High Speed Aerodynamics

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Lecture No. # 26

Linearized Flow Problems (Contd.)

So, we will take up couple of examples to illustrate these applications of linearized theory and the first problem that we will take up is 2-dimensional flow past wave shaped wall, a very well known classical problem also popularly known as Ackeret's problem.

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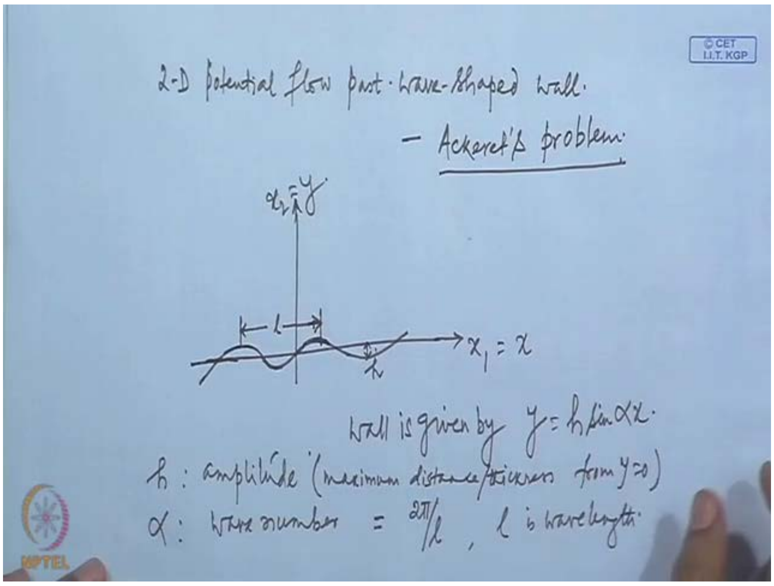

Now, let us consider flow over a wave shaped wall, we will now denote these to direction x 1, and x 2 by x and y. See instead of using x 1 and x 2, we will now use x and y and let us say, that this is the… and you consider 1 wavelength for the body and this amplitude is given by h. So, the wall is given by y equal to h sin alpha x where, so h is the amplitude and alpha is the wave number. Amplitude without the maximum from y equal to 0, and alpha is the equivalent wave number equal to 2 pi by l, where l is wavelength.

We will consider the flow is coming from left to right with undisturbed stream of m infinity.

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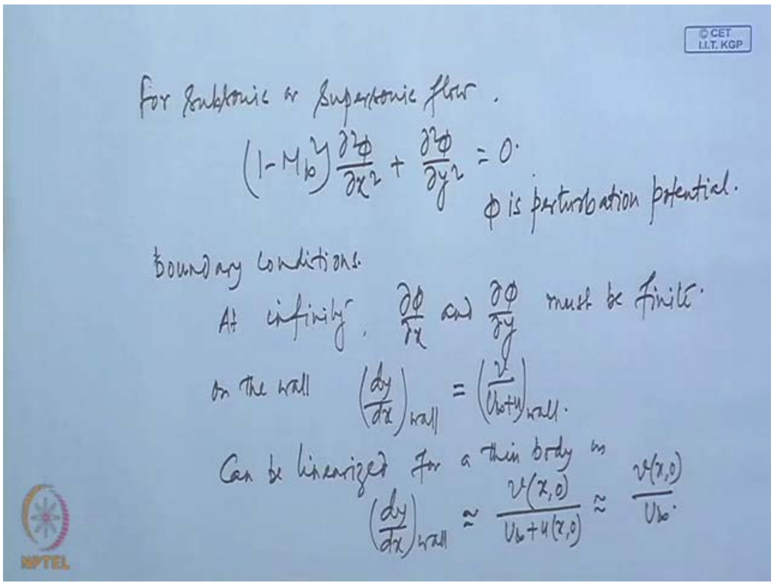

