High Speed Aero Dynamics

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Lecture No. # 27

Linearized Flow Problems (Contd.)

So as example example of solution for linearized flow problems, we consider subsonic flow first wavy wall and the solution gave us few important observations and the most important of them, that you found the perturbation is maximum on the wall itself, and as we move away from the wall the perturbation decreases, there all then attenuation factor we also saw that the attenuation reduces as Mach number increases. Now, we will consider this same example in Supersonic flow, that is a Supersonic flow past or wave shift wall.

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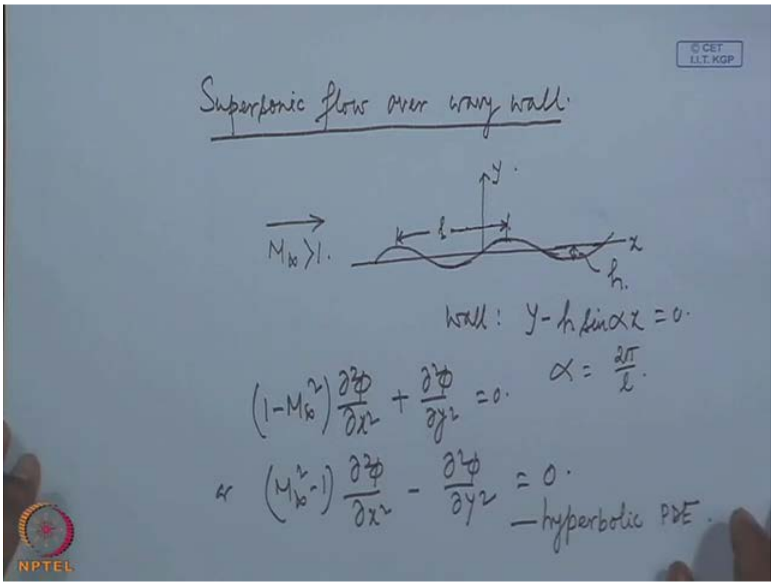

So, this is what we will be discussing today, a Supersonic flow past wavy wall the problem is essentially the same that is, we have an wave shaped wall the amplitude once again is h and length of 1 wave is once again, l that is once again the wall is given by y minus h sin alpha x equal to 0, where alpha is 2 pi by l, the governing equation in this case is once again 1minus m infinity square or since, 1 minus m infinity square is negative. We write this as m infinity square minus 1 d 2 phi dx 2 minus d 2 phi dy 2 equal to 0.

As, we have mentioned earlier that this equation is hyperbolic and of course, we do not need to specify boundary condition in all boundary, which is the property of hyperbolic partial differential equation and also they represent of propagation problem.

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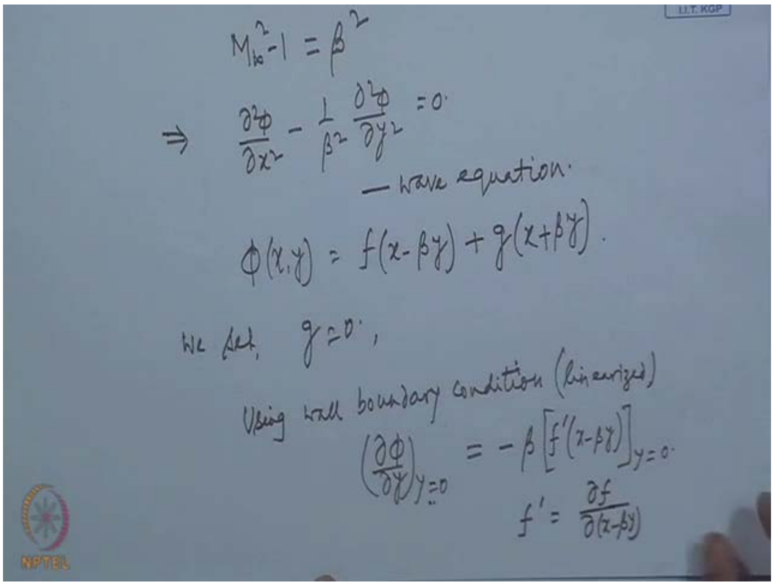

Now, this time we consider write m infinity square minus 1 equal to beta square, please note the change in the sin of beta square in the subsonic case, we consider 1 minus M infinity square to be beta square, but now, we take m infinity square minus 1 equal to beta square and then this equation becomes d 2 phi dx square minus 1 by beta square d 2 phi dy square. Once again, it must be remembered that phi in this equation is perturbation potential; that is phi is perturbation potential and it is gradient gives the only the perturbation velocity, not the total velocity. So, for to find the total velocity pre stream velocity, must be added to this perturbation velocity which is gradient of phi.

