High Speed Aerodynamics

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Lecture No. # 30
Linearized flow problems (Contd.)

We saw that, to solve linearized supersonic flow problems we can have a potential function, which is analogous to the potential function for subsonic case.

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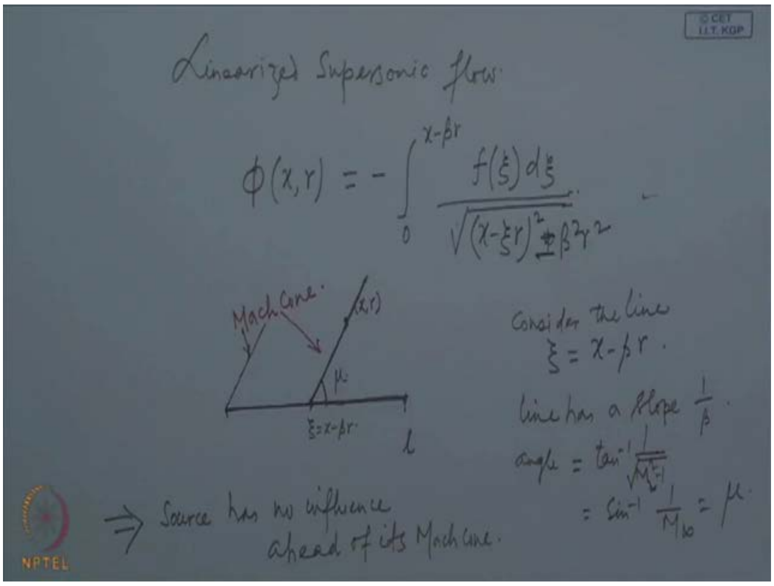

However, we also saw that, this potential is defined over a source distribution between 0 to x minus beta r. So, this is what we got is in the last class that, if we want to use that basic solution at a supersonic potential over a body, then the potential at any point must be given by the source distribution from 0 to x minus beta r, but not for 0 to l sorry.

What simply means that, we may have a source distribution from 0 to l that is the length of the body. However, the potential at any point x r will be given by the source distribution, which is distributed or spread over 0 to x minus beta r, the remaining portion of the source distribution x minus beta r to l, that does not influence the potential or condition at x r.

What it means that? Let us say, this is the axis of the body from 0 to l, the source may be distributed over the entire length l; however, for a potential at point x r (No audio from 02:53 to 03:20) that is the potential at this point is obtained or influenced by the source distribution over this part of the body.

However, this part of the body or this part of the source distribution does not influence or have no contribution to the potential at this point (Refer Slide Time: 03:41). Now, if we consider this Xi equal to x minus beta r, this line consider the line, now this line has a slope (No audio from 04:23 to 04:50) and the angle is, and this angle is tan inverse 1 by M square minus 1.

This is say M infinity (Refer Slide Time: 05:12), sine inverse 1 by M infinity, which we have earlier defined as the characteristic angle mu. So, this angle is happens to be mu and so this is the mu mach line form this point (Refer Slide Time: 05:28). And if we consider the complete axisymmetric body, then this defines a mach cone from this point (Refer Slide Time: 05:36). So, this line defines the mach cone from this point.

So, this solution has the interpretation that, the source distribution has no influence ahead of its mach cone. So, you can say that, the solution has the interpretation that, source has no influence ahead of its mach cone. Let us say, this is the mach cone from the leading edge or mach cone from the origin. We should see (( )) difference with the linearized two dimensional flow problem that, in linearized two dimensional problem we have seen that, the disturbance has no effect either upstream or downstream of its mach line, the effect is confined to the mach lines. However, in this axially symmetric case, there is effect over the whole region downstream of the mach cone.