Now, for subsonic, or supersonic flow we have 1 minus m infinity square d 2 phi d x square plus d 2 phi d y square equal to 0. And these are phi is perturbation potential these are subjected to boundary conditions at infinity. at infinity what you know about the perturbation? That they must be finite d phi nothing more than that d phi d x and d phi d y must be finite. And on the wall on the wall the slope of the wall that is d y d x on the wall is the slope of the streamline on the wall, which for can be linearized for very thin geometry d y, d x on the wall is…

Where, we have made this assumption that since the body is very thin, the velocity components on the wall is same as the velocity components y equal to 0, for the body very thin body this approximation is first order accurate and consistent to our first order perturbation theory, and then we have made further assumption that which of course, again consistent with the small perturbation theory that u is much smaller compared to u infinity and hence, there sum is close to infinity. So, here there are, two assumptions involved; one that v component of v, and u components of velocity on the body is same or, very close to the velocity that would have been at y equal to 0 and then sum of u infinity and perturbation along x is very close to u infinity itself. Both these assumptions are consistent with first order perturbation theory and are acceptable.

Now, to complete the solution of this problem at hand that is flow over a wave shaped wall, we need to solve this equation subjected to these boundary conditions, you know first of all we before solving the equation, we should see that the equation changes in nature as m infinity changes from less than 1; to more than 1.

When we see that when m infinity is less than both the term are of same sign; however, when m infinity is more than 1, the 2 terms are opposite sign, and this essentially changes the nature of the partial differential equation, that we have here, when both the terms are positive this equation is of elliptic nature while, when the 2 terms are of opposite sign the equation become hyperbolic. As now, if we may call back that when the equation is elliptic to solve a partial differential equation we need to satisfy boundary condition at all boundaries.

However, for solving the hyperbolic partial differential equation we do not need to satisfy boundary condition in all direction only the initial condition saw sufficient, also the hyperbolic equation supports discontinuous, or solution while, the elliptic equation supports only continuous, or smoothly solution. So, we see that the equation changes its nature, in case of subsonic flow; the equation is elliptic, when the flow is supersonic the equation become hyperbolic. So, we need to solve these 2 equation separately for subsonic or supersonic flow.

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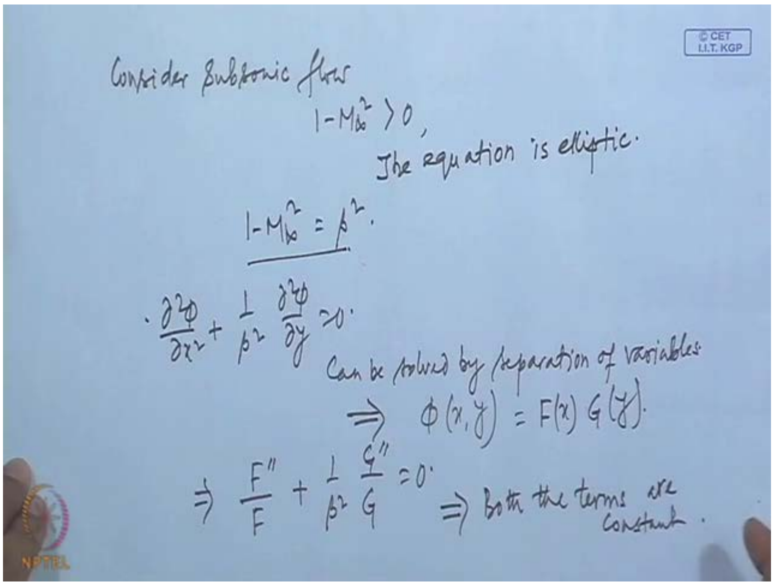

So, the first case we will consider a subsonic flow;

So, let us consider a subsonic flow, we know have 1 minus m infinity square is greater than 0 and the equation is elliptic, that is in this case the solution will be smooth over the entire flow domain, or in the over the entire flow field and we need to satisfy all the boundary conditions together to get the solution. And we let us say 1 minus m infinity square is beta square for convenience and the equation can be written as d 2 phi d x square plus 1 by beta square d 2 phi d y square equal to 0. This equation can be solved by separation of variables, which says that phi which is a function of x y the perturbation potential can be separated into 2 function each of 1 variable say f x g y and this when you substitute gives us f double dot by f plus 1 by beta square g double dot by g 0.

