High Speed Aerodynamics

Prof. K. P. Sinhamahapatra

Department of Aerospace Engineering

Indian Institute of Technology, Kharagpur

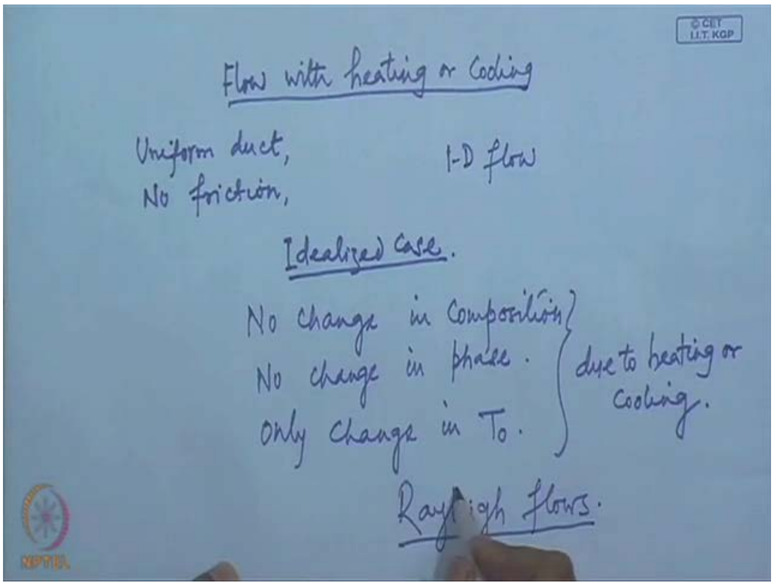
Lecture No # 22

Flow in uniform duct with heating

The third example, in the flow in a duct that we will be considering now is flow with heating or cooling. So, these are the three common factors that can produce continues changes in a flow in a duct that is the variation in area, effect of all friction, and energy effects; such as external heating or cooling which includes combustion condensation and so on.

Now, we have already consider flow with area variation without friction and heat exchange then we have considered flow with friction without area variation and heating exchange, and now we will flow with heating or cooling, but no area variation and no friction.

(Refer Slide Time: 01:29)

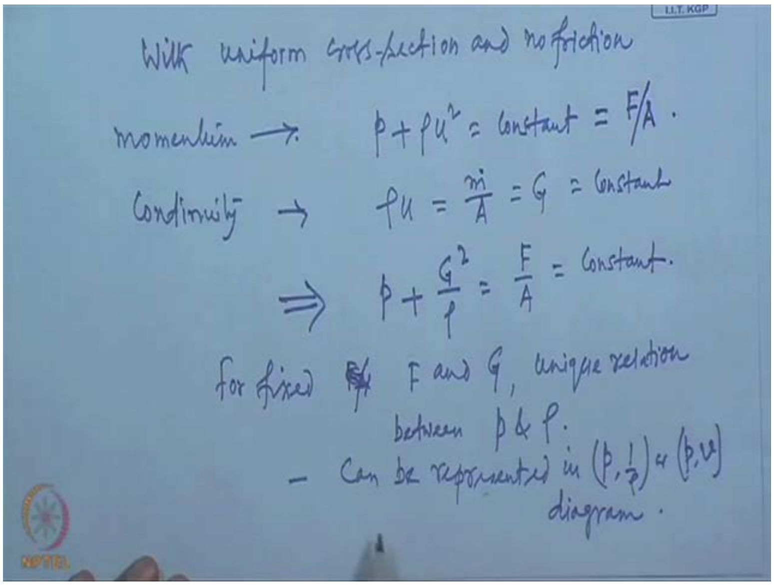


And of course, as before the duct is uniform. So, flow with heating or cooling is the topic of our discussion today, and we will assume uniform duct, no friction.

And of course, as before we will be using one dimensional flow that is any cross section we are considering only the average properties. Now, usually the mechanism of heating or cooling will be associated with many other effects they may produce friction and also when there is combustion or condensation involved there is a possibility of chemical reaction and change in species or change in phase also and consequently heating without these additional effects in real case is usually not possible; however, in this idealized case we will consider the heating is such that it produce no changes in composition or phase and it produces no frictional effect.

