High Speed Aerodynamics

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Lecture No. # 02

Review of Thermodynamics (Contd.)

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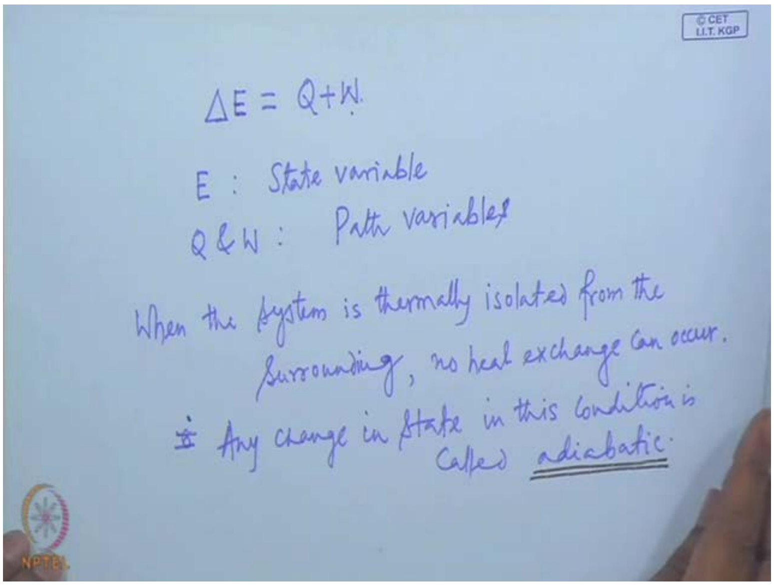

$\Delta E = Q + W.$

E : State variable

Q & W : Path variables

When the system is thermally isolated from the surrounding, no heat exchange can occur.

$\therefore$ Any change in state in this condition is called adiabatic.

So, we have written the first law of thermodynamics which states that the rate of change of internal energy of the system per unit mass. The change of internal energy per unit mass of the system is some total of the heat added to the system plus work done on the system.

Now the internal energy as, we have already mention is a state variable that it depends only on the initial and final state of the system irrespective of the process or path through which this change is taking place.

However this amount of heat added and amount of work done, they are measure of external effects and each of them depend on the path or in which the transition has taken place; that is changes from one state to other state. However it is there law states that,

that these two path dependant variables; some of these path to path dependant variables is independent of the path. This is the first law of thermodynamics as stated in classical thermodynamics and refers to a system which is in equilibrium. That is, if we consider the system to be certain mass of gas that is, when the gas is at rest. So of course, if you want to apply this for a flowing fluid or a moving fluid, then the energy content is not only the internal energy but, also say the kinetic energy and maybe other, change in other form of energy like potential energy and all are possible; so, this energy will then contain the kinetic energy and if necessary potential energy as well.

Now as we said that Q and W are path dependent variables; E is state variable, state variable; Q and W are called path dependent variables or path variables. Now if somehow, that the system is thermally isolated from the surrounding; that is when the system is thermally isolated from the surrounding, no heat exchange can occur; no heat exchange can occur. And if then any change take place, then the take the change will be due to the work done only and any change in state in such a situation in this condition is called adiabatic. That is any changes in the system which takes place, without heat transfer then the change is called adiabatic.

Similarly, if there are various other changes like isothermal, where no change in temperature takes place; the change in the process or the change maybe isobaric; that is the pressure is remaining constant while other changes, other variables are allowed and so on. Now looking to the work, see the most important way of performing work on the system is by compression. So most important approach of performing work is compression and if the compression is allowed in such a way that, the system persist a succession of equilibrium states; then this work done by compression can be expressed analytically.

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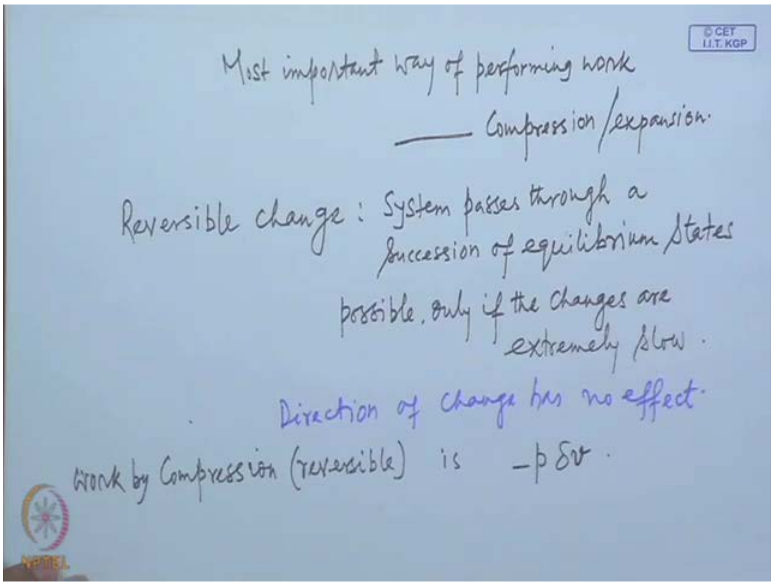

Most important way of performing work

— Compression/expansion.

