High Speed Aerodynamics

Prof. K. P. Sinhamahapatra
Department of Aerospace Engineering
Indian Institute of Technology, Kharagpur

Lecture No: # 25

Linearized flow problems

We have discussed, and derived the equations for irrotational multidimensional flow problems. And you have seen that when the flow is irrotational all the equations can be combined to give 1 equation either in terms of the velocity components or the velocity potential. The equation is strongly non-linear and it is formidable and quite difficult to solve and for general boundary conditions close form solutions are not possible. However before we discuss, the further simplification that we can try on these equation and the boundary conditions.

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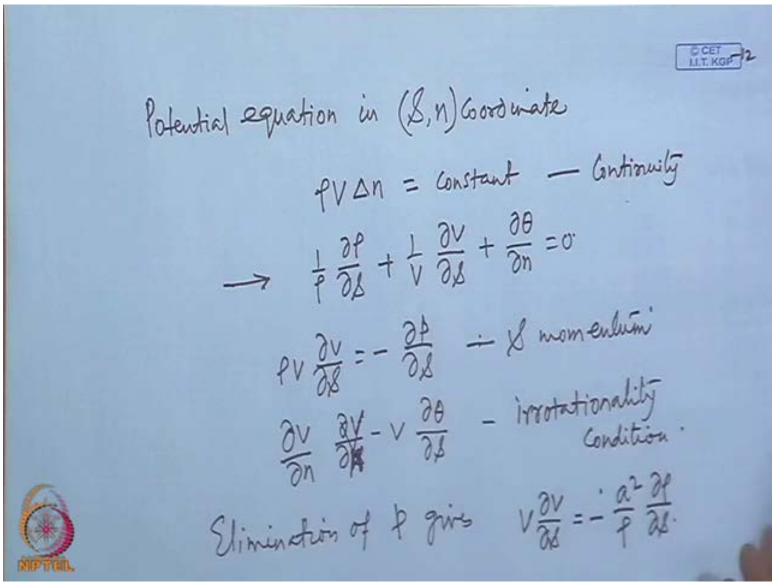

We will first see what the corresponding equation looks in streamline coordinate system. So, the velocity potential potential equation potential equations in streamline coordinate system. To do do this we have the continuity equation rho v delta n equal to constant, and this can be written as 1 by rho d rho d s plus 1 by v d v d s plus d theta d n 0 . The momentum equation as you have already written the stream wise momentum equation rho v d v d s equal to minus d p d s. Which is s momentum and the flow is irrotational. So, the vorticity is zero which is oh sorry d v d n d v which is irrotationality. Now, if we eliminate rho as before let us say elimination of the pressure from this equation gives us.

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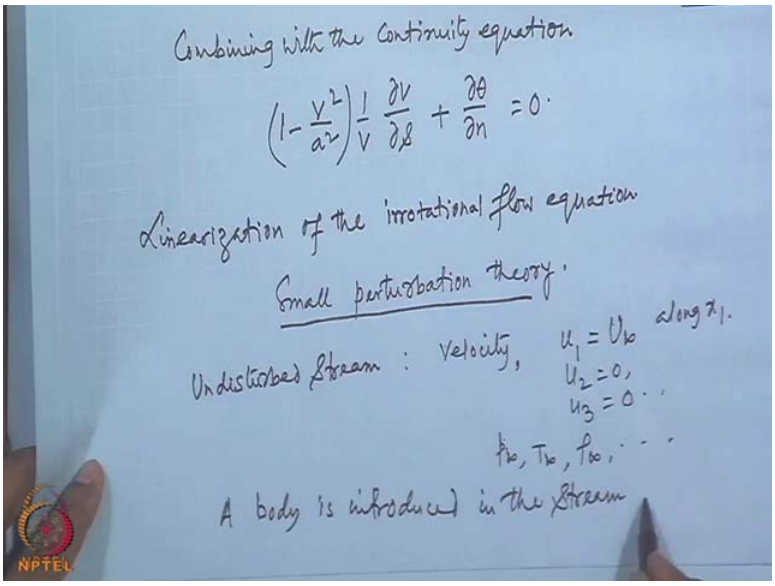

And these can be combined with the continuity equation. So, combining that with the continuity this gives us 1 minus. So, this is the equation of form of the equation in streamline normal coordinate system, with this now we will look further simplification of the equation. Linearization of the irrotational flow equation this linearization is achieved by using what is known as small perturbation theory. (No audio 06:41 to 07:17).