The heating in this case produces only a change in T 0. So, these flows also can be considered simple T 0 change, say idealized we are considering an idealized case idealized case; no change in composition, no change in composition of the gas ignition, no change in phase, and only change in T 0, due to heating or cooling. Now these flows are commonly called as Rayleigh flows.

(Refer Slide Time: 05:07)

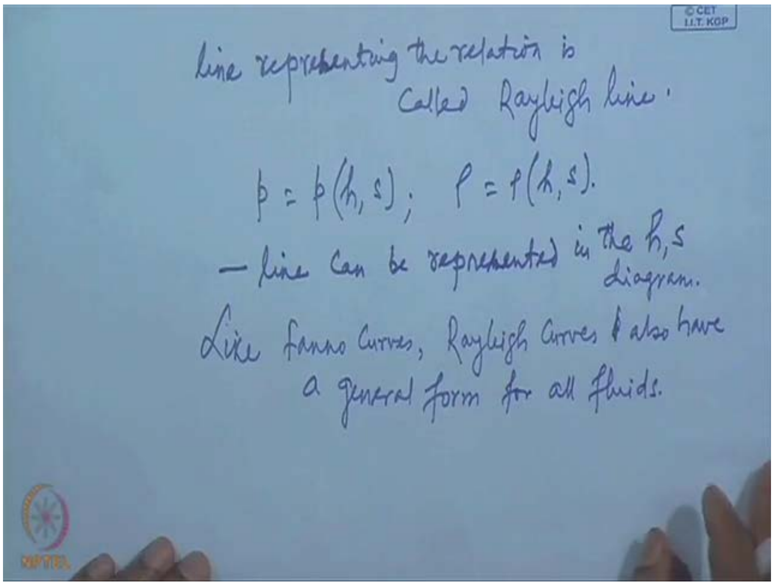


Now, with constant area and no friction, the momentum equation is; with uniform cross section and no friction. The momentum equation is simply gives us P plus rho u square equal to constant and as we have defined earlier that P plus rho u square is called the impulse per unit area, that is impulse per unit area is constant.

Now, continuity of course, rho u that is muss flow rate per unit area is constant and, if we combine these 2, this gives us P plus g square by rho equal to F by a equal to constant.

So, for a fixed mass flow and fixed impulse this is a unique relationship between pressure and density and it can be plotted in the p, P b diagram. So, for fixed F and g unique relation between P and rho, can be represented in P 1 by rho or P b that is the specific volume. P b diagram and of course, in P 1 by rho the diagram is basically a straight line and these lines are called Rayleigh line.

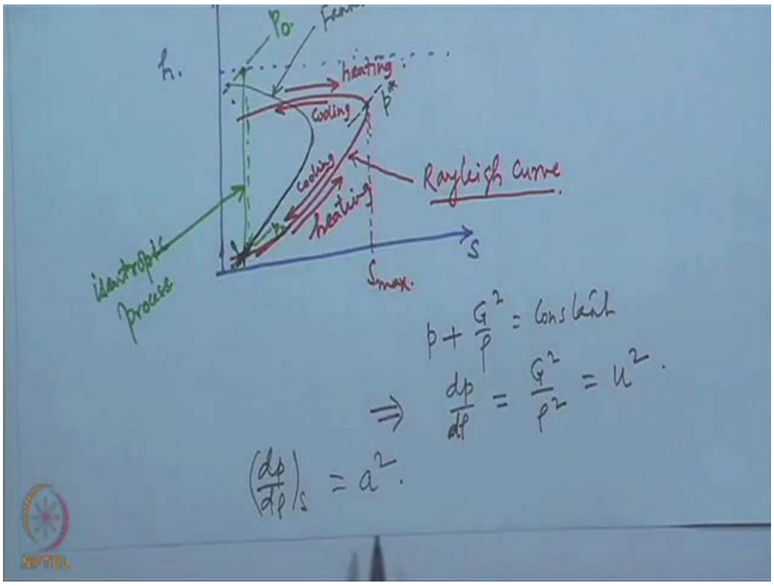
(Refer Slide Time: 08:43)



So, those lines are line representing the relation representing the relation then, is called Rayleigh line. Of course, we will have separate Rayleigh line for separate values of F and g that is the separate values of impulse function and mass flow rate. Now for a pure substance this P and rho can also be expressed as a function of h and s and they can be converted to a relationship between h and s.