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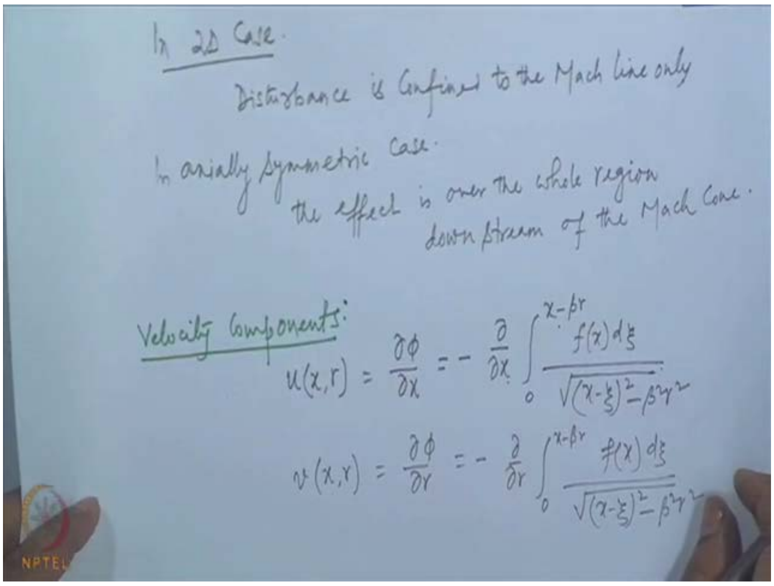

So, we see that, in 2D case disturbance is confined to the mach line only. In axially symmetric case (No audio from 08:39 to 08:53), the effect is over the whole region (No audio from 08:57 to 09:12) downstream of the mach cone, that is disturbance created by any part of the source is confined downstream of its mach cone; but, the disturbance has no influence ahead of the mach cone or this has a limited upstream principle.

Now, this part of the solution we obtained mathematically and this is the physical interpretation of this mathematical solution. And we clearly see that, this is what is required in case of a supersonic flow as we have discussed earlier that, the supersonic governing equation, even for linearized perturbation problem is hyperbolic, which has characteristic reduction; and which represent propagation problem and for which boundary condition at infinity boundary is not required.

And as you have seen in many other solutions, which you have earlier obtained for one dimensional flow cases and also for that two dimensional flow cases that, there is a limited upstream influence, in case of a supersonic flow. A disturbance at one point does not affect the flow ahead of it. And so we see that, this solution or this function needs to be satisfy this condition, if it has to be a solution and (( )).

Now, we will try to evaluate this integral; however, to find the velocity velocity component to find the velocity components (No audio from 11:43 to 11:57) or the perturbation velocity component to be precise we have, u x r is d phi d x (No audio from

12:13 to 12:37) root over x minus Xi square minus beta square r square. And similarly, v x r is d phi d r (No audio from 12:56 to 13:23), now this evaluation of this integral and its special attention first of all that, the upper limit is a variable. It is a function of x consequently, this integration is to be carried out using Leibniz rule and also (( )) see that, at the upper limit the integrand is singular.

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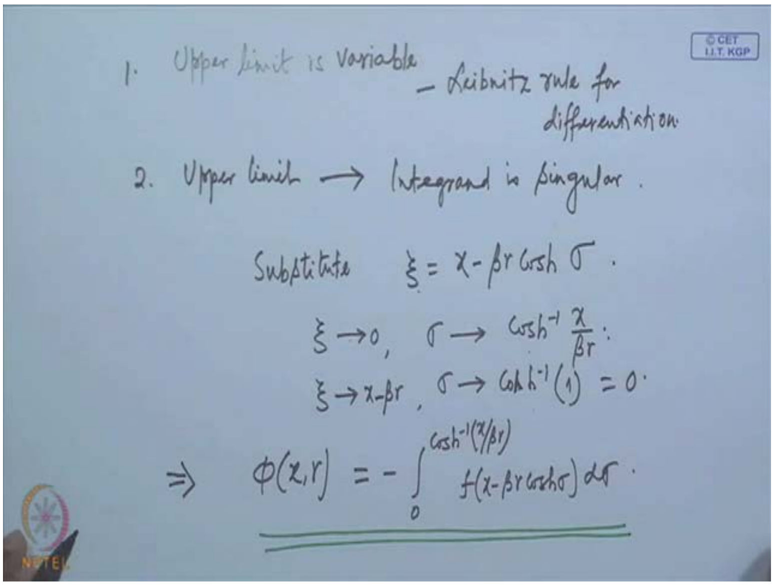