Reversible change: System passes through a succession of equilibrium states

possible, only if the changes are extremely slow.

Direction of change has no effect.

Work by Compression (reversible) is $-p\,\delta v$.

So let us look to that, say most important way of performing work; that is compression or expansion. And analytic expression for this work is possible, if the changes are, changes are made in such a way that the system persist through a succession of equilibrium states. That is the system allows in some equilibrium state and that type changes are called reversible. So reversible change, that is system passes through succession of equilibrium states.

Now this is possible, if the changes are very slow, if the changes are extremely slow. Possible only, if the changes extremely slow and in this case the direction of change has no effect; the direction of change has no effect. Now if we consider then say a compressive work, then that extremely slow compressive work. Then at each stage during each stage of change, that is during an infinitesimal amount of change the pressure practically remains uniform and the work is due to a small decrease in volume. So the compressive work then work by compression of course reversible, reversible compression is p delta v and since the volume is decreasing so, this is p delta v.

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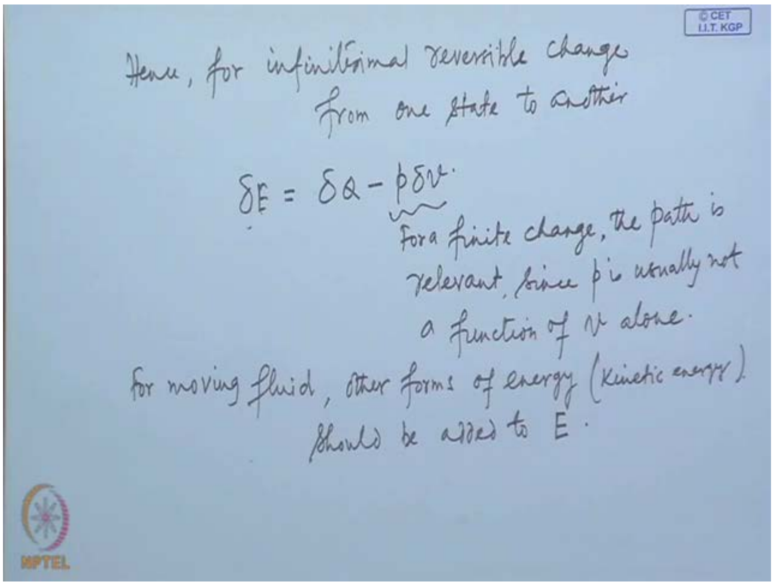

Hence, for infinitesimal reversible change from one state to another

$$\delta E = \delta Q - p\delta v.$$

For a finite change, the path is relevant, since p is usually not a function of v alone.

For moving fluid, other forms of energy (kinetic energy) should be added to E.

So hence for infinitesimal reversible transition from one state to other; hence for infinitesimal reversible change, reversible change from one state to another, the first law of thermodynamics now becomes delta E equal to delta Q minus p delta v, of course, we can write in differential form. Now again see this particular path by which initial and final state are joined is relevant; for a finite change, the path is relevant. Since p is usually not the function of v alone.

So, this is the first law of thermodynamics in differential form or infinitesimal form. And in classical thermodynamics apply to a fluid at rest. For moving fluid, this internal energy must contain the other form of energy also. We can write that for moving fluid other forms of energy say kinetic should be added to E and the equation is then the energy equation for flow. So the basic energy equation or basic form of the energy equation for a moving fluid is the same; where E now represents as the internal energy plus kinetic energy.

With this now, we will try to define a very important practical quantity known as the specific heat.