Now, the first term is function of x; and second term is function of y only now sum of these 2 functions can be 0 for all cases only when, both of them are constant this is function of x this is function of y and sum of them is 0, that can happen only if both of them are constant.

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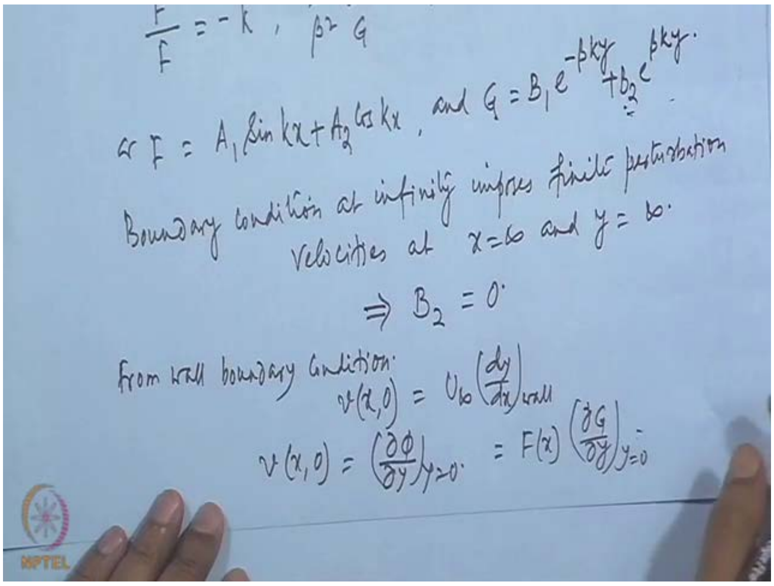

And let us, set that constant as we are taking the constant as a square just for convenience. Now these gives us a solution that f equal to a 1 sin k x plus a 2 cos k x and this gives g equal to b 1 e to the power minus beta k y plus b 2 e to the power beta k y.

Now, the boundary conditions at infinity needs that the perturbation velocity components must remain in finite at infinity that is at x is equal to infinity as y equal to infinity. So, boundary condition at infinity imposes finite perturbation velocities at x equal to infinity and y equal to infinity now x of course, remain finite sorry f remain finite for all values of x even at x equal to infinity this f remains right, but looking to g we see that this goes on increasing with y. So, if this has to remain finite then this b 2 must be 0 this implies b 2 equal to 0.

Now, from the wall boundary condition the wall boundary condition we have written as v x 0 equal to u infinity d y d x wall now v x 0 which is d phi d y at y equal to 0 that is f x.

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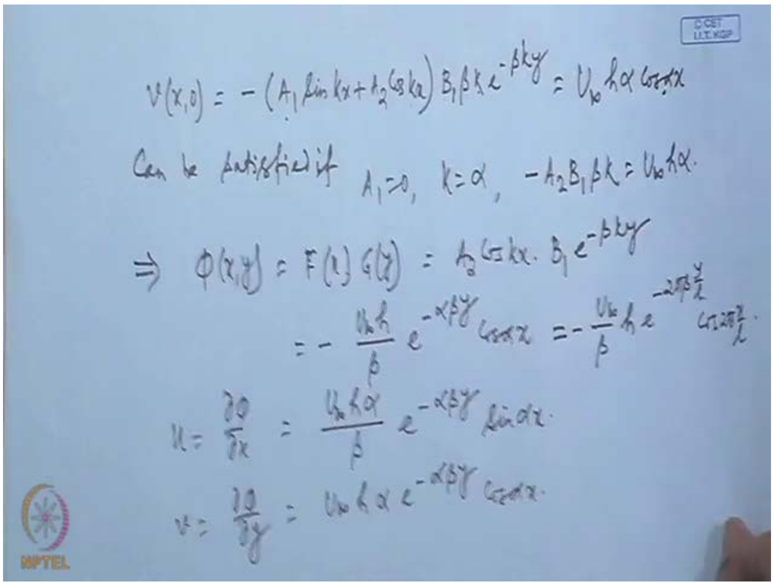

