High Speed Aerodynamics

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Module No. # 01

Lecture No. # 19

Adiabatic flow in ducts with friction

In the last few classes, we have discussed adiabatic frictionless flow in duct with varying cross sectional area. And as we have discussed, they are simplified flow representing flow in nozzle, ducts, wind tunnels and many such.

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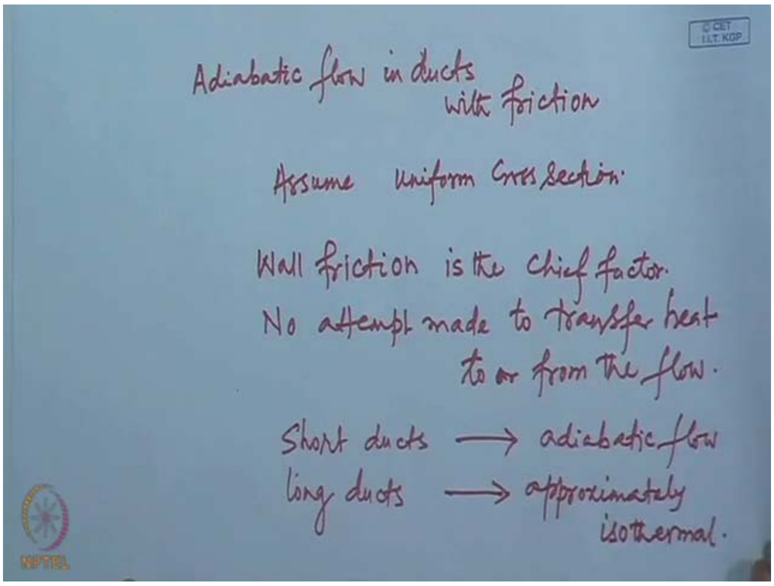

In our next example of flow through ducts, we will consider adiabatic flow with friction, adiabatic flow in ducts with friction and we will assume uniform area, uniform cross section. Now, flow in duct with friction is important in many practical applications. As an example in stationary power plants, in aircraft propulsion, high technology chemical process plant and natural gas transport through long pipes. We will consider that the wall friction is a chief factor, wall friction is a chief factor and no attempt made to transfer heat to or from the flow.

If the ducts are reasonably short, the flow is approximately adiabatic, but if the ducts are very long then sufficient area is available for heat transfer to make the flow non adiabatic. However, in that situation the flow can be treated as approximately isothermal. So, you can say that if the ducts are short ducts - short ducts the flow are adiabatic, long ducts heat transfer is possible, that is enough area to heat transfer take place and it is approximately the flow is an approximately isothermal .

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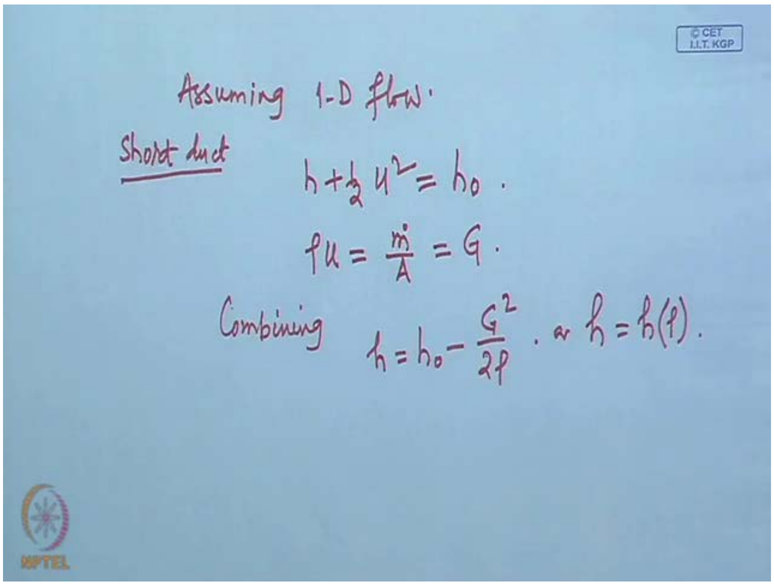

Once again, we will assume the flow to be one dimensional that is at any cross section, we are considering about the average flow velocity. Then the governing equation assuming 1 – D flow, since no heat is added and there are no other type of heat dissipation, the energy equation remain unchanged which is h plus half u square. So, and which we denote as the total enthalpy h naught. The mass flow rate is in a constant, this being the mass flow rate per unit area and let us denote this by G. Now, substituting this here what we get is that is, if we combine the two h 0 minus G square by two rho that is the enthalpy, is now expressed as a function of density alone, h 0 and G being constant.