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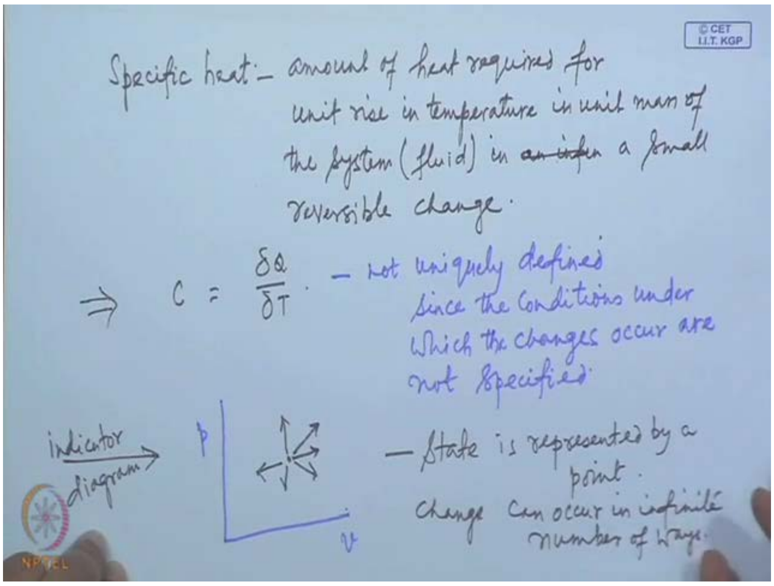

Specific heat:- amount of heat required for unit rise in temperature in unit mass of the system (fluid) in ~~an infin~~ a small reversible change.

$\Rightarrow \quad C = \frac{\delta Q}{\delta T}$. — not uniquely defined since the conditions under which the changes occur are not specified.

indicator diagram →

— State is represented by a point.

Change can occur in infinite number of ways.

Of course, amount of heat required to rise a temperature, unit rise in temperature in a small reversible change. So, amount of heat required for unit rise in temperature in unit mass of the system in an or say in a small reversible change. So, if we define the specific heat by C then, C becomes delta Q by delta T.

Now of course, is this is not uniquely; this is of course, not uniquely defined, because we have not specified how this change has taken place; not uniquely defined. Since the conditions, conditions under which the changes occur are not specified. See this, we can explain this way that a state of the system is just a point in the indicated diagram say, p v indicated diagram and a point represents a state, because in the indicated diagram a point fixes pressure and specific volume.

And as we have mentioned earlier that for a pure substance, if we fix any two variable then all other remaining state variables are fixed automatically; consequently a state represents a point; state is represented by a point.

Now the change of this state can occur in an infinite number of ways; change can occur in infinite number of ways. That is, see this is the current state of a certain system and let us see we change it in infinitesimally; this change can occur along this direction or along this direction or along this direction, along this direction or any such direction. So, these are all direction of change and since there are infinite numbers of neighbourhood points of this particular point so, we have infinite number of ways of this change.

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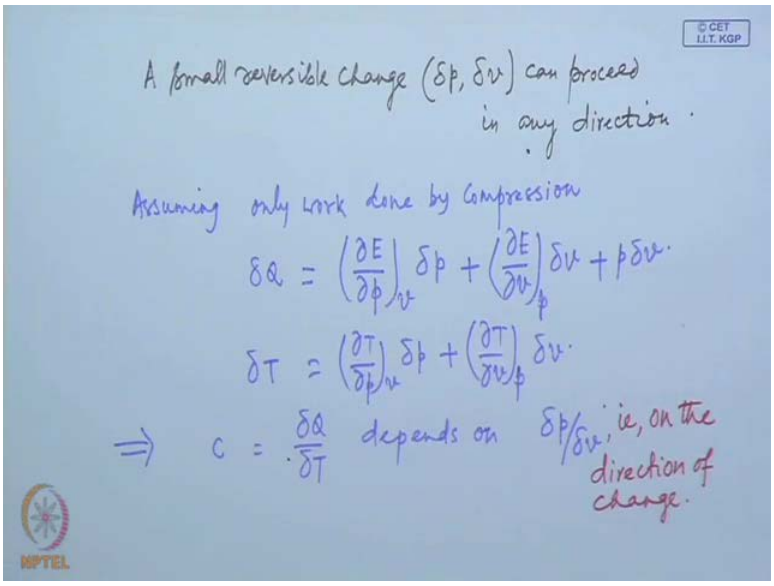

A small reversible change $(\delta p, \delta v)$ can proceed in any direction.

Assuming only work done by compression

$$\delta Q = \left(\frac{\partial E}{\partial p}\right)_v \delta p + \left(\frac{\partial E}{\partial v}\right)_p \delta v + p\,\delta v.$$

$$\delta T = \left(\frac{\partial T}{\partial p}\right)_v \delta p + \left(\frac{\partial T}{\partial v}\right)_p \delta v.$$

$\Rightarrow$ $C = \frac{\delta Q}{\delta T}$ depends on $\delta p/\delta v$, ie, on the direction of change.