So, v x 0 becomes minus a 1 sin k x plus a 2 cos k x into b 1 beta k e to the power minus beta k y that equal to u infinity into d y d x that makes u infinity each alpha cos alpha x. Now, this can be satisfied only, if a 1 is 0, there is no sin term on the right hand side. So, there must be no sin term on the left hand side.

So, 1 must be 0 k must be alpha and a 2 b 1 beta k minus is u infinity h alpha. So, can be satisfied if a 1 equal to 0 k equal to alpha and minus a 2 b 1 beta k is u infinity h alpha . So, we can now substitute these values of we have b 2 is 0 a 1 is 0 and we have got the product of a 2 b 1 and this an imply that phi x y, which is f x into g y a 1 is 0. So, f 2 remains only a 2 cos k x into g y b 1 e to the power minus beta k y and then we can substitute, what they are minus u infinity h by beta e to the power minus alpha beta y cos alpha x or in terms of the wavelength, instead of wave number this can also be written as minus u infinity by beta h e to the power minus 2 pi beta y y l into cos 2 pi x y l the perturbation velocity components u is d phi d x that becomes u infinity h alpha by beta e to the power minus alpha beta y sin alpha x and v equal to d phi d y u infinity h alpha e to the power minus alpha beta y cos alpha x, the total velocity filed of course, can be obtained by adding u infinity to the u component .

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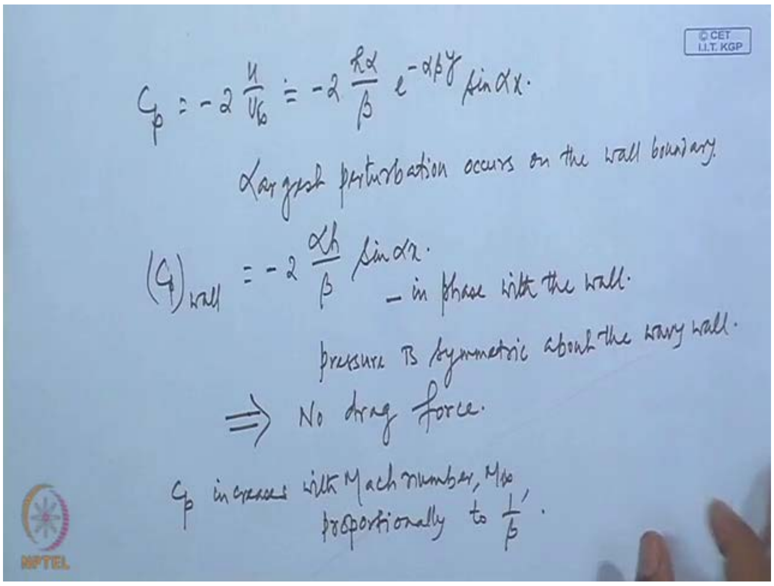

The pressure coefficient is…Now, looking to these properties perturbation velocity is u v and, pressure coefficient c p, we can see that the perturbation decays with y as y increases; perturbation velocity components decreases.

So, is the pressure coefficient? So, the largest perturbation occurs at the boundary all these show that largest perturbation occurs on the boundary, and the pressure coefficient on the wall that is pressure coefficient on the wall is minus 2 alpha h by beta sin alpha x you see that the pressure distribution on the wall is in phase with the wall. The wall is also given by h sin alpha x. Consequently the pressure is pressure distribution is symmetric about the wavy wall.