Now P is a function of enthalpy and entropy and similarly, rho is also function of enthalpy and entropy. So, the line can be represented in the h s diagram. And as we have seen in case of fanno curves, we can see these Rayleigh lines or Rayleigh curves have also general form. So, like fanno curves Rayleigh curves also have a general form for all substance.

(Refer Slide Time: 11:46)



Now, let us see this. So, this is the general form of Rayleigh curve. Now for continuous changes in duct as we mentioned earlier that there are most commonly these factors heating or cooling of the flow or wall friction or change in area which is in a sense is isentropic flow.

So, all can represented here this… (No audio 13:16 to 14:03) So, this is that. Now let say that the isentropic change. So, these are the three different path that the fluid will follow in case of the respective change for an isothermal change sorry for an isentropic process or isentropic change, as it happens in a duct of varying area without heat addition and friction. The states of the fluid will all Rayleigh on this path in case of uniform, adiabatic flow with friction in a duct. The flow will the flow states will follow these, fanno curve and when there is heat addition, but no change in area or no friction the process will follow this Rayleigh curve.

Now, as we know that in heating the entropy changes and while in cooling the entropy decreases. So, the path that the system will follow that, if cooling is allowed then, this will be the system in direction, and if heating is done then, the system will follow this path. That this is the maximum entropy for the given value of F and g, that is the given value of the impulse function and the mass flow rate.

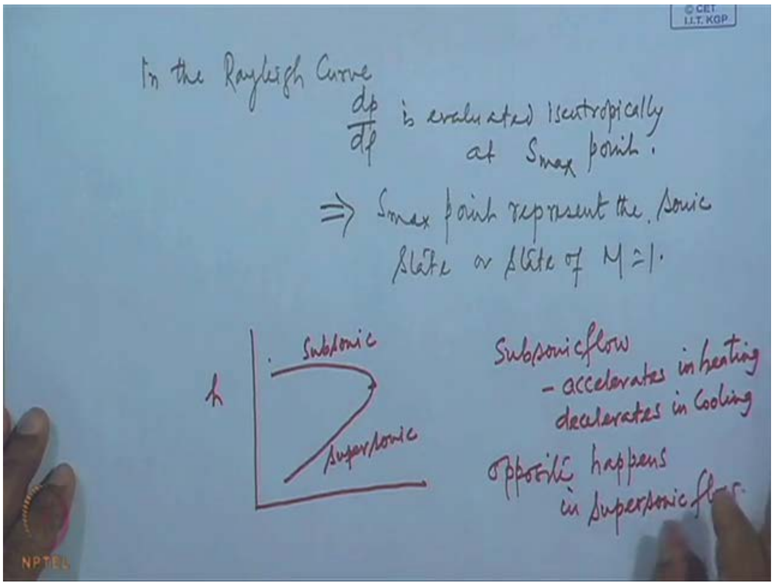
So, we see that in case of a heating the fluid takes the path to reach the maximum entropy condition, and while cooling it goes away from the maximum entropy condition.

This is the constant special line through the maximum entropy point, and let us say this is the similarly, you can have constant pressure line at this point shown as P 1.

Now, let us coming back to the relation that you already had that P plus g square by rho equal to constant, and this implies d P d rho equal to g square by rho square. Now, we know that d P d rho, if evaluated isentropic ally, represents speed of sound. So, d P d rho evaluated isentropic ally equal to a square or rather that, this represents the speed of sound only in the circumstances when, a change in pressure, a very small change in pressure; causes a very small change in density such that there is no change in entropy .

Now, we can see that this situation is fulfilled at the point of maximum entropy. So, that is d P d 0 represent speed of sound only for the special condition when a infinitesimal variation in pressure changes the density infinitesimally in such a way that there is no change in entropy, and from this figure we can see that this happens at the point of maximum entropy .