So if we a small reversible change, a small reversible change which can be a small reversible change which can be denoted by say delta p delta v can proceed in any direction.

Now assuming that the only work is done by, assuming the only work done is by compression say assuming only work, work done by compression. We have delta q is d E d p at constant volume into delta p plus d E d v at constant pressure into delta v; that is we have written E as a function of pressure and volume plus p delta v. And similarly, the change in temperature, temperature is also a function of pressure and volume can be written as d T d p at constant volume into delta p plus. So, this shows that C equal to delta Q delta T depends on delta p, delta p by delta v; that is on the path direction of change.

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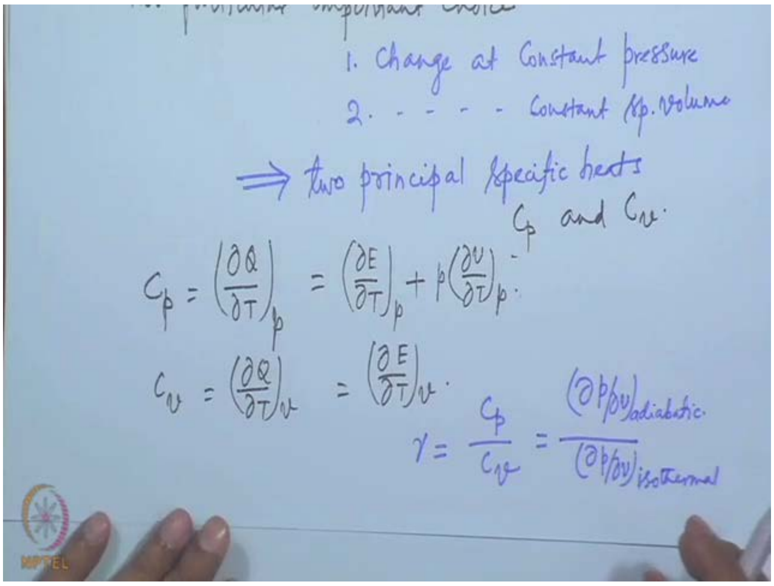

1. Change at Constant pressure
2. - - - - Constant Sp. volume

⇒ two principal Specific heats C_p and C_v.

$$C_p = \left(\frac{\partial Q}{\partial T}\right)_p = \left(\frac{\partial E}{\partial T}\right)_p + p\left(\frac{\partial v}{\partial T}\right)_p.$$

$$C_v = \left(\frac{\partial Q}{\partial T}\right)_v = \left(\frac{\partial E}{\partial T}\right)_v.$$

$$\gamma = \frac{C_p}{C_v} = \frac{(\partial p/\partial v)_{adiabatic}}{(\partial p/\partial v)_{isothermal}}$$

Now if we consider say two very important so, two particular choice or two particular important choice, two particular important choice: one is change at constant pressure, change at constant pressure and the other is change at constant volume, constant specific volume of course, constant specific volume and these two gives two principle, two principle specific heats. C p that specific heat at constant pressure and specific heat at constant volume; specific heat at constant pressure and specific heat at constant volume. That is c p is d t at constant pressure which from the earlier equation can be written as d E d T at constant pressure plus p d v d T at constant pressure. And C v is d Q d T at constant volume which is simply, d E d T at constant volume. The ratio of the two specific heats is a very important parameter in fluid dynamics; C p by C v which can be shown to be d p d v adiabatic, d p d v isothermal.

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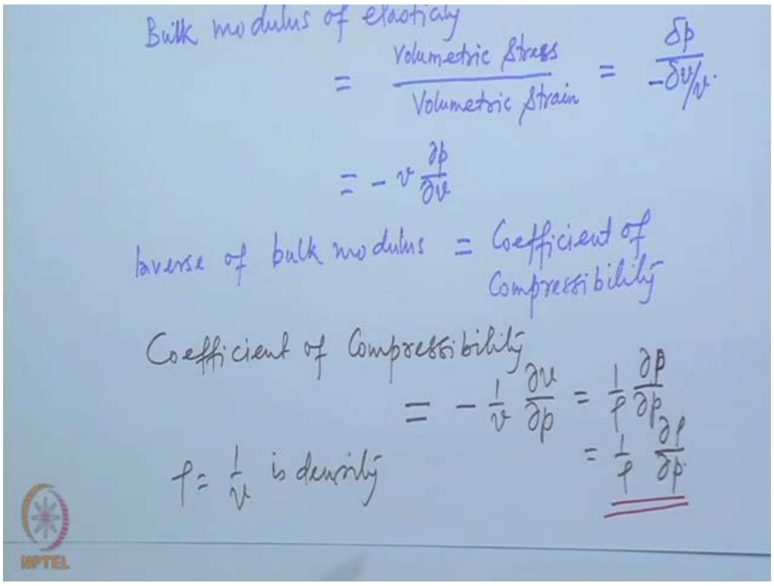