Now, this is familiar wave equation and we have very well known solution for this, which is phi x y is function of x minus beta y plus g into x plus beta y, the solution is quiet well known.

Now, for the time being, we set g equal to 0 and we see while g chosen to be 0 in fact, it has something to do with the direction of the flow and which makes a distinction between upstream and downstream, which is again a property of the hyperbolic partial differential equation. Now, to find the explicit form of this function f, we consider the boundary condition, the boundary conditions are of course, the same as in case of subsonic flow, there is no change in boundary conditions using the wall boundary condition. using the wall boundary condition

Which says, that the slope of the streamline is same as the slope of the body at which is of course, linearized as before. Which gives that d phi dy at y equal to 0. We see, here that instead of evaluating the velocity component on the wall itself, we are evaluating at y equal to 0 that is the linearization or approximation, but consistent with the linearized contribution theory and this gives minus beta x prime x minus beta y at y equal to 0, remember that f prime is that is, f is differentiated with respect to it is argument.

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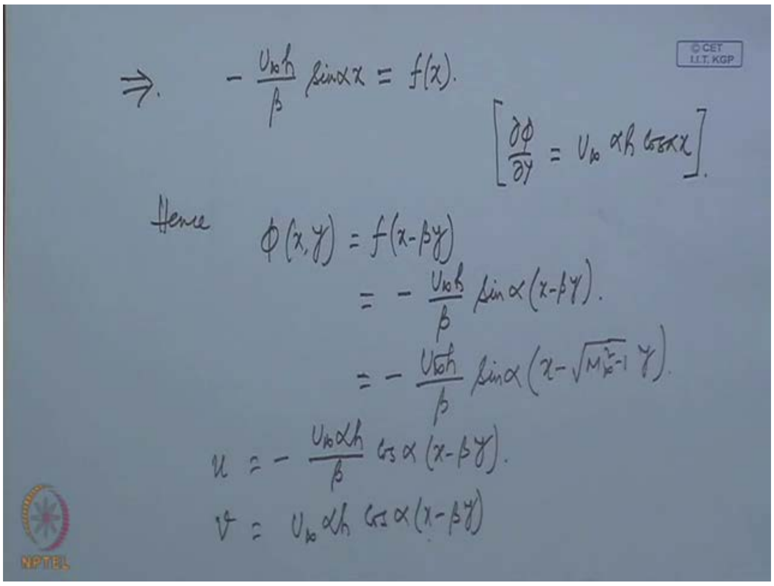

Now, d phi dy at y equal to 0 which, we have already seen is minus u infinity h by beta sin alpha x equal to this is what, we get substituting d phi dy y equal to 0 whereas, before d phi dy. ((no audio 09:59 to 10:32))

So, this is the function explicit form of the function f and hence or potential becomes perturbation potential is y x y equal to function of x minus beta y. Which is or in terms of mach number, this can be written as sin alpha x minus root over m infinity square minus

1 into y the perturbation velocity components are d phi dx that makes, infinity alpha h by beta cos of alpha into x minus beta y, u infinity alpha h cos of alpha into x minus beta y.

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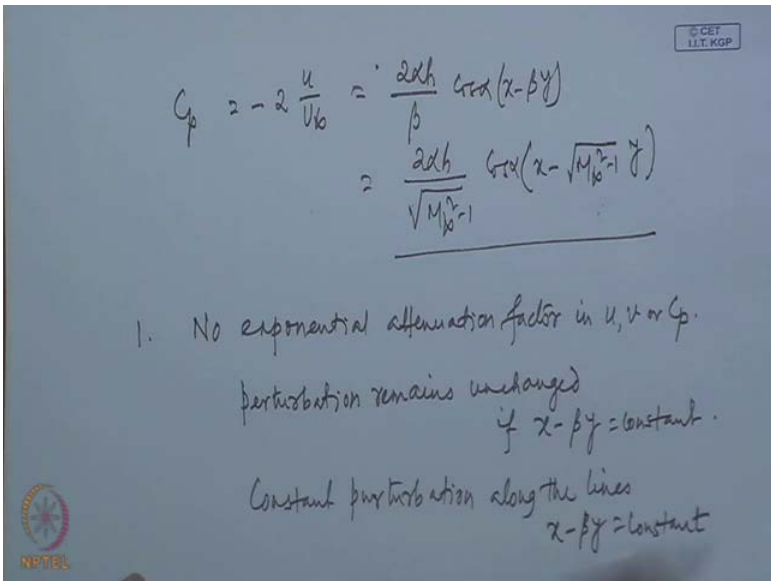

And the linearized pressure coefficient is minus 2 u by u infinity that gives 2 alpha h by beta cos of alpha x minus beta y or in terms of Mach number. It is root over m infinity square minus 1 for alpha x minus root over m infinity square minus 1 y.