First of all let us consider we have a free stream say undisturbed stream. The velocity is u 1 equal to u infinity u 2 equal to zero u 3 equal to 0. That is the undisturbed stream is along x 1 and the velocity component along x 2 and x 3 are zero. Of course, this is a simple we can achieve this by aligning or x 1 axis in the flow direction. So, that then this sub stream is along x x 1 and the other components of the velocity flow velocity are 0. It is associated with pressure temperature density and similar, other parameters. So, the undisturbed parameters we are denoting by the subscript infinity. Now let us say body is placed in this undisturbed stream. A body is.

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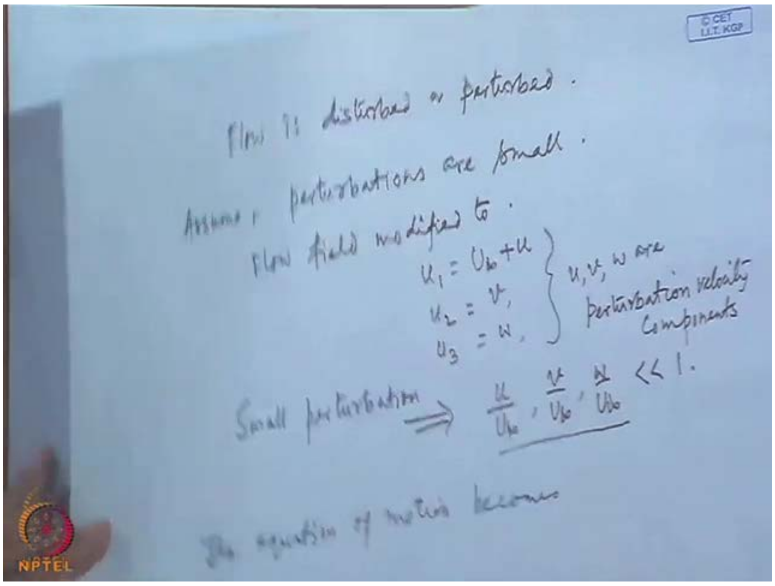

A body is introduced in the stream when the body is introduced the flow will be disturbed or perturbed. Flow is disturbed or perturbed and we assume that these perturbations are small. This is usually a quite justifiable assumption in aerodynamics, because most of the aero dynamical bodies are quite thin, and the perturbations that they produced in a stream are usually quiet small. Now the velocity field that now modified. So, the flow field now is u 1 equal to u infinity plus u u 2 is let us say we will call it v and u 3 equal to w. So, u v w are perturbation velocity components and that small perturbation means, that so this what we mean, by small perturbation that this ratios are very small.

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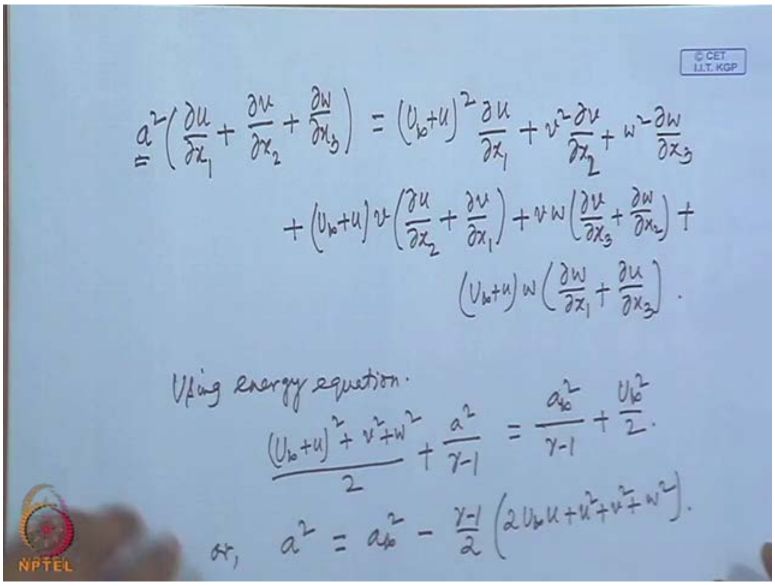