And these gives that there will be no drag force there will be no drag force acting on these body we also see that as mach number increases pressure increases actually this in increase in quantitative term is proportionally to 1 by beta this beta is known as prandtl-glauert factor or prandtl-glauert parameter also we see that that the perturbation velocities and pressure has an attenuation that is as you go away from the wall as y increases; the perturbation velocity and the pressure coefficient decreases. So, in the limit of y approaching infinity the perturbation velocities become 0 which again is an standard boundary condition used in incompressible flow where it says that the perturbation at infinity is 0. So, we see that we approach that situation subsonic flow.

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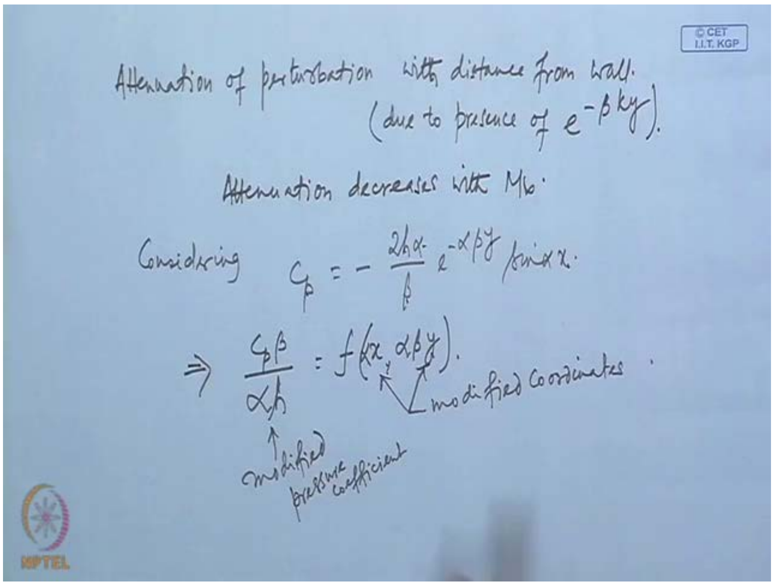

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However; since, the factor beta is present in the attenuation attenuating factor. So, the attenuation decreases with mach number. As m infinity increases; the attenuation decreases. So, that is when m infinity is larger within the framework of subsonic flow then the attenuation will be failed at a much greater distance compared to a low subsonic flow a high subsonic flow will be failed at a much greater distance compared to a low subsonic flow.

Now, considering the pressure term; pressure coefficient that c p is minus 2 h alpha by beta e to the minus alpha beta y sin alpha x, we can see that the parameter involved here are; 1, 2, 3, 4, 5, 6. There are 6 parameter involved c p, h, alpha, beta x and y. Now, these parameters can be arranged in this fashion that c p,beta by alpha h the constant we can forget is some function of alpha x and alpha beta y where the left hand side is modified pressure coefficient and these are of course, modified coordinate.

So, you see that it is possible to express; the pressure, a modified pressure in the form of some modified coordinates. So, that instead of 6 parameters, we now have 3 parameters;

that means, it is possible to reduce 3 parameters, and what it imply that, if we have 2 bodies with these coordinates being same then these pressure coefficient will also be same, this modified pressure coefficient will also be same.

And hence, knowing the solution for 1 such body we can obtain solution for similar other bodies. So, this is basically a similarity rule in which a similarity rule which imply that there is some possible combination when used then the total number of parameters involved will decrease. So, in this case of course, we are getting this from the known solution, not a priory; however, it shows that if we have this type of combination; that means, instead of coordinates x and y, if we express the coordinates in the form alpha x and alpha beta y then and the pressure in the form of c p, beta by alpha h then for all walls for which alpha x and alpha beta y are same they will have same modified pressure given by this relation.

We can have some further inside, or further assumption, further result; from this solution. First of all that our perturbation velocities are, such that u by u infinity and v by u infinity are small. According to small perturbation theory that is when this solution is valid u by u infinity and v by u infinity are smaller than 1.