So, this is the linearized pressure coefficient and what we see here, that we did not need to satisfy any boundary condition at infinity to obtain, this solution that is again as a special property of hyperbolic partial differential equation that, we do not need to satisfy the boundary condition at all boundaries. Now, look into this perturbation velocity and pressure coefficient what immediate, we can see is that there is no exponential attenuation factor here.

So, the first observation that we can make is no exponential attenuation factor in u, v or C p and it simply means that the perturbations do not vanishes. As, we increase our y which is unlike subsonic case however, we saw that the perturbation decreases as y is increased in this case you see that of course, the highest perturbation is obviously still on the wall, but it is not decreasing with increasing distance, meaning that in a supersonic flow the perturbation will not vanished. Even at very far distance; very far off from the body. So, supersonic perturbation will not vanish even at very great distance from the body. In fact, what we can see here that the perturbation remains constant perturbation remain constant as long as this remain constant.

So, now x minus beta y equal to constant is essentially, as straight line in the x y plane. So, the perturbation is constant along along the lines x minus beta y equal to constant.

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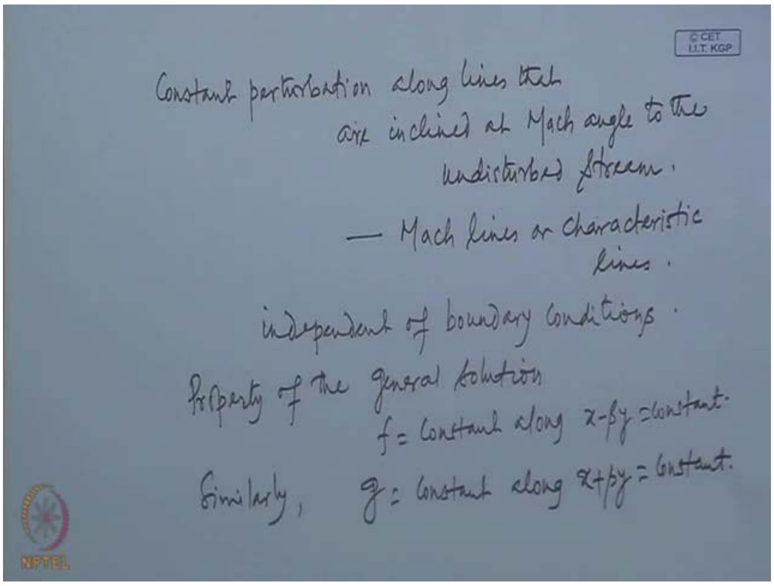

And obviously, these lines make an angle beta or we say that or we can say that the constant perturbation along lines that are inclined at Mach angle to the undisturbed stream. That is, along any line which makes Mach angle with respect to the undisturbed stream the perturbation remains constant, even up to infinity or in practical case to a very large distance. Now, the lines which makes Mach angle with the with respect to the undisturbed stream are called as before, we have called them the Mach lines. These are the Mach lines or the characteristic lines.

So, the perturbations remain constant along the Mach lines or the characteristics lines. Now, once again we see that this has nothing to do with the boundary conditions and in fact, this is the property of the basic solution itself. So, this is independent of the boundary conditions this fact is independent of boundary conditions independent of boundary conditions or whether that the flow flow is such that it is perturbation remain constant along Mach lines irrespective of whatever, body shape, we are considering in a supersonic flow. So, this is this property is contained within the specific solution, f equal to constant along x minus beta equal to beta y equal to constant.

So, property of the solution itself property of the general solution, f equal to constant along x minus beta y equal to constant and similarly, similarly, we can see g equal to constant along x plus beta y equal to constant.