Bulk modulus of elasticity

$$= \frac{\text{Volumetric Stress}}{\text{Volumetric Strain}} = \frac{\delta p}{-\delta v/v}.$$

$$= -v\frac{\partial p}{\partial v}$$

Inverse of bulk modulus = Coefficient of Compressibility

Coefficient of Compressibility

$$= -\frac{1}{v}\frac{\partial v}{\partial p} = \frac{1}{\rho}\frac{\partial \rho}{\partial p}$$

$$= \underline{\underline{\frac{1}{\rho}\frac{\partial \rho}{\partial p}}}$$

$\rho = \frac{1}{v}$ is density

With the specific heats, we will define a very important fluid dynamic of a parameter which is say known as the bulk modulus say bulk modulus of elasticity. Bulk modulus of elasticity which we know the volumetric stress by volumetric strain and so ((no audio 28:56 to 29:26)) Now the volumetric stress is delta p and the volumetric strain is the delta v by v. And we know that if it is a compression, then that volume will decrease. So it is actually the when pressure increase, volume decreases. So, it will have a negative sign or we can write this to be minus v d p d v.

Now of course in fluid dynamics, the inverse of this parameter is more important and which is called coefficient of compressibility. So, the coefficient of compressibility is defined as … and since the inverse of the specific volume is density; this becomes to be one by rho, one by rho d rho by d p; one by rho d rho by d p; rho equal to one by v is density. And is a more preferred variable than specific volume in fluid dynamics. You know, since this definition also involves d p d v hence, they are also path dependant

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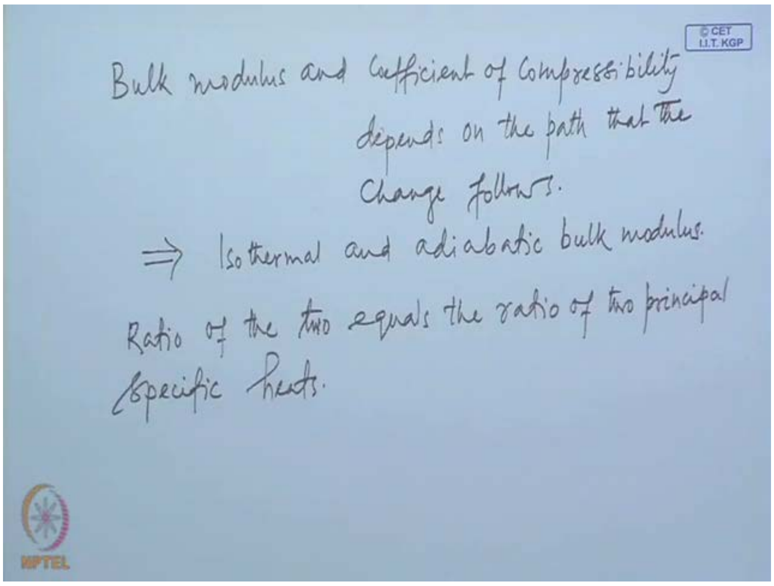

Bulk modulus and coefficient of compressibility depends on the path that the change follows.

⇒ Isothermal and adiabatic bulk modulus.

Ratio of the two equals the ratio of two principal specific heats.

So bulk modulus or coefficients of compressibility are also path dependant. So bulk modular and coefficient of compressibility depends on the path that the change follows. That is on each path, the values will have been deferent and again say two particular changes, two particular significant definitions are isothermal and adiabatic bulk modulus.

So, here also we will have isothermal and adiabatic bulk modulus and from first to again that their ratio will be equal to, the ratio of principle specific heats. ((No audio 34:54 to 35:44)) That is ratio of the two corresponding bulk modulus; that is adiabatic bulk modular by isothermal bulk modular is again C p by C v. Now each pure substance is represented by, a simple equation of state as we have already mentioned that for a, for a pure substance any state variable can be expressed via other state variables. And this equation is known as, the equation of state. So for a pure substance, we have one simple specific heat, one simple equation of state; it represents involving three parameters, three state variables. The most important equation of state or the thermal equation of state is called the perfect gas law or where p is given by rho r t. ((No audio 37:03 to 37:47))