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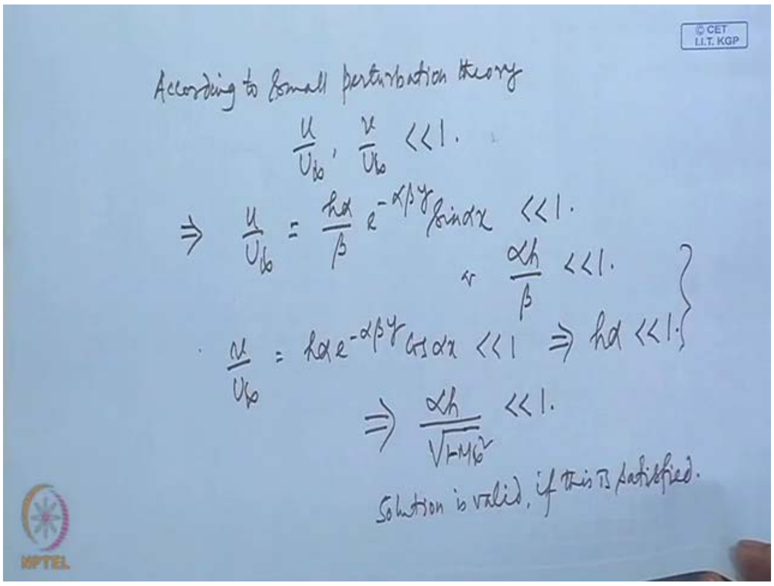

Now, looking back to our u by u infinity and v by u infinity the u by u infinity equal to h alpha by beta e to the power minus alpha beta y sin alpha x this much smaller than 1 or this is possible if alpha h by beta is much smaller than 1, the same thing we get from v by h v by u infinity is h alpha e to the power minus alpha beta y cos alpha x less than 1 with this implies h alpha is much less than 1 of course, if h alpha by beta now, if h alpha by beta is much less than 1, then of course, h alpha will be much less than 1.

So, together imply that alpha h by beta is root over 1 minus m infinity square is much less than 1. So, if this is satisfied only then the solution is applicable. So, what we see that for the solution to be valid it is not essential that h has itself has to be very small, it is required that the combination h alpha by root over 1 minus m infinity square that must be very small, if this condition is satisfied the solution is valid or that the solution is within the framework of small perturbation theory. Also we have used the linearized equation to solve this problem.

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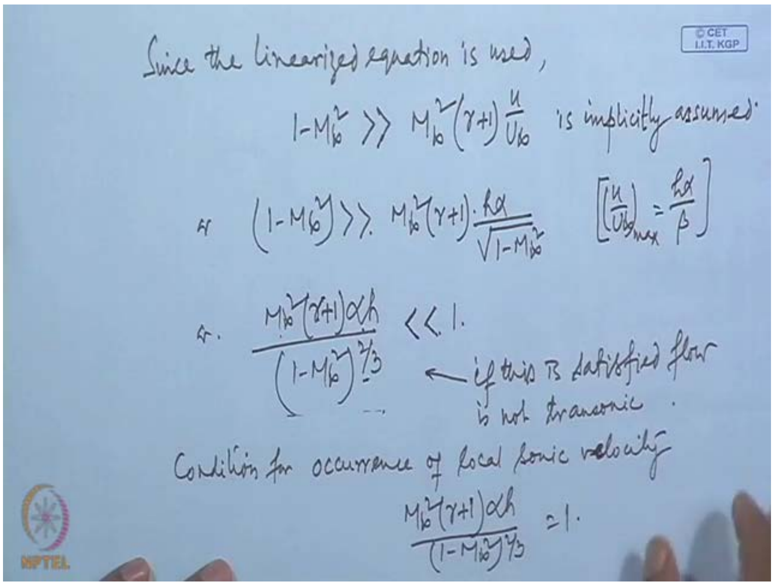

Since, the linearized equation is used we have implicitly assumed that 1 minus m infinity square is much larger than the term on the right hand side, which is required when m infinity is very close to 1 that is this term is much larger than this term this is implicitly assumed . So, we have implicitly assumed this, or in this case this will become m infinity square into gamma plus 1, the largest possible value for u by u infinity is h alpha by beta, (No audio 41:35 to 42:05) or what we have is m infinity square into gamma plus 1 alpha h by 1 minus m infinity square to the power 2 by 3 is much less than 1. So, we have implicitly assumed this.