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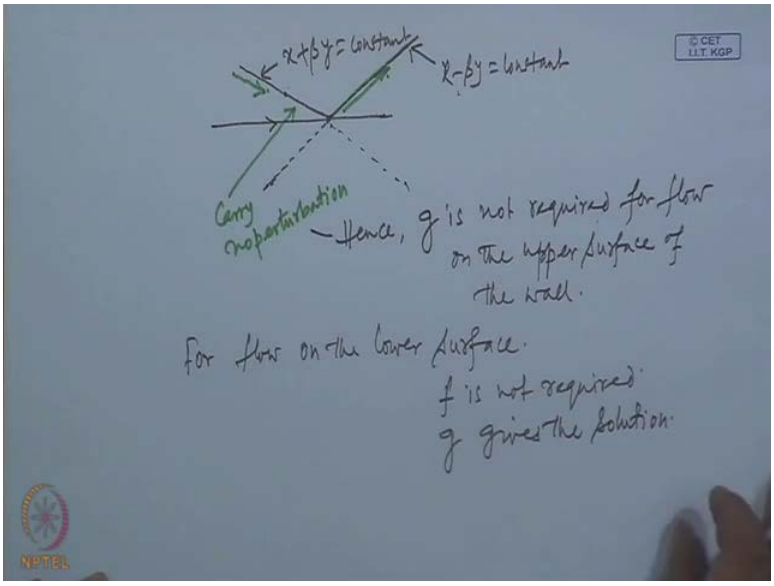

As, we can see that with respect to any wall or any streamline. Now, these set of characteristics are as you can see inclined downstream and meaning that, they are originating from this wall they are originating from the wall and they carry the information from the wall to the field flow field, while the other set of characteristics. Which are inclined upstream, meaning that they bring in information from infinity and hence, they carry no perturbation.

So, carry no perturbation this set originates at infinity and brings in perturbation to the wall. However since, there is no perturbation the perturbation is not created on the wall it at the infinity. So, these are carrying no perturbation, while this originates at the wall and carries perturbation to the field. So, this is the reason that for flow over the wall only the function f is important hence, g is not required for flow on the upper surface of the wall. However if, we consider the flow on the lower surface, then this is what is originating at the wall and carrying the perturbation to the field. While this then is originating at infinity and there is no perturbation. So, for the lower flow for flow on the lower surface g gives the solution.

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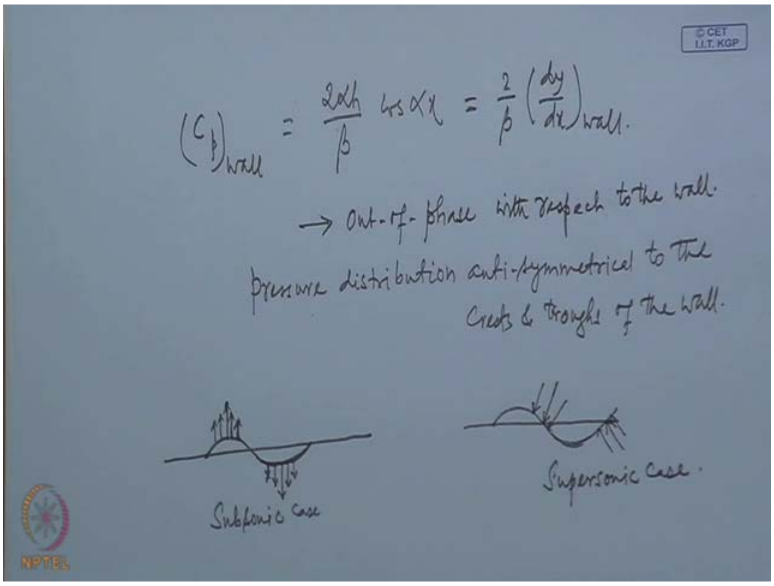

Now, let us see that C p on the wall, the pressure coefficient on the wall 2 alpha h by beta cos alpha x which can be written as 2 by beta dy dx wall, and if you remember, this is the same pressure coefficient that, we obtained in case of weak wave theory or linearized shock expansion theory. So, we get the same result here also, one more thing that, this is now, anti phase to the wall out of phase with respect to the wall. Maxima are maxima and minima are shifted by a phase of pi by 2 with respect to the maxima and minima of the wall.

So, we have pressure distribution antiphased pressure distribution anti symmetrical to the crests and troughs of the wall, ((no audio 27:33 to 28:30)) remember the pressure is these are of course normal reduction for the supersonic case however, this as ((no audio 28:42 to 29:38)) and this of course, clearly, shows that for if the wall is symmetric. As, we have considered, there will be no lift in both the cases. However, in this case there will be drag force acting, while there is no drag force here.