Now, with this the equation of motion can, the equation of motion becomes square d u d x 1 that is you are substituting u 1, u 2, u 3 these values in the equation that we have already derived and the resulting equation becomes u infinity plus u square, into d u d x 1 plus d square, d v d x 1 plus sorry d v d x 2 plus w square d w d x 3 plus u infinity plus u, into v into d u d x 2 plus, d v d x 1 plus, v w d v d x 3 plus, d w d x 2 plus, u infinity plus u into w d w d x 1 plus d u d x 3.

In this the small u v and w are the perturbation velocity which we are following our assumptions are small when compared with respect to the free stream undisturbed velocity u infinity. Now this equation contains only those perturbation velocities are unknown; however, it also contains the term the speed of sound a. The speed of sound a is also a local variable as the flow velocity pressure changes from point to point. So, would as the speed of sound and we now want to replace this speed of sound in terms of the speed of sound in the undisturbed stream a infinity.

Now, that is achieved by using the energy equation using energy equation for a perfect gas. We have u infinity plus u square, plus v square, plus w square by 2 plus enthalpy a square by gamma minus 1 which was same as before, since the flow is adiabatic in the undisturbed stream this was a infinity square minus gamma minus 1 plus u infinity square by 2. Now, simplifying this gives a square equal to a infinity square minus gamma minus 1 by 2 2 u infinity u plus u square plus v square plus w square. Now we substitute this a square in this equation and then divide throughout by a infinity square.

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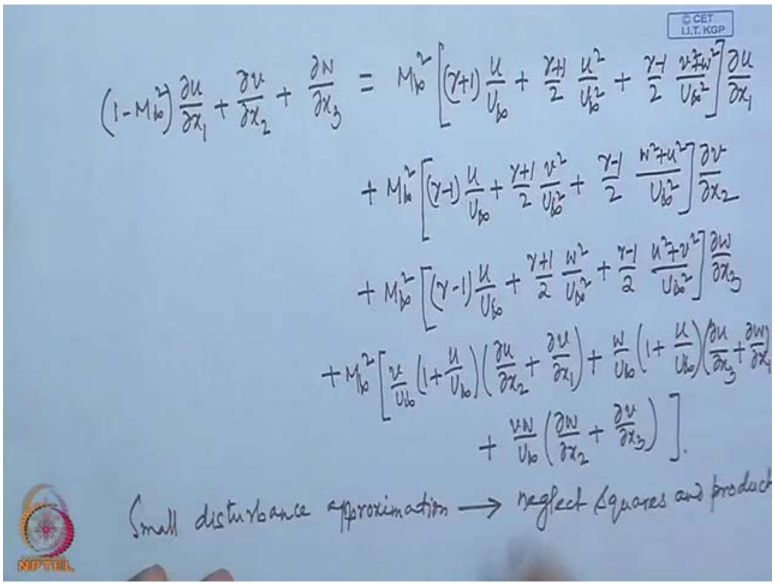

The result becomes when you substitute a square from this equation into the earlier equation and then divide by a infinity square. The resulting equation becomes 1 minus m infinity square, d u d x 1 plus d v d x 2, plus d w d x 3 equal to m infinity square into gamma plus 1 into u by u infinity plus gamma plus 1 by 2 u square by u infinity square plus gamma minus 1 by 2 v square by plus w square by u infinity square into d u d x 1.