So, we have (No audio from 14:05 to 14:15) upper limit is variable, which needs to use Leibnitz rule Leibnitz rule for differentiation. Also we have at upper limit, the integrand is singular. However, in this particular case, integration can be carried out by a substitution, substitute Xi equal to x minus beta r cos hyperbolic sigma. Then, when Xi is 0, when Xi is 0 sigma goes to cos hyperbolic inverse x by beta r.

When Xi is x minus beta r, sigma is cos hyperbolic inverse 1, which is 0 (( )) the substitution of this (Refer Slide Time: 17:01), in place of Xi result in change in the limits to lower limit to cos hyperbolic x by beta r and upper limit to 0. And if we substitute we see that, this phi x r now becomes cos hyperbolic inverse x by beta r into of f x minus beta r cos hyperbolic sigma d sigma.

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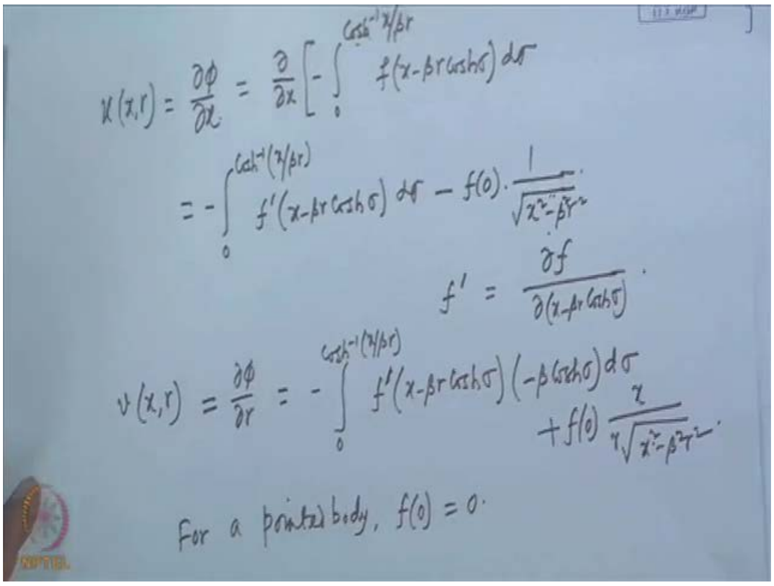

So, this is the potential function in the transformed coordinate. And this now, we can differentiate u x r now becomes (No audio from 18:40 to 19:12) f of x minus beta r cos hyperbolic sigma d sigma; and this gives (No audio from 19:29 to 19:40), cos hyperbolic inverse x beta r f prime x minus beta r cos hyperbolic sigma d sigma, where f prime is derivative of f with respect to its argument that is with respect to x minus beta r cos hyperbolic sigma; and term comes, because of the variable upper limit, which is f 0 1 by root over x square minus beta square r square.

This term comes due to the application of Leibnitz rule for differentiation under integration with variable limit (Refer Slide Time: 20:47), in this case only the upper limit is variable; and the term comes as the function at the lower limit and derivative of the upper limit, so this is it. And as we mention, this f prime here denotes differentiated with respect to x minus beta r cos hyperbolic sigma.

Similarly this, the radial component of velocity becomes (No audio from 21:47 to 21:58) 0 to cos hyperbolic inverse x by beta r f prime x minus beta r cos hyperbolic sigma into minus beta cos hyperbolic sigma d sigma plus f 0 x by r root over x square minus beta square r square. This happens to be the derivative of the upper limit with respect to r (Refer Slide Time: 22:41). While this is a derivative of the upper limit with respect to x (Refer Slide Time: 22:47).