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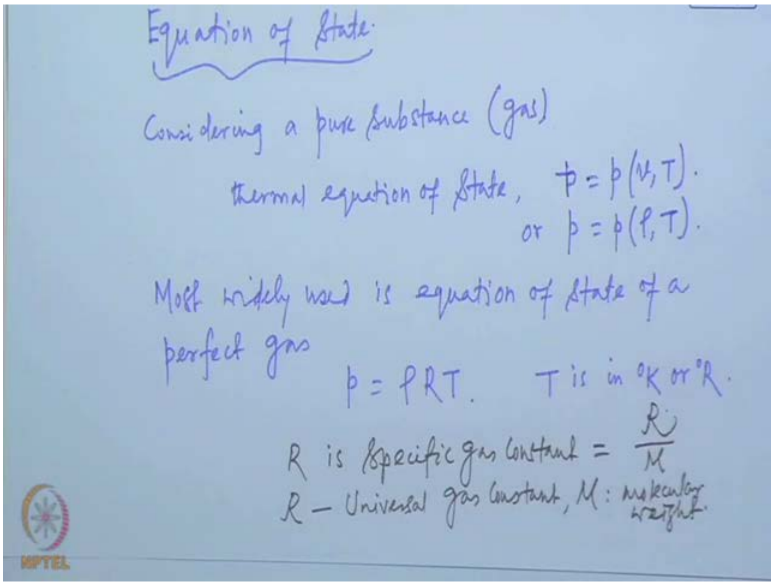

Equation of State:

Considering a pure substance (gas)

thermal equation of State, $p = p(v, T)$.

or $p = p(\rho, T)$.

Most widely used is equation of State of a perfect gas

$p = \rho R T$. T is in °K or °R.

R is Specific gas Constant $= \frac{\mathcal{R}}{M}$

$\mathcal{R}$ – Universal gas Constant, M : molecular weight.

Considering a pure substance of course, you are concerned only with gases so, it is gas. So thermal equation of state, T is or else say p is a function of specific volume and T or for fluid dynamics as you said that, density is a more preferred variable than specific volume; this the most important widely used equation of state of a perfect gas, which is p equal to rho R T. Where T is in degree Kelvin, T is in degree Kelvin or degree Rankine and R is specific gas constant which is universal gas constant by molecular weight; universal gas constant and M is molecular weight.

Now this equation holds for all gases, at low densities. Particularly, when the temperature is not very large and neither the pressure is too high. So, now this is the equation of state for a perfect gas.

Perfect gas are though the gases of course, these are hypothetical gases which is made up of molecules which are hard sphere without having any intermolecular forces and when they collide, they have they follows perfect collision. Of course, no gases under all conditions follows perfect gas law however, as we mentioned already that almost all gases particularly like low densities are so that.

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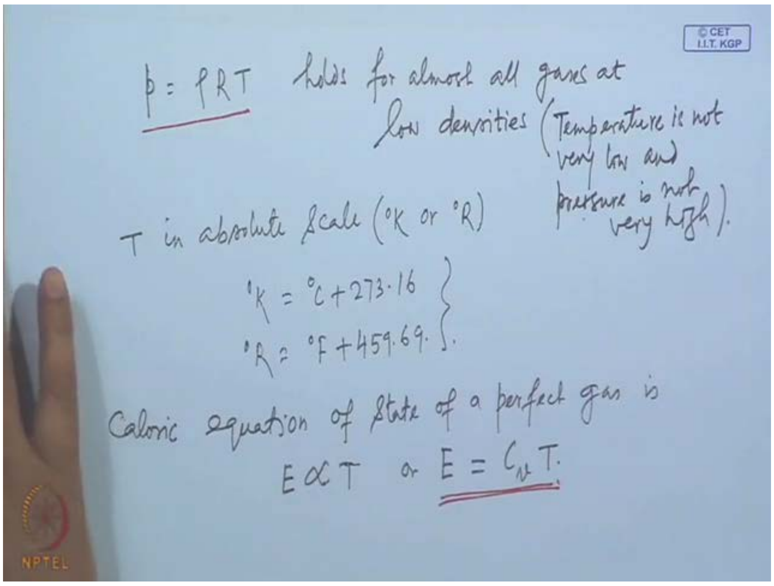

$p = \rho R T$ holds for almost all gases at low densities (Temperature is not very low and pressure is not very high).