(Refer Slide Time: 21:22)



So, in the Rayleigh curve (No audio 21:17 to 21:56) is evaluated isentropic ally at s max point.

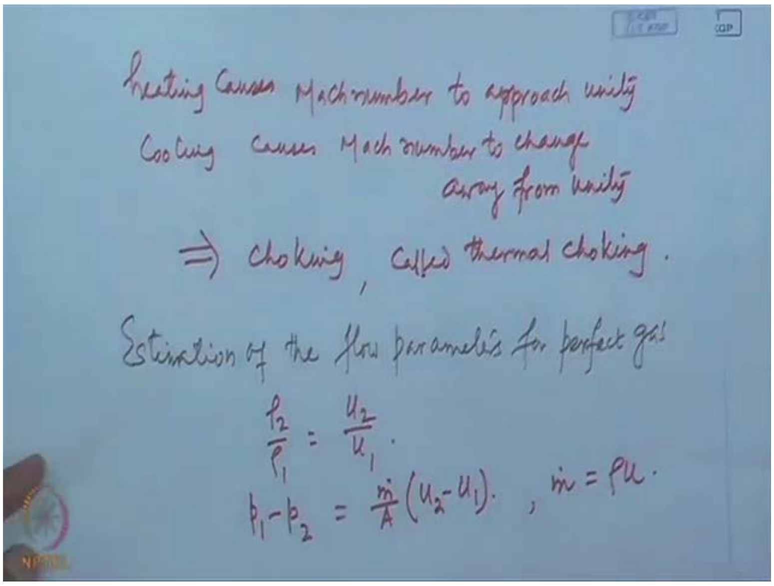
So, this implies that S max point represent sonic state, or state of M equal to 1. So, in the Rayleigh pressures are simple T 0 change process the point of maximum entropy represents the sonic state or state of mach 1.

Now, if we consider the state at point 1, beginning the state at point at 1 from here mach unity can be reached in several ways, they might be isentropic ally, they might be adiabatically at constant area, they might be at wheel changing friction is friction, they might be do with heat addition and; however, only for the simple heating process that maximum entropy point on the Rayleigh curve will represent the sonic condition or the sonic condition in the Rayleigh curve only be represented by that maximum entropy point.

Now, the branch of the Rayleigh curve, we once again come back to this Rayleigh curve. This is subsonic part this is supersonic part and we can see that a subsonic flow, if heated tends towards sonic case similarly, a supersonic flow when heated again tends towards the sonic state similarly, in case of subsonic flow, if cooling is allowed the mach number decreases while, it increases for supersonic flow. So, to from this you can say that a subsonic flow accelerate in heating in where, it decelerates in cooling. And opposite happens when the flow is supersonic, that is a supersonic flow will decelerate, if it is heated and it will accelerate if it is cool.

So, heating causes the mach number tends towards unity and cooling causes the mach number move away from unity.

(Refer Slide Time: 26:52)



So, heating causes heating causes mach number to approach unity. And cooling causes mach number to change away from unity. So, the heat addition or the amount of heat that can be added to a flow either subsonic or supersonic such that the maximum heat that can be added to a subsonic or supersonic flow can be such that the mach number become unity. The amount of heat cannot be greater than that for which the mach number that leaves the duct is more than 1 or different from 1.