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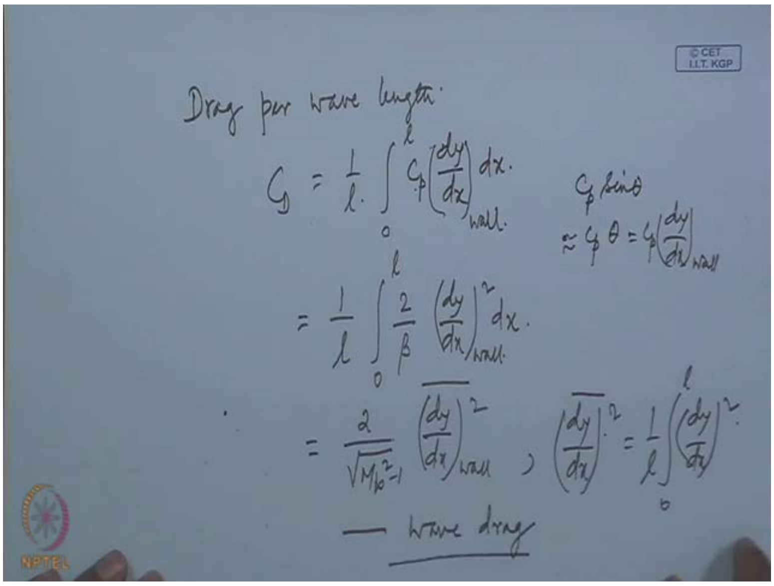

Now the magnitude of the drag force per wavelength can be written as C D equal to 1 by l 0 to l this gives the component along the flow direction component of the pressure coefficient along the flow direction which is say C p sin theta is nearly equal to C p theta that is C p d by dx of course, this dy dx of wall.

Now, as we have already seen that the C p is written as 2 by beta into dy dx. So, this becomes dy dx square dx and this, we can replace by a average or this average is defined. So, once again we have seen that this, there is a drag in viscid two dimensional supersonic flow, which is called the wave drag, because of the wave nature of the solution of supersonic flow again considering the range of validity.

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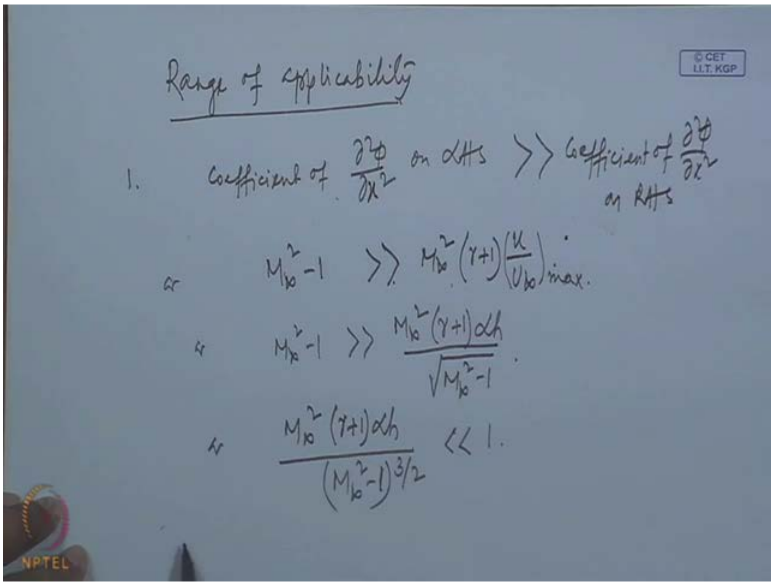

As, before we can see since first of all we have considered the linearized supersonic flow or there is supersonic flows. Which can be linearized and this implies that, the coefficient of d 2 phi dx 2 on the left hand side the coefficient of d 2 phi dx 2 dx square on left hand side is much larger than coefficient of d 2 phi dx 2 on right hand side. ((no audio 34:43 to 35:15)).

Now, u by U infinity maximum as, we have already seen is alpha h by beta, ((no audio 35:29 to 36:08)) which again gives and we see that, we have again obtain the same relation. As, we have seen in case of a subsonic flow, that the linearization can be used only if, this condition is satisfied and if not as this quantity moves towards one, the applicable validity becomes 4 and when this reaches very close to 1.This approximation cannot be used and this terms on the right hand side, cannot be neglected meaning this, suggest when the flow will become transonic.