T in absolute scale (°K or °R)

$$\left.\begin{aligned} {}^\circ K &= {}^\circ C + 273.16 \\ {}^\circ R &= {}^\circ F + 459.69. \end{aligned}\right\}.$$

Caloric equation of state of a perfect gas is

$$E \propto T \quad \text{or} \quad E = C_v T.$$

p equal to rho R T holds for almost all gases, all gases at low densities, T is not very low and pressure is not very high for a pure gas. We have already mentioned that, the T must be expressed in absolute scale, T in absolute scale; that is degree K or degree R. And as we know that, degree k is degree centigrade plus 273.16 and degree Rankine is degree Fahrenheit plus 459.69. In aerodynamics, the fluid concerned is usually air and for air this equation of state is a very very good approximations unless, the temperature is not extremely low or the pressure is not extremely high.

Of course, since air is not a pure substance; it is a mixture so, at very high temperature also due to the chemical reactions and thermal reactions that may occur. Then this equation of state can be modified accordingly for the mixture. However as I have already mentioned, that all gases at low densities approach a perfect gas. Now the caloric equation for a perfect gas is, caloric equation of state of a perfect gas is that E is a function of E varies, E simply of function of T or to be find that E equal to C v T.

Now let us say a few word, how this equation of state will change for a real gas? We know that real gases liquefy; these all real gases liquefy.

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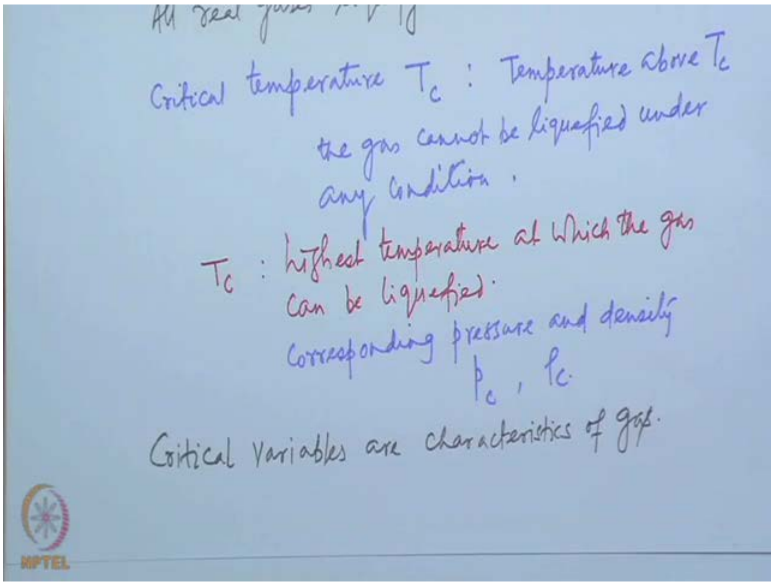

All real

Critical temperature T_c : Temperature above T_c the gas cannot be liquefied under any condition.

T_c : highest temperature at which the gas can be liquefied.
corresponding pressure and density
p_c , ρ_c.

Critical variables are characteristics of gas.

All real gases liquefy and there is a maximum temperature where this can occur; that is above this temperature under any condition, this gas cannot be liquefied. So, that temperature is called T c let us say called the critical temperature T c.

Critical temperature T c say at any temperature above T c, the gas cannot be liquefied; that is whatever pressure we apply, if the temperature is above this temperature, the gas cannot be liquefied. So T c is the highest temperature at which the gas can be liquefied; T c is the highest temperature, highest temperature at which the gas can be liquefied. And the corresponding pressure and density are corresponding to certain density are let us denote them as, density are p c and rho c.

Now this critical variables are characteristics of a gas; critical variables are characteristics of a gas that is they are fixed for a certain gas, like an identity of the gas, critical variables are characteristics of gas; that is each which gas is its own fixed critical temperature and critical pressure and of course, they depends on, the intermolecular forces that are present between the molecules of the gas.

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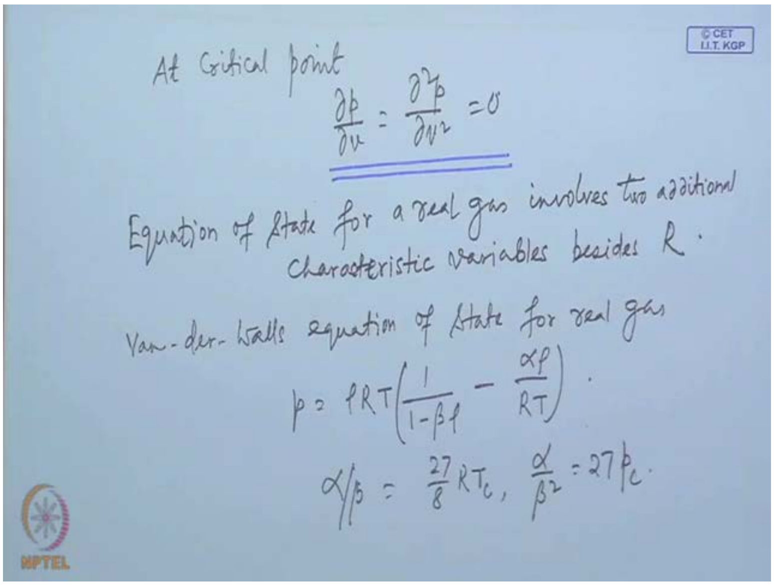