Plus m infinity square gamma minus 1 u by u infinity plus gamma plus 1 by 2 v square by u infinity square. Plus gamma minus 1 by 2 w square plus u square by u infinity square into d v d x 2 plus m infinity square gamma minus 1 into u by u infinity plus gamma plus 1 by 2, w square by u infinity square plus gamma minus 1 by 2 u square plus v square by u infinity square into d w d x 3, plus m infinity square v by u infinity into 1 plus u by u infinity into d u d x 2, plus d v d x 1, plus w by u infinity into 1 plus u by u infinity into d u d x 3 plus, d w d x 1 plus v w by u infinity square into d w d x 2 plus d v d x 3. The equation is still exact even though you have inserted perturbation, but we have not introduced any approximation as yet. So, the equation is still the equation still remains exact for irrotational flow now we will make use of this small disturbance approximation.

So, that small disturbance approximation neglects square and product of perturbation velocities. That is since u by u infinity is very small, then the square of it is still smaller or similarly, u by u infinity is small v by u infinity is small. So, their product is also much smaller and consequently we neglect such terms which are present here. So, you can see that from this equation this term can be neglected, this can be neglected here, also this can be neglected this term can be neglected here this term is negligible this is also negligible and in this product of this and this that is negligible, but not this 1 this is a single product similarly here also product of these and these is negligible, but not these and this term is negligible.

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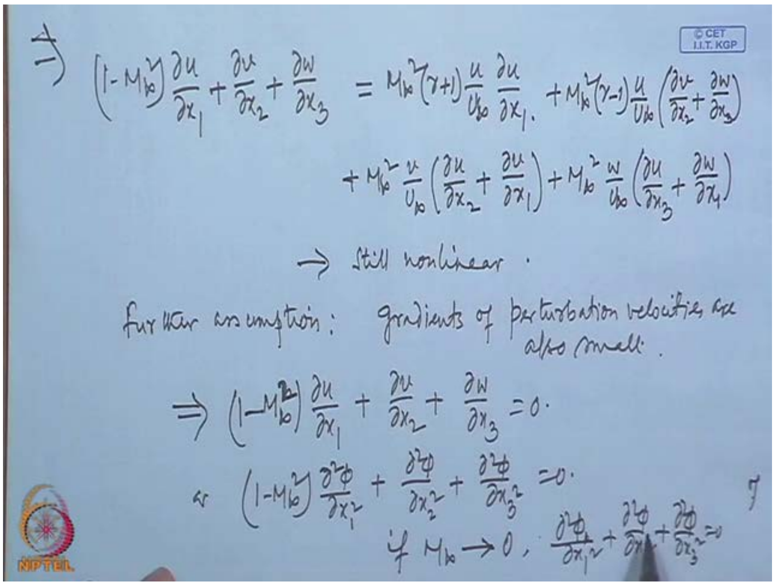

So, if we neglect all such terms the equation now becomes 1 minus m infinity square the left hand side of course, remain as it is there is no product or had other terms. The first terms keeps only m infinity square gamma plus 1 u by u infinity d u d x 1. The second and third term, the second and third here, the second keeps these and the third keeps these. So, the second and third together can be combined to m infinity square gamma minus 1 u by u infinity into d v d x 2 plus d w d x 3. And that last term keeps m infinity square v by u infinity d u d x 2, plus d v d x 1 plus m infinity square w by u infinity d u d x 3 plus d w d x 1. So, this is what remains when we neglect those higher order terms that is terms of second degree or higher of course, higher than second degree terms. Now this equation is still quite formidable and it is also non-linear, because of the product terms present on the right hand side

Now, this equation can again be further be simplified, if we further assume that the product of these perturbation velocity and the gradient of those perturbation velocity is also small. So, if we assume then. So, this equation is still non-linear further assumption gradients of perturbation velocities are also small. If we assume that then we see the on the right hand side all the terms are again product of 2 small terms the perturbation velocity itself is small and following these assumption these gradients are also small. So, this makes that all the terms on the right hand side are again now product of 2 small terms. And this results that the entire right hand side is negligible and sorry 1 minus m infinity square d u d x 1 plus d v d x 2, plus d w d x 3 equal to 0 or in terms of perturbation potential this becomes 1 minus m infinity square d 2 phi d x 1 square plus d 2 phi d x 2 square plus d 2 phi d x 3 square equal to 0. And when m infinity approaches 0. This becomes d square phi d x 1 square plus d 2 phi d x 2 square plus d 2 phi d x 3 square equal to 0.