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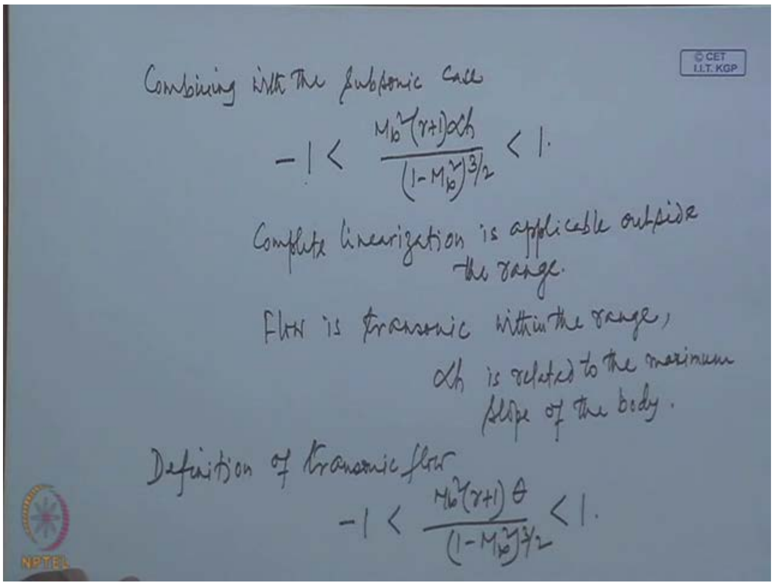

Now, if we combining with the subsonic case; combining the result with subsonic case. We have, ((no audio 38:00 to 38:32)) outside this range linearization is possible, complete linearization is applicable outside the range and the flow is transonic within the range, and as we have mentioned earlier or as you can see that alpha h the parameter, alpha h can be related to the maximum slope of the body. ((no audio 39:47 to 40:18))

So, we can replace this alpha h by the maximum slope, and this gives concise definition of transonic flow. So, we can see this is this gives us a definition of transonic flow, a theta is the slope of the body then, and this term is usually call the transonic parameter it is denoted by chi.

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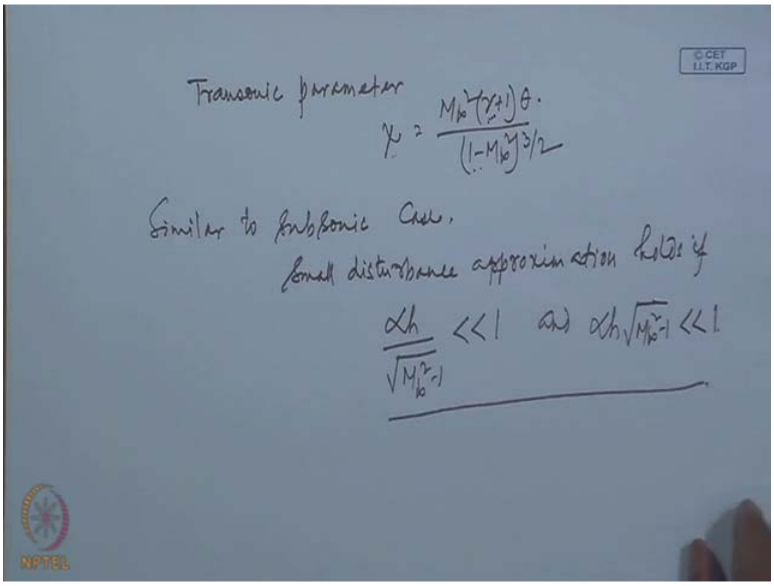

So, we see that whether a particular flow can be called transonic or not depends on the free stream mach number and particularly and 3 2 3 by 2 exponent for the factor 1 minus M infinity square the slope of the body, and also the specific heat ratio gamma that means, it we it depends on the gas itself. So, for a given body and given free stream Mach number the flow may not be transonic, if gamma is if gamma is such that this condition is not satisfied.

We can also see that the small perturbation, also leads the small perturbation; also leads to the earlier assumptions that, and similar to subsonic case small disturbance approximation. If alpha h by root over M infinity square minus 1 is much less than 1, and alpha h root over M infinity square minus 1 h much less than 1. This also you can see as before using the same approach that is u by U infinity and v by b b infinity M v by U infinity of much smaller than 1 to get this relation and the higher order term in the wall boundary conditions is negligible, if this is satisfied. So, hence these are the requirement for the small disturbance approximation to be valid.