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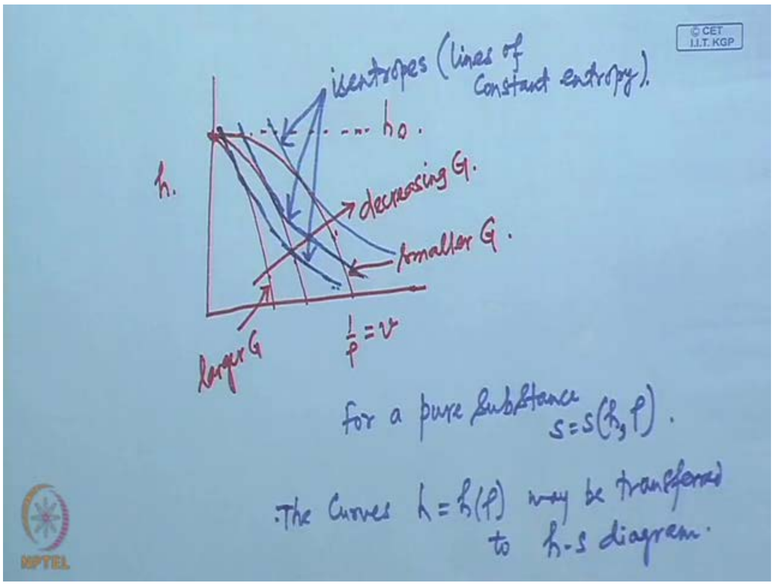

Now, this can be plotted in the specific volume for fixed h 0, we have different curves for different value of G and this is the direction of decreasing G, meaning that this curve corresponds to a smaller value of G compared to these and these. So, this is for as a larger G this curve represents a larger value of G and this is a smaller G. Now, if we plot the isentropes that is lines of constant entropy lines of constant entropy ((no audio from 08:19 to 08:50)). So, these are all these are the isentropes lines of constant entropy.

So, all possible states of the fluid for a given adiabatic constant area flow lie on one of these lines. Now, since we know that for a pure substance entropy or any flow property any flow property is a function of any other two state property. So, for a pure substance so the curves may be transferred to h s diagram and since all possible states of the fluid for a given adiabatic uniform area flow, lie in one of these curves that are either in the h rho diagram or in the h s diagram. Then, each of these diagrams represents a particular flow and these curves are called Fanno curves or Fanno lines.

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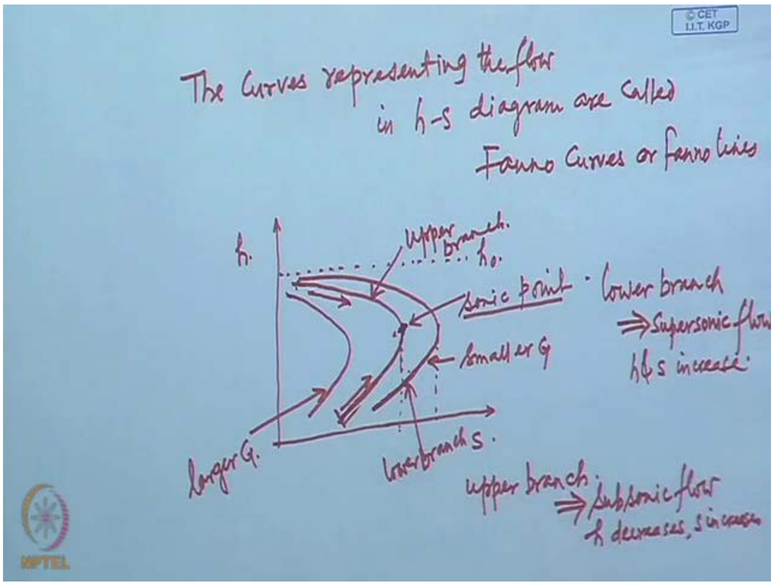

So, the curves representing flow in h s diagram are called Fanno curves or Fanno lines. And for all substances, these Fanno curves have a general shape which unlike this ((no audio from 12:43 to 13:25)) as before this represents a smaller G and these represents a larger G. So, these are the general shape of Fanno curves for all substances or for all fluids. Now, see that in an adiabatic flow, the entropy cannot decrease and as a consequence considering n of n any Fanno curve, let us say these the flow can be either follow this part of the path, along this direction or along this the flow cannot be in this direction or in this direction, that is not possible.