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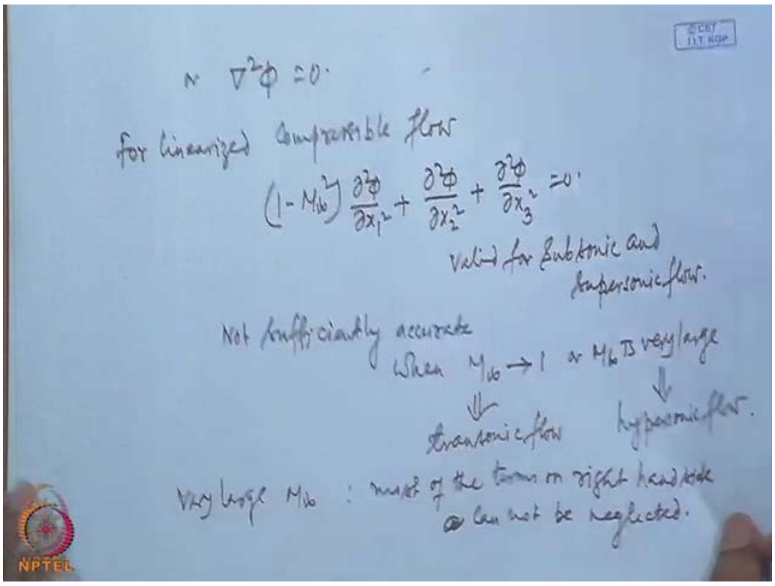

Which is the well known or laplacian phi equal to 0. Phi is now the perturbation potential phi in this is equation is perturbation potential, phi in this equation is perturbation potential. And we see that this equation results our well known equation for incompressible flow, that for a small perturbation or linearized flow equation that laplacian of the perturbation potential is 0. Now as it happens that this equation is quite acceptable for subsonic and supersonic flow.

So, for linearized compressible flow (No audio 31:41 to 32: to 32:24) ; however, as it happens this holds good or valid for subsonic and supersonic flow, but the equation is not accurate not accurate up on m infinity is very close to 1 or m infinity is very large. Not sufficiently accurate when we call this at transonic flow for the time being and this is a hypersonic flow. When m infinity is very large you can see that many terms on the right hand side cannot be neglected, for very large most of the terms on the right hand side terms on right hand side cannot be neglected.

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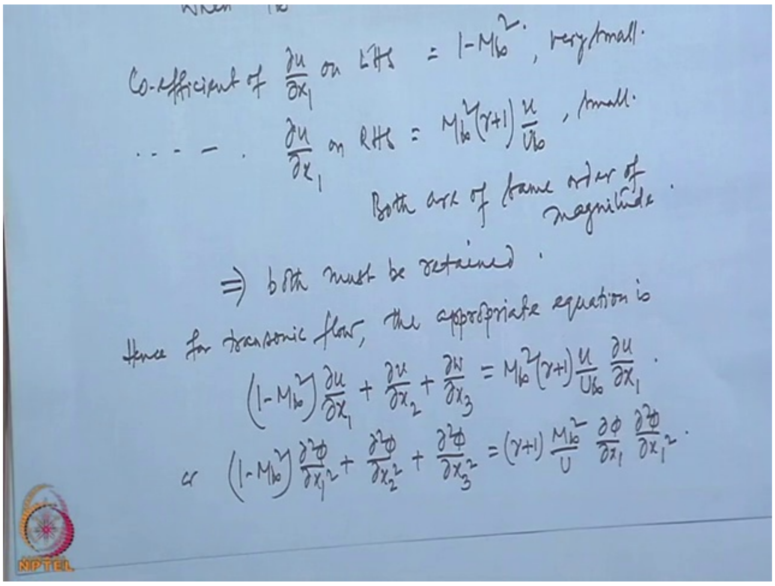