Now, we will see that, f 0 which represents the value of the source distribution at the origin or at the tip, it becomes 0 if we have a pointed body. So, we will have a little simplified relation for u and v, (( )) you have a pointed body (No audio from 23:32 to 23:49) and once again then substituting back.

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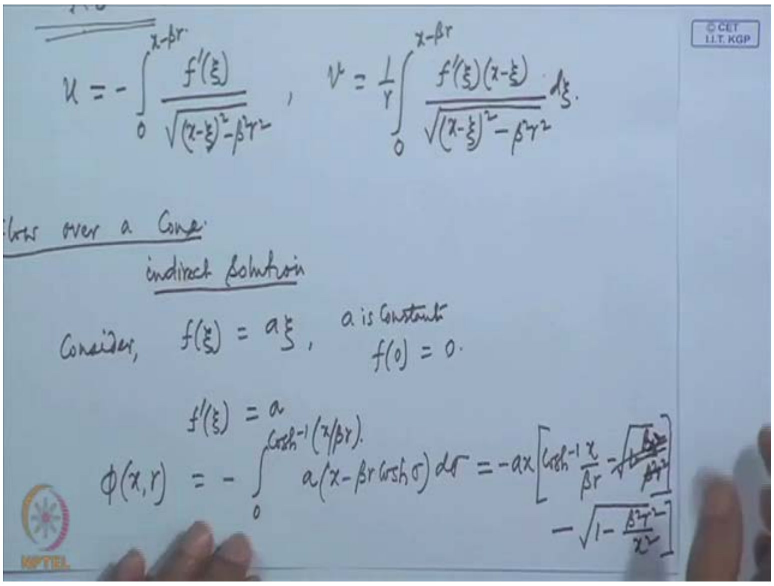

So, when we have, when f 0 equal to 0 we have if we substitute Xi again, u will become (No audio from 24:23 to 24:44), and v will become 1 by r (No audio from 24:54 to 25:29), these are the simplified relation for the two velocity components, the axial and radial, when f 0 is equal to 0 that is for a pointed body, that is body with pointed (( )), which can be represented by a source distribution with source strength 0 at the leading edge. The velocity components or the perturbation velocity component at any point x and r are given by these relations.

Now, this f 0 equal to 0 is also consistent with the incompressible flow we have seen earlier that, to model or simulate thickness we use a source distribution that is as an example the very famous example of point source in a uniform stream or a two dimensional point source in a uniform stream gives a (( )) body or (( )) body with thickness.

So to simulate flow over a body with thickness we always use source distribution. So, here also that is indicated that, if there is no thickness at the (( )), then the source strength at the (( )) are at the origin becomes 0 and these are (( )) the velocity components.

Now, directly using a source distribution and then, satisfying these the boundary conditions to evaluate a prime is usually a numerical procedure. In rare case, where the body shape can be expressed mathematically, analytic solution may be possible. And one such case is flow over a cone, so we will now consider flow over a cone. Of course, the solution is basically an indirect one that is we will assume a source distribution and then, we will show that, that particular source distribution are the flow that we get represents flow over a cone.

So, it is indirect solution like what we deal in case of incompressible flow (( )) a circular cylinder, infinite circular cylinder either lifting or non lifting. We started with a uniform stream with a point doublet at the origin and we saw that, the resulting flow represents the flow over a circular cylinder without any lift. So here also, we will have a special source distribution, which will give rise to a flow, which can be seen to be the flow over a cone.

So, let us start with f Xi equal to a Xi we consider a very simple source distribution, f Xi equal to a Xi varying linearly with Xi, a is a constant a is constant and as you can see here directly, f 0 equal to 0, the f prime Xi is simply a. Then, the potential can be written as, potential function at any point can be written as minus 0 to cos hyperbolic inverse x by beta r.

What we had is? f of Xi, which is simply a Xi and Xi is then replaced as, a x minus beta r cos hyperbolic sigma d sigma, which can be written as minus a x cos hyperbolic x by beta r minus root over 1 minus x square by beta square r square sorry one minus beta square r square by x or let us say, minus square root of 1 minus beta square r square by x square.