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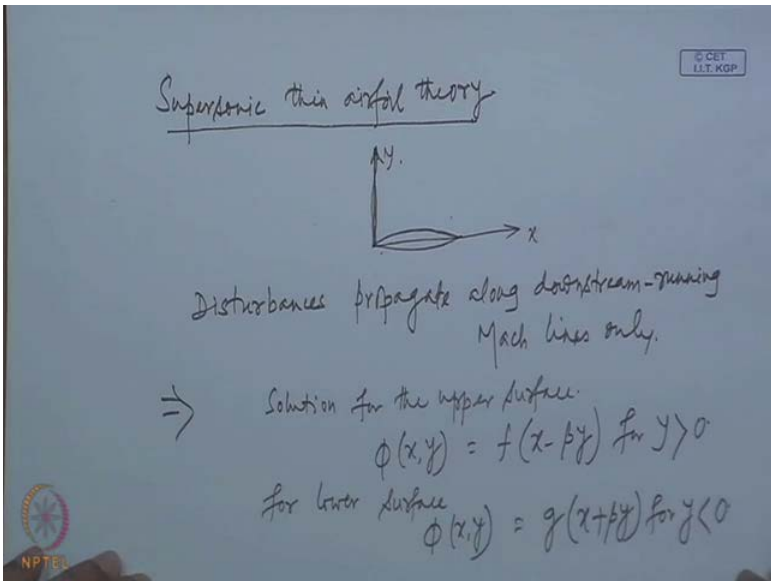

Now, with this we can find what is a supersonic thin airfoil theory, this will give us supersonic thin airfoil theory ((no audio 44:42 to 45:22)) from the solution, our wavy will we, have seen that disturbance propagated along downstream running Mach lines, only disturbances propagate along downstream running Mach lines only hence, the solution for the solution for the upper surface, phi x y equal to function of x minus beta y for y greater than 0, for lower surface. Now, the specific form of f and g can only be obtained by satisfying the wall boundary condition.

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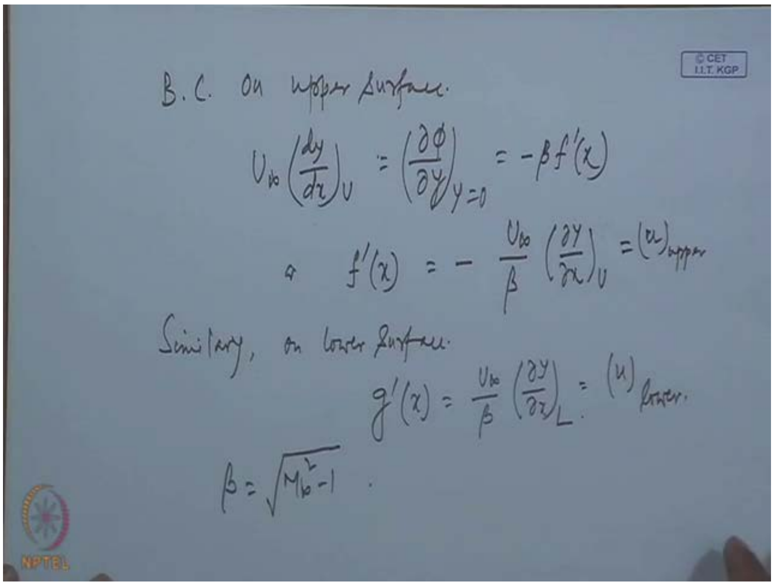

And boundary condition on upper surface gives us U infinity dy dx on the upper surface is again approximated to d phi dy at y equal to 0, and which is d phi dy is minus beta a prime x or f prime x is minus U infinity by beta dy dx on the upper surface. Similarly, on the lower surface and once, this must be remembered that beta in this case is root over M infinity square minus 1. These are also the velocity component on the wall that is u on upper wall.

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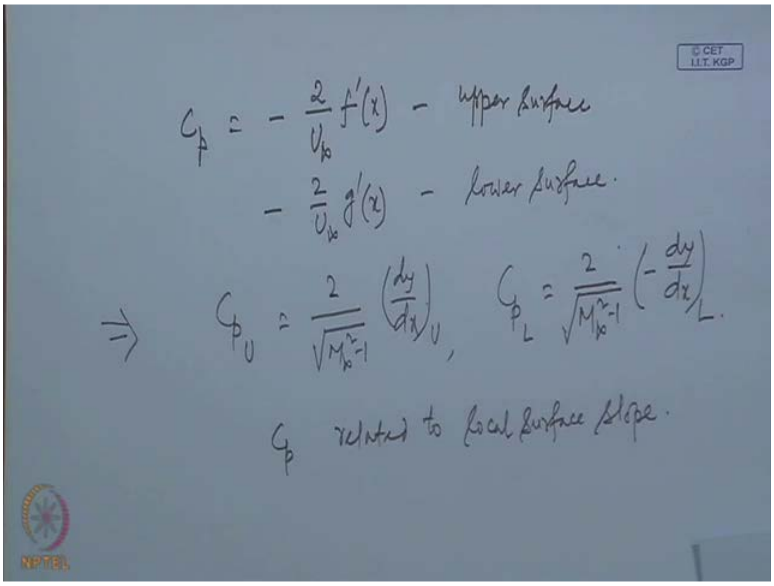