This represents so the maximum entropy point, consequently this upper branch of the Fanno curve in which is at the enthalpy continuously falls, while entropy increases this represents a subsonic flow. So, the upper branch represent upper branch in which these represent a subsonic flow, where h decreases, s increases, s of course will increase always and similarly the lower branch, that is this part or in this part this is the lower branch and this is the upper branch.

This lower branch now represents supersonic flow, where the enthalpy increases and entropy also increases. So, both h and s increase that is a supersonic flow through a uniform area duct with friction, both h and s increases and the maximum entropy point represents sonic point. Sonic point that is both the subsonic flow and supersonic flow they approach to the sonic point or the point at which the entropy is maximum. So, what we see that a subsonic flow accelerates to sonic condition and a supersonic flow decelerates to sonic condition. So, the effect of friction in a uniform duct when the flow is adiabatic is to accelerate, a subsonic flow to the sonic condition and decelerate a supersonic flow to the sonic condition.

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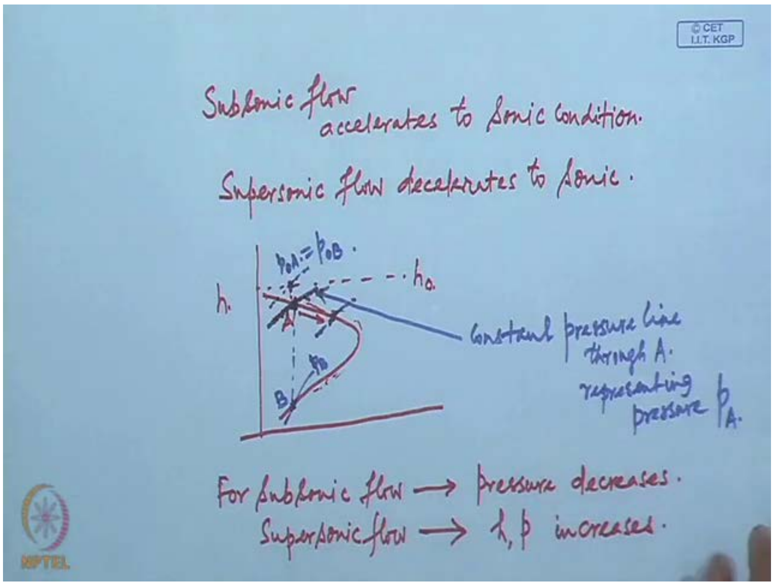

So, subsonic flow accelerates to sonic condition and supersonic flow decelerates sonic well in other way, the effect of friction is to accelerate subsonic flow with decrease in enthalpy and supersonic friction decelerates supersonic flow to sonic condition with increase in enthalpy. What happen to the pressure; to see what happen to the pressure we can consider a particular flow with fixed G. Let us say, the duct the flow is at condition A the flow is at condition A. Then, the pressure at this point can be obtained from the constant pressure line passing through that point. Let us say, this is the constant pressure line passing through that point. So, this point represents constant pressure line through A and this represents a pressure of P A.

The corresponding point on the total enthalpy line that is this h 0 line. Similarly, here also you can have a constant pressure line so this represents P 0 A. Now, since the flow can move only along this path. So, at a downstream station in the duct if the initial point, initial state is A, the subsequent point will be say somewhere here and this will of course, be a lower value of pressure. So, in this case pressure decreases ((no audio from 23:03 to 23:37)) this is also of course, obvious from other facts that if for the subsonic flow the enthalpy is decreasing, because the flow is accelerating. So, the energy of the flow is being converted to kinetic energy part consequently the internal energy is decreasing which results in a decrease in enthalpy and Pressure.