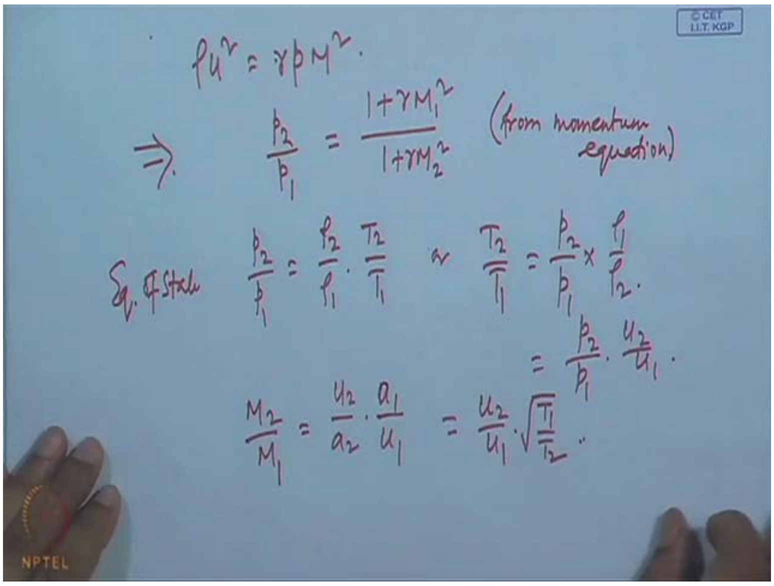
Now, if the addition is if the heat added is more than that heat or the maximum amount of heat that is required to take the mach number to unity then, there will be then, the flow will be chocked and the mach number at the exit will remain at unity, but there will be change in the inflow condition. And this chocking by heating is called as thermal chocking.

So, we have a chocking process here, that is, if we have a given duct, and we have a fixed amount of mass flow specified amount of mass flow and impulse function then there is an maximum amount of heat that can be added to the flow such that at the exit the mach flow mach number becomes unity. Now once this flow flow mach number reaches unity if more heat is added then, additional mass flow is not possible no more change in mach number, the mach number at the exit still remain unity; however, the inlet condition changes and the mass flow rate decreases.

So, what we say that the amount of maximum mass flow rate for a given duct depends on a the amount of heating for which the mach number will become unity at the exit and this is the condition of choking that is the maximum of heat, maximum amount of mass flow that can be allowed when heat is added, and since in this case choking is taking place due to heating it is called as thermal choking.

Now, let us try to estimate or evaluate all the parameters, considering the gas to be perfect. So, we will call the estimation of all parameters. Estimation of the flow parameters or perfect gas, we have from mass conservation, 1 relation from momentum equation, we have P 1 minus P 2 equal to mass flow rate per unit area divided by multiplied by u 2 minus u ,1 and to say that M dot is basically rho u .

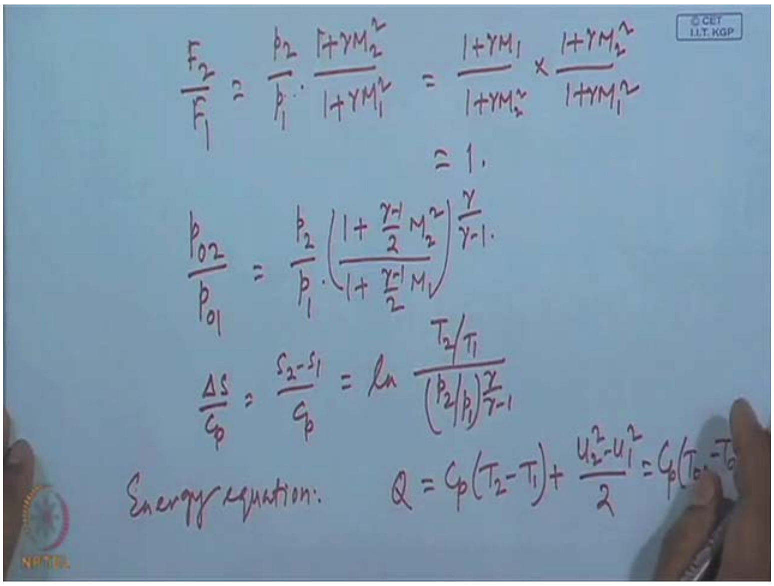
(Refer Slide Time: 32:19)



Now, we know rho u square is as we have seen is equal to gamma P M square. So, the momentum equation can be written as 1 plus gamma M 1 square by 1 plus gamma, M 2 square. The equation of state can also be written as equation of state P 2 by P 1 equal to rho 2 by rho 1 into T 2 by T 1 and or we have t 2 by t 1 equal to P 2 by P 1 into rho 1 by rho 2, that equal to P 2 by P 1 into u 2 by u 1.