When m infinity is close to 1, when m infinity is close to unity. What you see that coefficient of d u d x 1 on the left hand side on the left hand side is 1 minus m infinity square, and when m infinity is very small very close to 1 this is very small. And the same thing coefficient of d u d x 1 on the right hand side is m infinity square gamma plus 1 into u by u infinity. Now, this is also small and both are of both are of same order both are of same order of magnitude. That is 1 term cannot be neglected in comparison to the other. So, both must be retained. So, this implies both must be retained; however, for the coefficient of the other derivative that is d v d x 2 and d w d x 3. There is no such problem and that term on the right hand side can be neglected. So, hence for transonic flow the appropriate equation is (No audio 37:23 to 38:18) or of course, that can be expressed in terms of perturbation potential or in terms of perturbation potential. This becomes 1 minus m infinity square d 2 phi d x 1 square plus d 2 phi d x square.

So, even in this equation there is only 1 unknown variable which is phi; however, the equation is still non-linear. So, we can see that even with small perturbation approximation the governing equation cannot be linearized for transonic flow and of course, also for hypersonic flow; however, the it is possible to linearize the equation both subsonic and supersonic flow. And so, this is the equation appropriate for transonic flow and also of course, if necessary this can be used for both subsonic as well as supersonic flow. So, valid for subsonic to supersonic range valid for subsonic to supersonic range.

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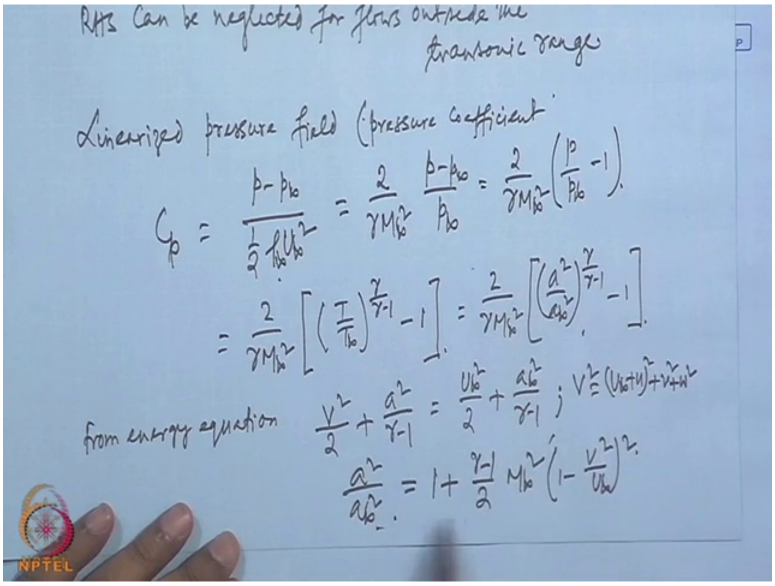

And the right hand side can be neglected when the flow is outside the transonic range. That is the right hand side can be neglected or flows outside the transonic range. Now as in case of incompressible potential flow that once the potential function is solved we can very easily find the velocity component, there is the perturbation velocities are simply the derivative of those perturbation potential. And then we need to find the pressure coefficient or pressure from the velocity from the known velocity field. Now, let us see what we can do here. So, how to find the linearised pressure coefficient, that is linearised pressure field or pressure coefficient c p, because you are more interested in pressure coefficient than the absolute pressure. Now this pressure coefficient by definition is p minus p infinity by half the rho p infinity square. Which we have seen that half rho infinity u infinity square can be written as gamma p infinity m infinity square. So, this becomes 2 by gamma m infinity square, by p minus p infinity by p infinity which you write as 2 by gamma m infinity square p by p infinity minus 1.