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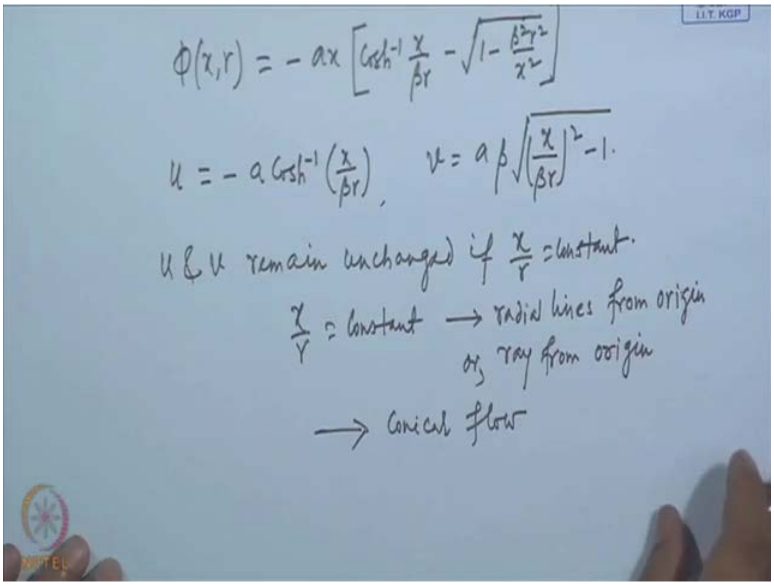

Let us write the expression once again, phi x r what we find is, minus a x cos hyperbolic inverse x by beta r minus square root of 1 minus beta square r square by x square. The velocity components can obtained directly by differentiation or also of course, using those general expressions for u and v and integrating. However, these we can write as, equal to minus a cos hyperbolic inverse x by beta r, and v equal to a beta into root over x by beta r square minus 1.

So, you can see that, both u and v are function of x by beta r. And since, beta is square root of M infinity square minus 1 is a constant, so you can see that, u and v are constant or u and v remain unaltered, if x by r is constant. Now, (( )) x by r are basically a line or radial line originating from the origin.

So, u and v remain unchanged, if x by r equal to constant. Now, x by r equal to constant, these are radial lines from origin also called ray from or ray from origin. This (( )) that, along all these radial lines from the origin, the flow velocity remain unchanged and hence, this is a conical flow hence this is this is a conical flow.

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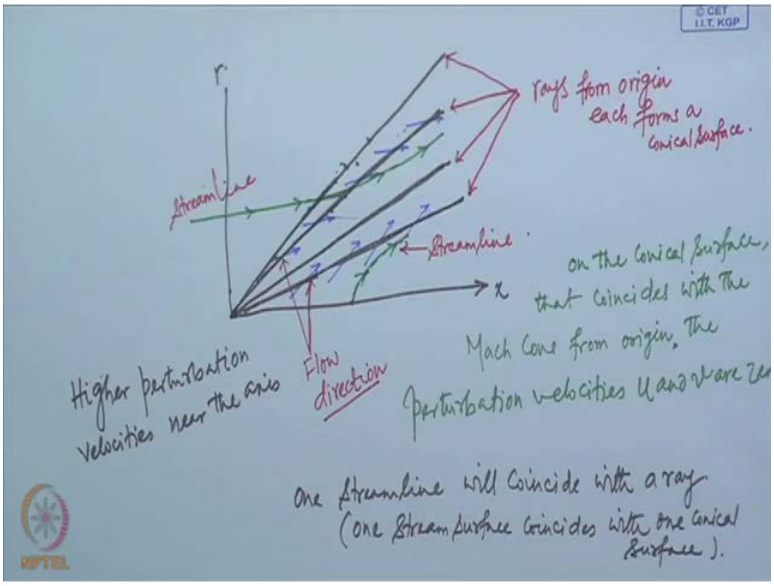

(No audio from 36:19 to 36:30) See, these are the x by r equal to constant line (Refer Slide Time: 36:32) (No audio from 36:42 to 36:57) and if we consider the complete axisymmetric body or complete axisymmetric case, so all these lines are generators and each of them form a conical surface. So, these are (No audio from 37:11 to 38:00), these are the rays from origin each forms a conical surface.