Now, what is the meaning of this? We have seen; we have we earlier stated that this term is negligible when m infinity is not close to 1, in particular when m infinity is close to 1 only then this term become comparable to this 1. So, this term cannot be neglected for m infinity close to 1, or for transonic flow.

Now, what it says that this can be negligible if this is satisfied. So, if this is satisfied then the flow is not transonic. So, if this is satisfied. So, we see that this particular parameter decides or sets whether the flow is transonic or not. So, we can call it a transonic parameter and what we see that this transonic parameter includes; gamma that is the gas. So, just the value of free stream mach number, or even the geometry is not sufficient to decide whether the flow is transonic or not, also what the gas is that depends that decides whether a particular flow is transonic or not; that means, flow at a some same mach number, over same geometry may be transonic, if the gas is a particular gas, but may not be transonic if it is other gas depending on the value of gamma, another important is that, it contains the parameter power 2 by 3, 3 by 2 and this is what is the condition for occurrence of local sonic velocity. (No audio 45:32 to 46:43)

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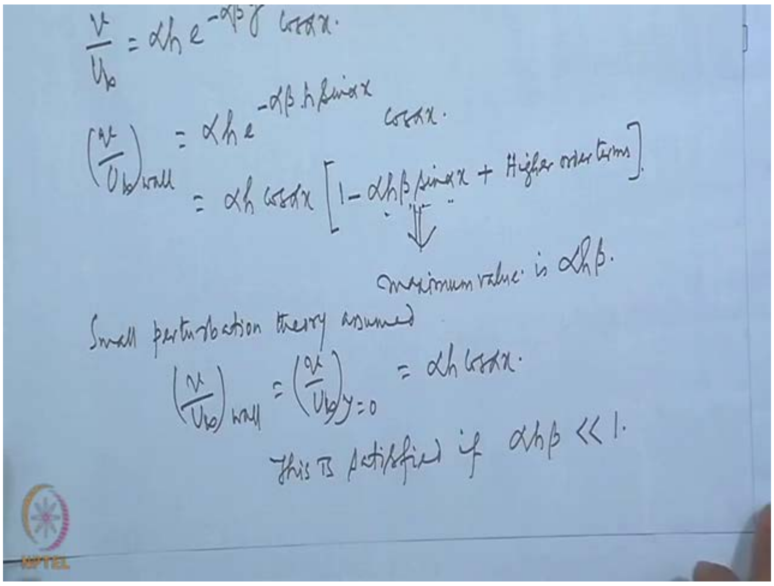

Let us, now consider the other component of free stream velocity that is v sorry perturbation velocity v by u infinity as alpha h e to the power minus alpha beta y cos alpha x. On the wall this term becomes, on the wall we know y equal to h sin alpha x so minus alpha beta h sin alpha x into cos alpha x, this we can write alpha h, cos alpha x and this term e to the power alpha beta h sin alpha x we can expand in power series to that 1 minus alpha h beta sin alpha x plus higher order terms.

Now the maximum value of this second terms is alpha h beta. So, maximum value f this second term is maximum value is alpha h beta. Now, you know small perturbation theory we have approximated that v by u infinity, this term v by u infinity wall is v by u infinity y equal to 0 that is only this much. The small perturbation theory assumed v by u infinity wall is v by u infinity y equal to 0 and in this case that equal to alpha h cos alpha x.