Now, the flow is isentropic. So, this can be written as 2 by gamma infinity square p by p infinity can be written as t by t infinity to the power gamma by gamma minus 1 minus 1. And t by t infinity is again expressed in terms of speed of sound. You know using which we have already d 1 and b square is u infinity plus u square plus u square plus w square. And this gives a square by a infinity square to be 1 plus gamma minus 1 by 2 m infinity square into 1 minus this now we can substitute here.

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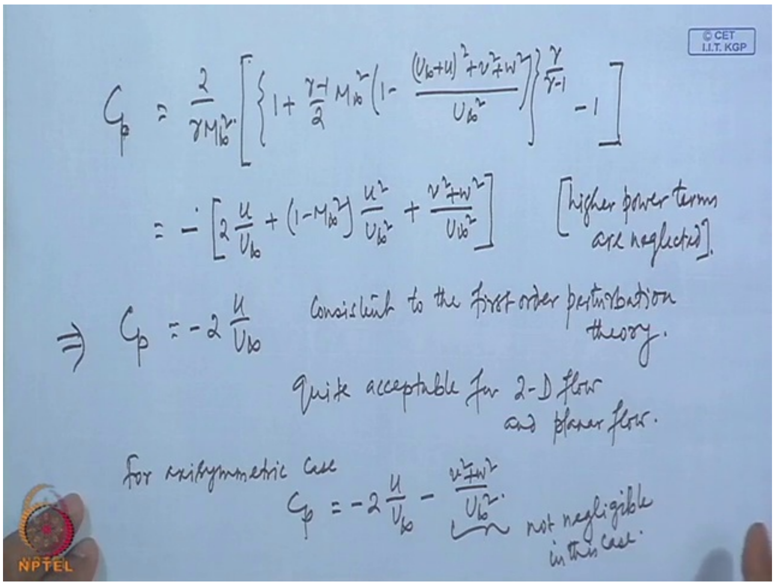

So, we have the pressure coefficient 2 by gamma m infinity square, 1 plus gamma minus 1 by 2 m infinity square 1 minus where b square we write the term u infinity plus u square, plus v square, plus w square by u infinity square through the power gamma by gamma minus 1 minus 1. Now, using binomial theory and neglecting terms of higher order, because these being smaller than 1 we can neglect the terms of higher order. So, we can expand this in binomial using binomial theorem and neglect the terms of higher order and this finally, gives to minus 2 into u by u infinity plus 1 minus m infinity square, u square by u infinity square, plus higher power terms are neglected.

Now, again going back to our small perturbation theory where we neglect even the second order term. So, we have minus 2 u by u infinity consistent to our first order for a first order perturbation theory. That is we keep only the first order terms not the product and square terms, and this is quite acceptable for 2 dimensional flow and also for planar flow. Planar flow means, that like flow over a wing over the wing is all most like a plane; however, we later on see that for axisymmetric case, this c p is minus 2 u by u infinity and this last term is not negligible. So, not negligible this of course, we will come later that why it is not negligible.

So, we see that for 2 dimensional and planar flow this linearized pressure coefficient become simply, twice the x 1 component of normalized perturbation velocity with negative sign. So, that is what is our linearization and we have see that the problem is greatly solved when we have linearized the equation of course, for transonic and hypersonic cases, that if for free stream mach number very close to 1 and very large this linearization is not possible, even with the small perturbation approximation the equation still remain non-linear for subsonic and supersonic case. All though we get a completely linearized equation and that equation can be solved quite easily as we have been d 1 for incompressible flow where the equation is simply laplacian of phi is 0. However this to solve these equations we are also need in need of the boundary conditions.