Now we we know that, a particular ray or the particular conical surface, which is the mach cone from the vertex, the perturbation velocities are 0. So, the particular on the conical surface, that coincides with the mach cone from origin, the perturbation velocity u and v are zero. Rather, the flow is still at the free stream condition the flow has the flow is still in the free stream condition.

Also looking to the expression for u and v we can say that, the higher perturbation velocities are near the axis and smaller perturbation velocities are away from the axis. You can say that, higher perturbation velocities are (No audio from 40:27 to 40:43) that is the perturbation velocities on this ray will be much higher than the perturbation velocities on this ray and so on. And assuming that, this is the particular ray (Refer Slide Time: 41:02), which coincides with the mach cone here.

So that, the flow is undisturbed until the flow expects or flow interacts with these this way. So, if we consider any particular stream line, that stream line will not change, it will remain unaltered, until the streamline intersects this line and once this is intersects after this perturbations are non zero and are gradually increasing towards the axis anyway. Since, the perturbation are non zero here (Refer Slide Time: 41:52), the streamline will change.

We will also be able to see that, that the direction of the perturbation velocity or the velocity deduction changes from one ray to the other. So, it my happen that the, let us consider first of all, one (No audio from 42:32 to 42:47) this is one (Refer Slide Time: 42:49). Let us say, this cone this represents the mach cone as well, so consequently no perturbation on this line. However, in this line (Refer Slide Time: 43:03), there are perturbation and let us say the perturbation velocities are along these direction (No audio from 43:10 to 43:44).

So, these are flow direction flow direction, this is one streamline. So, you can have another streamline also, originating from the origin (No audio from 44:40 to 45:03) also what we see that in these region, the flow direction changes. Let us say, on this ray, this is the flow direction, then on this ray, the flow direction will be this and there will be an intermediate ray somewhere between these two, on which the flow is flow direction is along the ray.

So, one streamline one streamline will coincide with a ray or for the complete axisymmetric picture that of stream surface will one stream surface will coincide with one conical surface. For the complete axisymmetric case, one stream surface coincides with one conical surface. As you know that, in an inviscid flow any streamline can be replaced by a solid wall. So, in this case, this complete conical surface can be thought of as a solid cone.

And if say assume that, this is this is that particular ray just that on this ray the flow is along the ray or that this itself is a stream surface and consequently, this the cone that this ray makes is a stream surface and we can replace this by a solid cone; so then, the flow velocity on this solid cone become tangential, which is the required boundary condition. And these streamlines, which are inside this cone is of immaterial is of no (( )).

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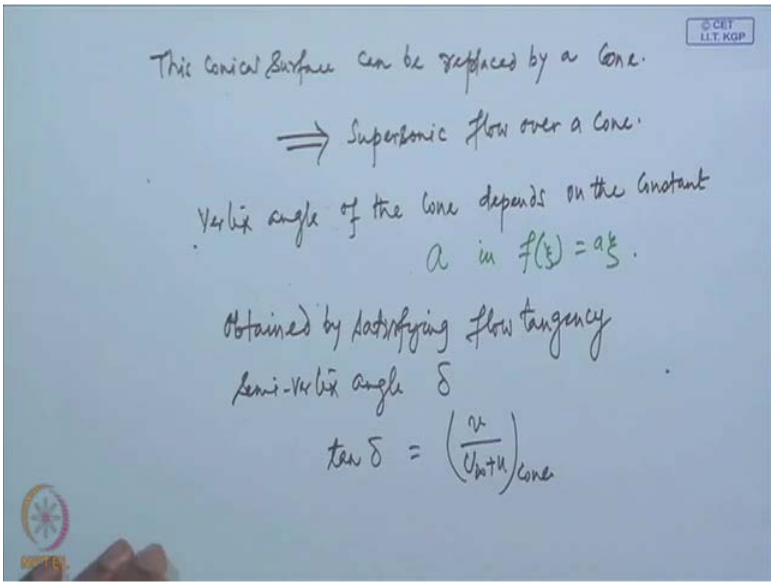