And, obviously the opposite happens in case of a supersonic flow if we consider again a corresponding point, here this is a curve for P B and this is the point B and this is of course, P 0 A equal to P 0 B. So, in supersonic flow the flow is decelerating consequently the kinetic energy is being converted to internal energy. So, enthalpy and Pressure increases so h and P increases. And, since in this adiabatic flow the entropy cannot decrease. So, the flow cannot come flow from this side cannot come to this side and flow from this side also cannot go to that side. Consequently a subsonic flow cannot accelerate to supersonic flow; similarly a supersonic flow cannot decelerate to a subsonic flow.

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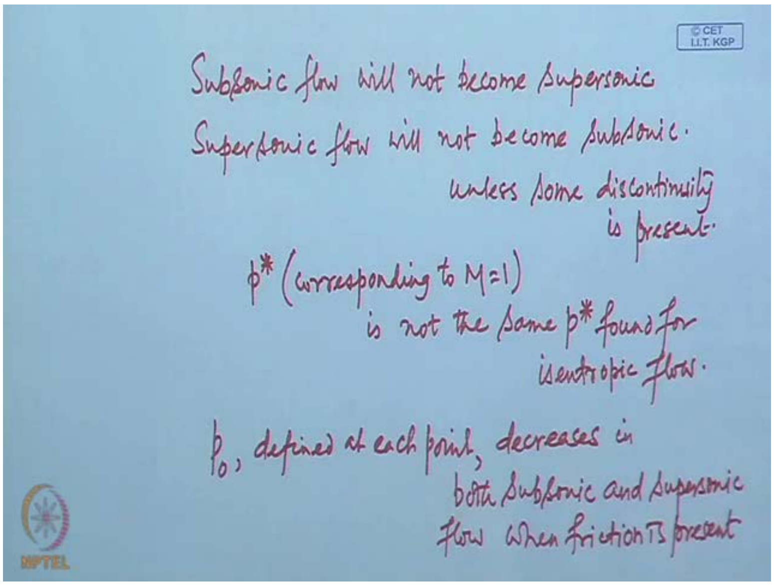

So, subsonic flow will not become supersonic and supersonic flow will not become subsonic unless some discontinuity is provided, unless some discontinuity is present. Now, the limiting pressure coming back here the limiting pressure beyond which entropy would suffer a decrease of constant Mach number unity and the pressure is again denoted by P star. However, this must be remembered that this P star is not the same P star as in isentropic flow. The value of P star that we derived earlier is not this particular P star. So, that also this P star corresponding to M equal to one is not the same P star found for isentropic flow.

Now, also remember that the isentropic relations are also not applicable in this case because this flow is not isentropic. However, at before as we have discussed that the stagnation pressure or stagnation temperature can be defined at any point, assuming that the flow there is brought to rest isentropically. So, we can define a isentropic stagnation pressure at each and every point in this flow also and we see that this isentropic stagnation pressure delays decreases, as a result of friction and it decreases irrespective of whether the flow is subsonic or whether the flow is supersonic. So, P 0 defined at each point decreases in both subsonic and supersonic and supersonic flow when friction is present.

Now, since we see that the flow here also reaches to the sonic condition, if of course the required length is present or the required amount of frictional force is there and so a chocking occurs due to friction also so there is a chocking due to friction. Now, let us consider that stagnation enthalpy flow per unit area A length of duct are such that the Mach number unity is reached at the end of duct. Now, if the duct length is further increased what we have discussed earlier from that, we can say that no further increase in Mach number if the flow were earlier subsonic is possible. Similarly if the flow were earlier supersonic, no further decrease is Mach number is possible.

So, some sort of adjustments in the flow is necessary. If the flow were if the flow were subsonic then the adjustment will be in the form of reduction in the flow rate, that is the flow rate will decrease and we can say that the earlier the flow was chocked. If the flow is supersonic then this adjustment will involve appearance of shock waves and for sufficiently large increase in duct length, there will be a wave propagation mechanism and ultimately the flow is chocked. So, if we have an appropriate length so for a given stagnation enthalpy and flow rate and the length of the duct, if the mach number of unity is reached at the end of the duct. Then, we will further increase in the duct length the mass flow rate will fall and again the maximum Mach number will shift to the tip to the end. So, you can say that when Mach number unit is reached that will always reach at the end of the tube and that corresponds to the maximum flow rate, for the given total enthalpy and length of duct and the flow is chocked.