We also have from the definition of mach number for a perfect gas. And using perfect gas relationship this becomes u 2 by u 1 into root over T 1 by T 2 using perfect gas relation for speed of sound.

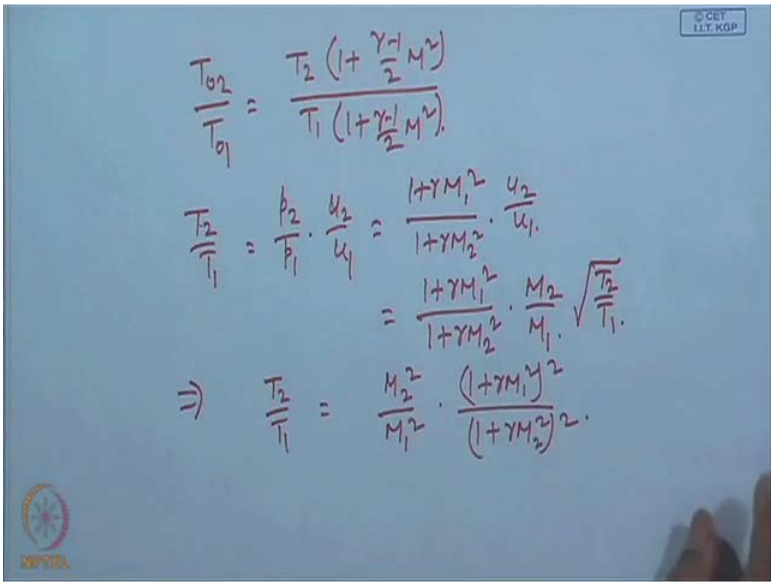
(Refer Slide Time: 34:56)



The impulse function relation can be written as F 2 by F 1 equal to P 2 by P 1 into 1 plus gamma M 2 square by 1 plus gamma M 1 square and since, F 2 by F 1 as you can see this P 2 by P 1 using the earlier relation P 2 by P 1 is this. So, we can write P 2 by P 1 already we have 1 plus gamma M 1 by 1 plus gamma M 2 square into 1 plus gamma M 2 square by 1 plus gamma M 1 square equal to 1 which of course, we have seen in the beginning that the impulse function is constant for flow in a uniform area duct without friction.

Now, from isentropic relations stagnation pressure relation to the power gamma by gamma minus 1 and the change in entropy delta s by c P that is s 2 minus s 1 by c P can be written as log T 2 by T divided by P 2 by P 1 to the power gamma by gamma minus 1. The energy equation can now be written as amount of heat added is C P into T 2 minus T 1 plus u 2 square minus u 1 square by 2 and this is C P into T 0 2 minus T 0 1.

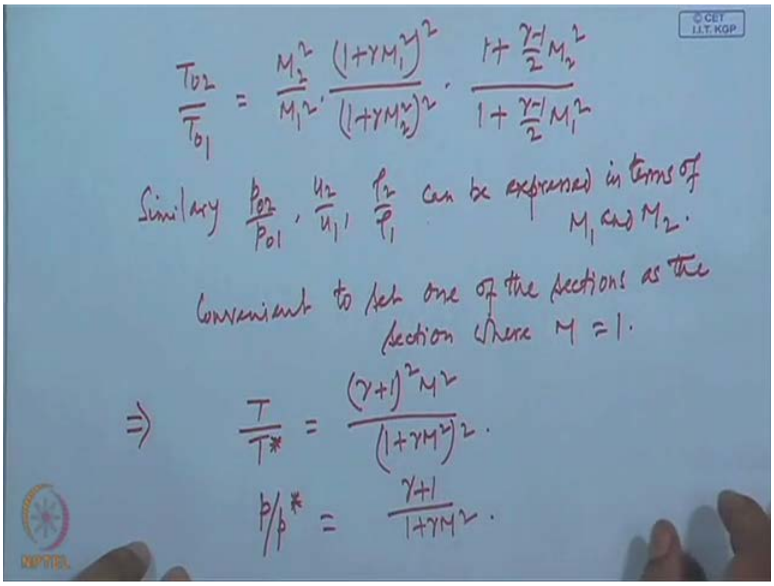
(Refer Slide Time: 38:29)