So, this cone surface this conical surface can be represented replaced by a solid cone (No audio from 48:02 to 48:13), can be replaced by a cone. And for all rays, which have r by x higher than these, then we will represent the flow field and what we get is, supersonic flow over a cone.

Now, in this problem we still have one unknown that is a, we do not know what is that value a, a is a constant. Now that, value of constant a gives us cone of different size or different semi vertex angle or vertex angle. That is the vertex angle of the cone the vertex angle of the cone depends on the constant constant a in f Xi equal to a Xi. This of course, can be obtained by satisfying the obtained by satisfying the flow tangency condition, flow tangency.

Now, let us come back to this figure once again (Refer Slide Time: 50:39), if this is the cone surface as an example, then this becomes its vertex angle and the flow is tangential to this, now we mean that, this is what is the streamline slope (Refer Slide Time: 50:55). So, if this semi vertex angle is delta, define this semi vertex angle to be delta, then we have tan delta is v by U infinity plus u cone.

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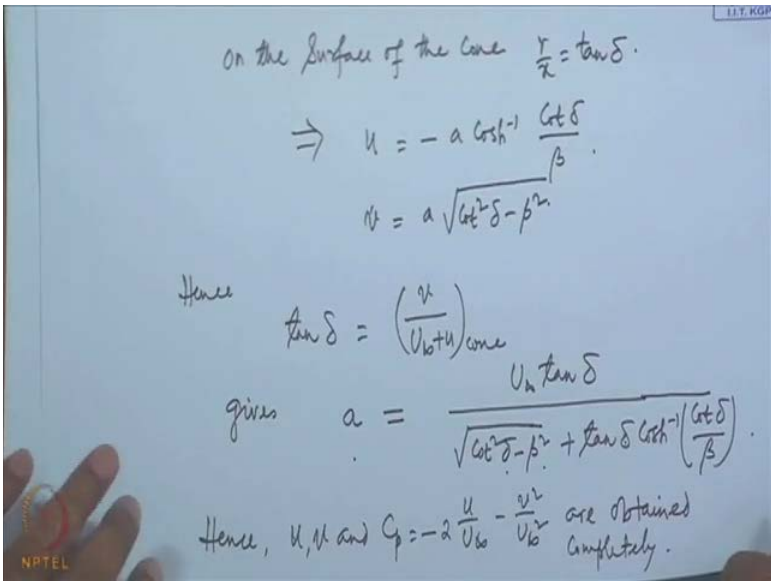

Now, on the cone surface, on the surface of the cone we have x by r is or r by x equal to tan delta, this gives us u equal to minus a cos hyperbolic inverse cot delta by beta; and v equal to a into cot square delta minus beta square. And substituting these, we get (No audio from 52:44 to 53:05) we substitute this v and u here (Refer Slide Time: 53:07), and it gives a equal to U infinity tan delta divided by cot square delta minus beta square plus tan delta cos hyperbolic inverse cot delta by beta, which of course can be evaluated, if we know the free stream speed and free stream mach number and the semi vertex of the cone angle.

So for a given cone, since delta is known and for the given flow you know U infinity and (( )) and hence be the we can find out, what would be the value of a or what would be the required source distribution to obtain the solution of the flow. And once, a is obtained u and v are obtained completely and C p equal to (No audio from 54:30 to 55:04).

So, we get the complete solution for flow over supersonic flow over a cone or rather the irrotational potential flow supersonic flow over a cone. We can see here, that this coefficient of a is not a very simple form; however, this form can be simplified or this expression can be greatly simplified, if we consider very very cylinder cone and which we will do in our next class.