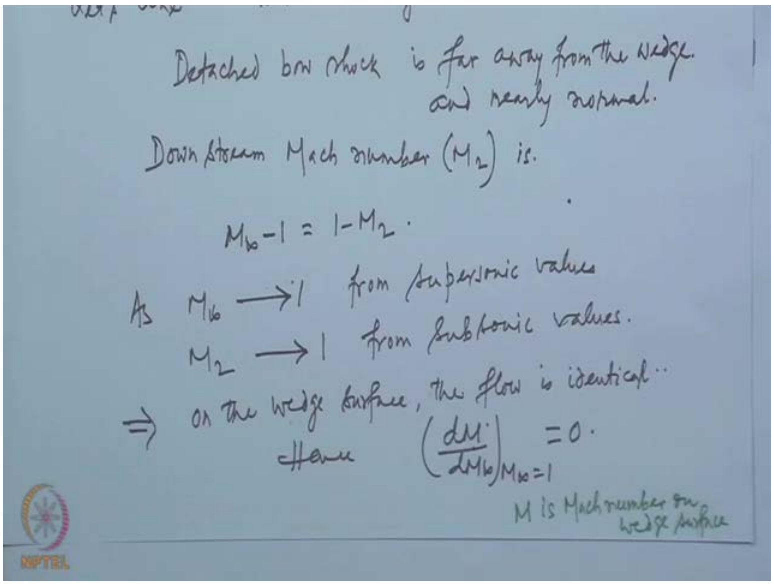

Say, first of all let us take, let us take M infinity, little larger than M infinity, little larger than, littler larger than 1. In that case, the detached bow shock will be far away, so detached bow shock is far away from the wedge and nearly normal. So, the downstream of the shock, the flow is subsonic. So, downstream Mach number, which we always call M 2, downstream Mach number M 2, this will be given by, is 1 minus M 2.

So, what we see here, that as M infinity approaches to 1 from supersonic, so as M infinity approaches to 1 from supersonic values, M 2 approaches 1 from subsonic values. That is, say as an example, if we have M infinity 1.02, the M 2 is 0.98. So, as we see, that now the wedge is facing free stream at Mach 0.98, so whether the free stream is at 1.02 or 0.98, the wedge is actually experiencing the same flow and so, on the wedge surface the flow is identical. And mathematically we can say, d M, where M is the Mach number on the surface of the wedge; M is Mach number on the surface of the wedge, Mach number on wedge.

This is of course, based on the qualitative description and of course, also supported by physical observation in different experiment, that this is to, that when free stream Mach number is very close to 1, whether it is subsonic or supersonic, the flow on the wedge surface is identical. This simple result, of course, can be used to obtain some other important relation as regard to flow over a wedge particularly. We can even show, that the pressure coefficient or force coefficient are also almost invariant, very close to M infinity equal to 1.

And this, of course, we will continue in the next class and we will conclude this discussion with, that we have qualitatively analyzed the flow over thin wedge in the transonic range. We have clearly defined the lower critical and upper critical Mach number and we have also discussed briefly some of the very basic physical nature of transonic flow.