So, this is the form of the energy equation. Now, using the definition of stagnation temperature T 0 2 by T 0 1 is simply T 2 into 1 plus gamma minus 1 by 2 M square by T 1 into 1 plus gamma minus 1 by 2 M square. Now, these if we substitute this relation in particular the result that you have obtained from the momentum equation and from equation of state, and continuity, if we substitute into the equation of state what we get is…

Now, we already have T 2 by T 1 equal to P 2 by P 1 into u 2 by u 1 and for P 2 by P 1 we have 1 plus gamma M 1 square by 1 plus gamma M 2 square into u 2 by u 1 also we have u 2 by u 1, as M 2 by M 1 into root of T 2 by T 1 and this gives T 2 by T 1 equal to .

(Refer Slide Time: 41:34)

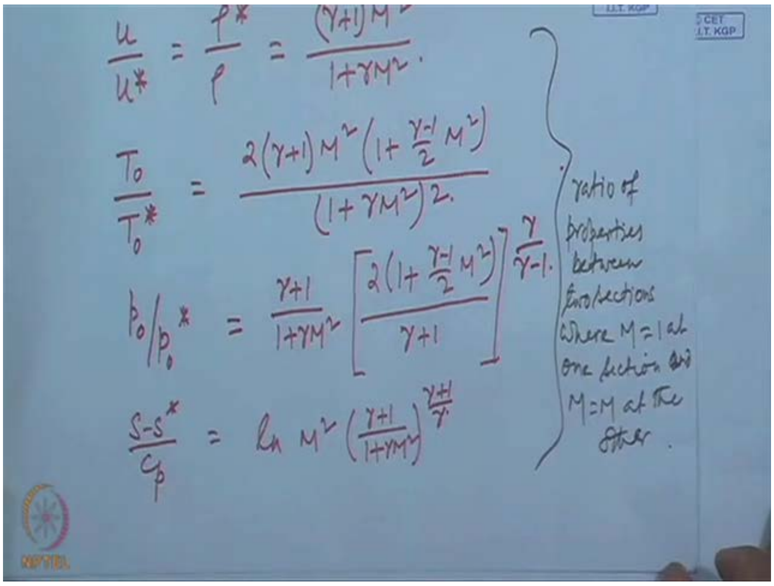


Now, this temperature ratio can be substituted in the into the stagnation temperature ratio. To find the stagnation temperature ratio in terms of mach numbers alone, and this gives us T 0 2 by T 0 1 as M 2 square by M 1 square into 1 plus gamma M 1 square 1 plus gamma M 2 square whole square into 1 plus gamma minus 1 by 2 M 2 square divide by 1 plus gamma minus 1 by 2 M 1 square.

Similarly, other relations for P 2 by P 1 and rho 2 by rho 1 and u 2 by u u 1 can be obtained for rather in terms of mach number. Similarly, P 0 2 by P 0 1 u 2 by u 1 rho 2 by rho 1 can be expressed in terms of M 1 and M 2.

Now, it is quite useful or convenient to set 1 of these step that is either the condition 1 or condition 2, as the sonic condition and convenient to set 1 of the sections as the section where M equal to 1, and this gives us then T, T star is gamma plus 1 square into M square divided by 1 plus gamma M square whole square P by P star equal to gamma plus 1 by 1 plus gamma M square.

(Refer Slide Time: 45:17)