And similarly, this is U on lower wall hence the pressure coefficient can be written as minus 2 by, U infinity f prime x on the upper surface and minus 2 by U infinity g prime x on the lower surface.

Now, substituting here a prime and g prime, this is this gives C p on the upper surface as 2 by and on the lower surface. So, see the pressure coefficient on the airfoil surface is quite easily obtained from the solution of the wave shift wall or using the linearized problem of course, and we can see that, this leads to a local inclination theory, that C p at any point on the surface is simply, related to it is slope surface slope at that point. So, C p related to local surface slope; C p at any point on the surface of their foil. Simply, depends on the slope at that point and of course, the free stream mach number within the frame work of thin airfoil or linearize theory.

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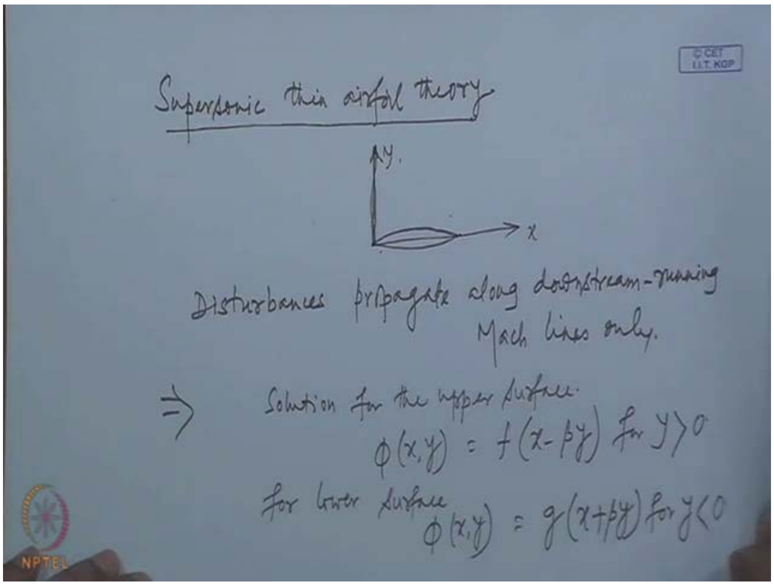

So, to summarize what, we have done today is solved linearized supersonic flow first wave view shift wall, and the most important thing that, we have observed that there is no exponential attenuation factor in case of a supersonic flow, that is the perturbation in a supersonic flow extends to far away theoretically. It infinity and the perturbation remain constant along the characteristic lines also, we have seen that that the disturbances propagate, along only downstream running Mach lines hence, solution of the solution for the over the upon however the flow the only function equal in the functionate and surface the solution that is required is the function g, considering the validity of this linearized approximation.

We have come to an explicit definition of transonic flow and we have defined a transonic parameter. Which, we have seen the depends the free stream mach number in particular to the prandtl glauert factor, we rise to the exponent 3 by 2, it also depends on the slope of the geometry, but surprisingly. It depends on the gas itself, whether a flow at a particular mach number, over a particular geometry is transonic or not depends also on the gas itself also, we have seen that as before, we have found a drag force acting, even in two dimensional on in viscid flow over over a body.

So, we have confirm the fact again that, if Supersonic flow. There is a drag even, if the flow is in viscid two dimensional the drag, which we called wave drag and in this case also every valuated the wave drag then finally, we have explain extend this solution of flow over a wavy wall to flow over a thin airfoil and we have developed what is known as the supersonic thin airfoil theory. Where, we have seen that the pressure coefficient at any point on the airfoil surface depends only on the local slope and which also depends on the free stream Mach number.

So, with this we conclude our definition that, this thin airfoil theory within the framework of small perturbation theory leads, us to a very useful, but very simple result for flow past on a foil and of course. As you can see that this solution will, we also applicable two or three dimensional geometry except near the t or the effect of three dimensionality will come in.