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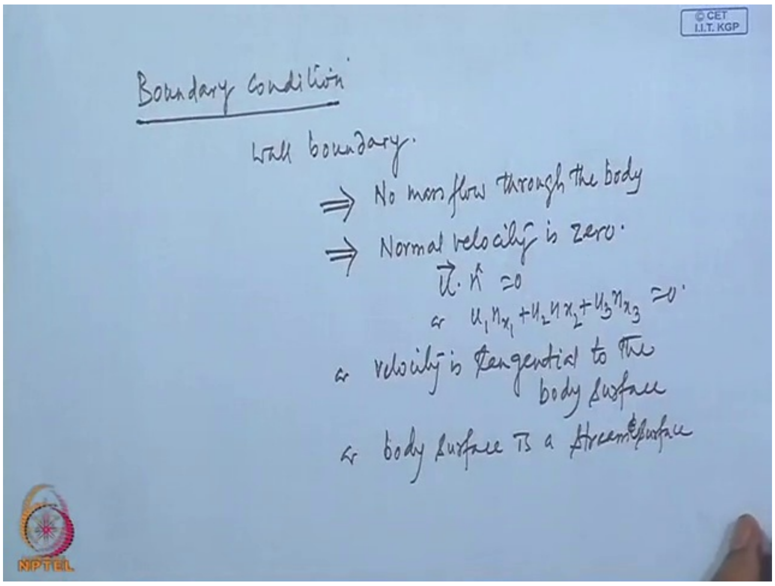

Now the most of most encountered boundary condition is the wall boundary, because we are always interested flow around a body of interest in case of aerodynamics the most interesting or most important bodies are air falls and wings. So, we are most often encountered with wall boundary. And in wall boundary it is that no mass flow through the body no mass flow through the body which is quite obvious and this implies that normal velocity is 0. That is the normal velocity u dot n equal to 0 or u 1 n x 1 plus u 2 n x 2, plus u 3 n x 3 equal to 0 or velocity is tangential to the body surface. Which is also equivalent to saying that the body is body surface is a stream surface. And with this boundary condition the equation is to be completed and solved for appropriate boundary condition

So, what we have done in this lecture, is that we have derived the linearized form or the of the irrotational flow equation, or the potential flow potential equation for the irrotational problem. We know how only 1 equation in terms of a single unknown variable that is the perturbation potential or the velocity potential. And we also have the boundary condition which if necessary can be linearized for a particular given problem. In the first problem that we will be handling is 2 dimensional flow past waves shape to all.

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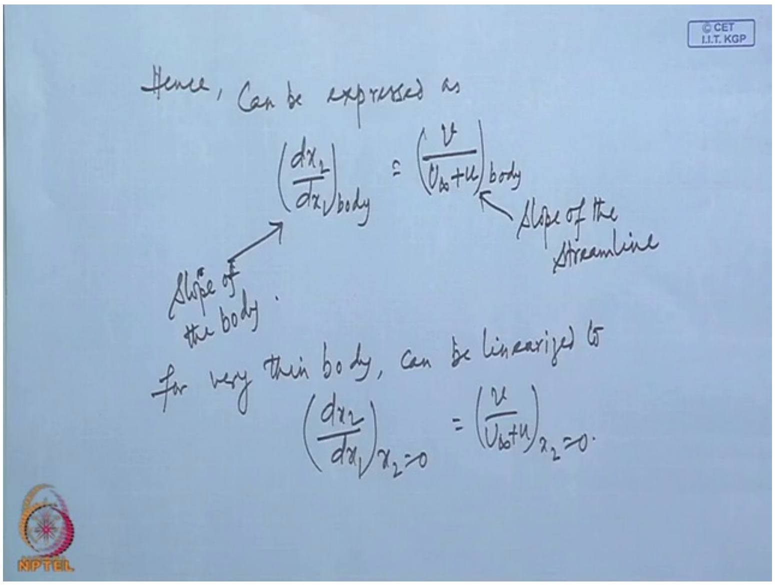

So, the first problem that we will consider is 2 d potential flow past waves shape to all which we I am second. So, this body surface is stream surface, this is for 2 dimensional flow, this implies that body is or stream line and hence, the boundary condition can be expressed. Hence this can be expressed as body slope which is slope of the stream line approximately slope of the stream line and this is slope of the body. So, when the body slope is known that is body is known, then you can equate that body slope to the slope of the stream line which is v by u infinity u of course, calculated on body for very thin body can be linearized to d x 2 by d x 1.