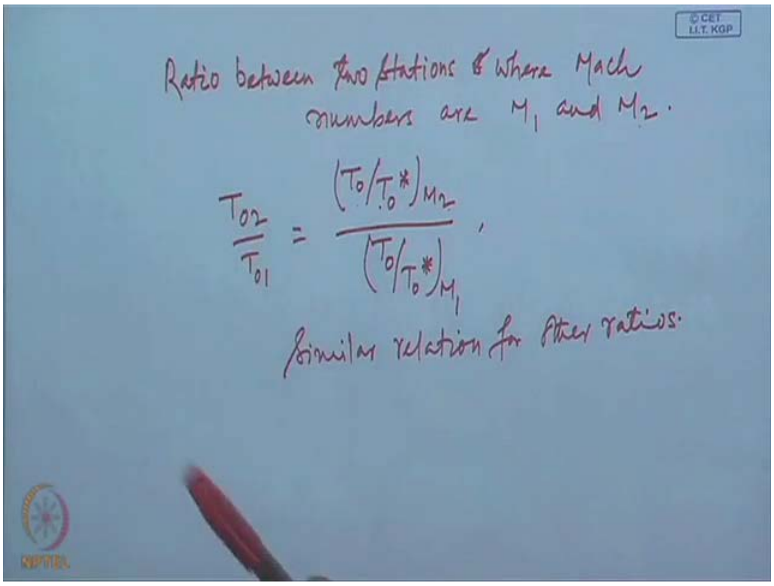


u by u star that is of course, a inverse for density ratio rho star by rho that is gamma plus 1 into M square by 1 plus gamma M square. (No audio 45:34 to 46:25) The stagnation pressure ratio between 2 station. 1 station is where mach number is 1, the other station and any other station where, mach number is M the stagnation pressure ratio becomes gamma plus 1 by 1 plus gamma M square into 2 into 1 plus gamma minus 1 by 2 M square divided by gamma plus 1 whole to the power gamma by gamma minus 1.

And this gives the entropy change essentially in these forms only. In the Rayleigh flow tables that are usually available in all books, in all text books. In this form only all the parameters are given that is parameters are expressed as a ratio of ratio between the properties at two sections in which, at mach number at 1 of the section is unity, and the other section is the value M. And the tables are available for different values of M, and once the ratio of these two properties are obtained.

So, we can say these are the ratio of the properties between two sections; the ratio of where properties between two sections, where M equal to 1 at 1 section, and M equal to M at the other. And as you and once ratio in these form are known the ratio between,

(Refer Slide Time: 50:03)



So, ratio between any two stations, ratio between two stations where mach numbers are M 1 and M 2.

Say we can find, T 0 2 by T 0 1 as an example, as T 0 by T 0 star corresponding to mach 2 and T 0 by T 0 star and similarly, for other ratios that is T 0 2 by T 0 1 where, T 0 2 is the stagnation temperature at station 2 where, mach number is M 2 and t 0 1 is the station 1 where, mach number is M 1 and that ratio can simply be obtained by T 0 by T 0 star evaluated at mach 2 that is in this relation, if M is used as M 2 that is what T 0 by T 0 star at M 2 and T 0 by T 0 star evaluated at M 1 similarly, for other relations also, for other ratios also same relation holds similar relation holds and all the properties be obtained and solution be completed.

So, to summarize we have considered now in today's discussion flow in a uniform duct with simple t 0 change that is we have considered heating and cooling in such a way that this heating and cooling simply changes the t 0 of the flowing gas, it does not add friction it does not change the property, or it does not change the composition and phase of course, it can be thought of that if there is a change in composition, but to we can an equivalent process is considered in which, the composition change or other effects of composition changes are not there only the change in t 0 happens, and for these flows we have seen that if the flow is subsonic or supersonic when it is heated it tends towards mach unity; however, if it is cooling then, the mach number flow mach number always moves away from unity that is a supersonic flow in cooling become more supersonic or its mach number increases, while for subsonic flow it is decelerate its mach number decreases.

In case of a heating both subsonic and supersonic flow approaches towards the sonic condition or mach unity, also we have seen that for a given mass flow rate there is a the exit mach number for a duct 1 and, if more heat is added the mass flow rate only adjusted. So, that the mach number at the exit remain unity, or rather there is a for a given duct there is an maximum amount of heat that can be added without changing the initial conditions, and this is called thermal choking, and then considering perfect gas, we have derived the relationship that can be used to find the properties between different sections and as you've seen that like isentropic flow, fanno flow, normal shock, and oblique shock flow. There are flow tables for this Rayleigh flow or simple T 0 change type of flow where, ratios are usually expressed in the ratio between two stations in which at 1 station mach number is unity the other station is a general station and from these ratios also considering 2 ratios at 2 different mach number we can find the desired property also.