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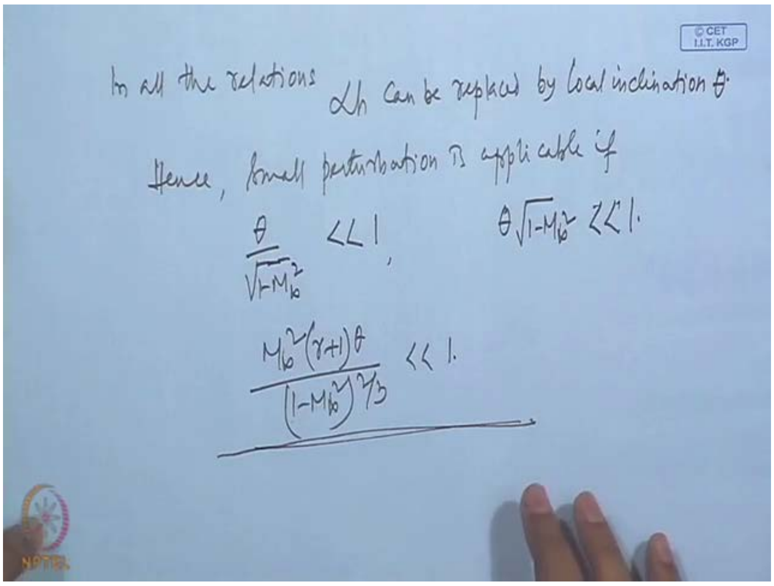

Now, this to be… now this is satisfied if alpha, h beta is much less than 1 and in all these relations in all these relation alpha h can be replaced by in all the relations alpha h can be replaced by local inclination theta and this then all these conditions then can be retained as… hence, small perturbation is applicable, if theta by root 1 minus m infinity square much less than 1 theta into root over 1 minus m infinity square is much less than and finally, m infinity square into gamma plus 1, into theta by 1 minus m infinity square to the power 2 by 3 might be less than 1.

So, if all these conditions are satisfied then the small perturbation theory is quite applicable. So, we have consider a subsonic flow past a wave shaped wall and obtain its solution, and we have also seen the validity of these conditions and this result have shown us; that of flow can be transonic for certain mach number, and body shape for a particular gas, but may not be transonic for another gas that is a flow transonic flow parameters, or transonic condition depends on the particular gas itself along with the flow speed of flow mach number and the body geometry.

Also we have seen that, the parameters involved in these problem can be arranged of arranged in 3 groups, or 3 parameters, where 1 is modified pressure; and 2 are; modified coordinates and that modified pressure can be expressed in terms of the modified coordinates, and hence, we can obtain our similarity rule for this type of problem, which gives the hint that for high speed flow problem perhaps it is possible to get similarity rules for many other situation particularly when the problem is linearized, or small within the framework of small disturbance theory.

Also we have seen that for a wave shaped wall, what would be the amplitude of the perturbation and what maximum thickness, or maximum amplitude of the wall can be permitted. So, that the flow can be treated as subsonic, in particular we are we have seen that the perturbation has attenuation factor in it, that is the perturbation dies away as we go away from the wall; however, the attenuation decreases; as mach number increases.

So, what we see that as the flow approaches incompressible there be no perturbation at all at infinity, which is the incompressible flow boundary conditions that at infinity, all the perturbation velocities are zero; however, that is applicable even for low subsonic flow, but as mach number goes on increasing; the attenuation decreases.

We have also seen that the perturbation is maximum on the wall and the pressure distribution on the wall is in phase with the wall itself. So, that the pressure on considering a particular wavelength of the wall the pressure, on half of the wave balances the pressure; on the other half of the other half of the wall consequently there is no drag force acting in this case.

So, we see again in inviscid 2-dimensional flow there is no drag force, which is true for incompressible flow as well; however, as we saw that in supersonic flow there is such drag, and we will next consider the solution of supersonic flow pass this wavy wall, and we will see gain that a drag force is present. There in other way that the pressure will not be in phase with the wall pressure will be anti-phase with the wall and consequently a drag force will be present, which we found in earlier cases also that in a supersonic flow there is drag even in inviscid flow which is due to the wave nature of the supersonic flow. So, that we will be seeing here also however; we will consider that solution of supersonic flow pass this wavy wall in our next lecture.