At Critical point

$$\frac{\partial p}{\partial v} = \frac{\partial^2 p}{\partial v^2} = 0$$

Equation of State for a real gas involves two additional Characteristic variables besides R.

Van-der-Walls equation of State for real gas

$$p = \rho R T\left(\frac{1}{1-\beta\rho} - \frac{\alpha\rho}{RT}\right).$$

$$\alpha/\beta = \frac{27}{8}RT_c, \quad \frac{\alpha}{\beta^2} = 27p_c.$$

At critical point, at critical point this is the condition that is satisfied and an equation of state for a real gas, equation of state for a real gas involves two additional, two additional characteristic variables of course besides R is a characteristic variable. And equation of state for a real gas will involve two additional characteristic variable say one of these, two of these, three T c P c and rho c or V c. And as you know that, most widely used real gas equation of state is the Van-der-walls equation, of course. There are many such equations ((no audio 53:19 to 54:00)) and these two constant alpha and beta square beta are given as, alpha by beta is 27 by 8 R T c and alpha by beta square is 27 P c.

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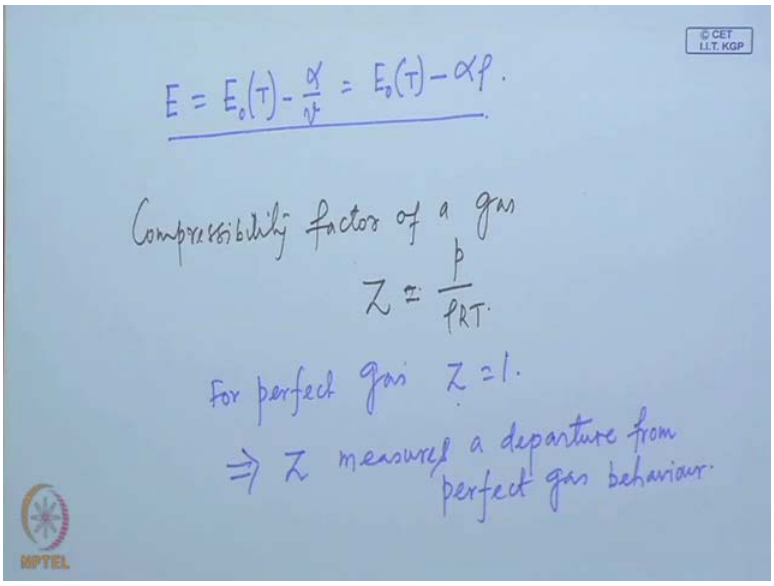

$$E = E_0(T) - \frac{\alpha}{v} = E_0(T) - \alpha\rho.$$

Compressibility factor of a gas

$$Z = \frac{p}{\rho R T}.$$

For perfect gas $Z = 1$.

$\Rightarrow$ Z measures a departure from perfect gas behaviour.

And the internal energy of a van- der- wall gas is given as ((no audio 54:57 to 55:33)).

Now we define a very important parameter which is known as the compressibility factor of a gas usually, denoted by Z is p by rho R T. So the perfect gas has compressibility factor Z equal to one. So you see that, it actually also measures the departure from perfect gas behaviour; Z equal to one. So, you can see that Z measures a departure, Z measures a departure from perfect gas behaviour.

So before we proceed further let us say, what we have discussed today? We have discussed the first law of thermodynamics or the statement of energy conservation. We have defined several processes; adiabatic isothermal and so on. Also we have discussed what are reversible processes? And we have discussed about the specific heats; different type of specific heats in particular to principle specific heats and again to principle bulk modulus and coefficient of compressibility. And finally, we have discussed about equation of state, in particular perfect gas equation of state and have also discussed deeply how this perfect this equation of states should be changed? If we consider the real behaviour of the gas particularly, at low temperature and high pressures and with this we will now, move on to a discussion on second law of thermodynamics and try to develop some additional relationships which are particularly useful in high speed aerodynamics.

And but, so our next discussion will be on second law of thermodynamics; but that is in the next lecture.