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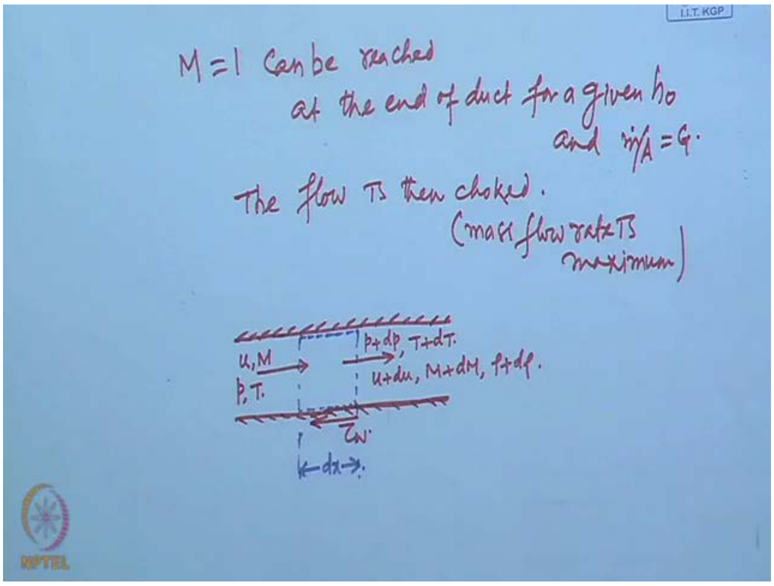

So, M equal to one can be reached at the end of duct, for a given h naught and M dot by A G. The flow is then chocked, that is the mass flow rate is maximum; mass flow rate is maximum that is possible for the given condition and as you have seen that, if we increase duct then there will be adjustment and mass flow rate will decrease. And, if the flow were subsonic, if the flow were supersonic a shock will appear and again a mass flow rate will decrease.

Now, let us come to the full mathematical analysis for this problem let us consider a uniform area duct. Let us consider a control volume and these are the control surfaces let us say, the flow that enters here has flow velocity u, mach number M, the pressure is P, temperature is T and so on. And the flow that comes out here has a pressure P plus d P, a temperature T plus d T, flow velocity u plus d u, mach number is also M plus d M, density is rho plus d rho and so on. Let us say that is the direction of the frictional force and let us consider this length to be d x.

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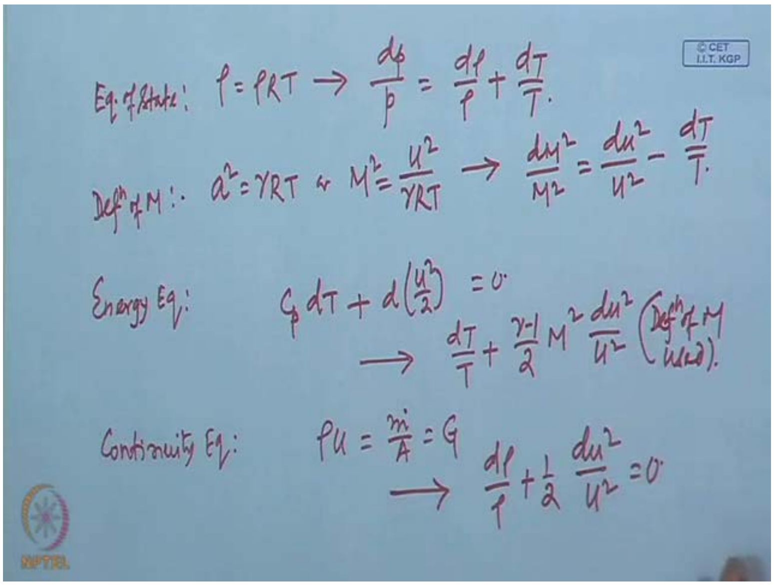

Now, writing the appropriate relations for these we have from equation of state P equal to rho R T or in the differential form, we have d P by P equal to d rho by rho plus d T by T. We defined speed of sound T or M square equal to u square by gamma R T which gives us d M square by M square equal to d u square by u square minus d T by T. So, write equation of state, definition of mach number can write the energy equation in the form since you are considering the perfect gas C p d T plus d of u square by two that equal to zero and this results in d T by T plus gamma minus one by two M square definition of mach number that is used to obtain this relation. The conservation of mass or continuity equation gives us G constant or which in the differential form becomes d rho by rho plus half d u square by u square equal to zero.

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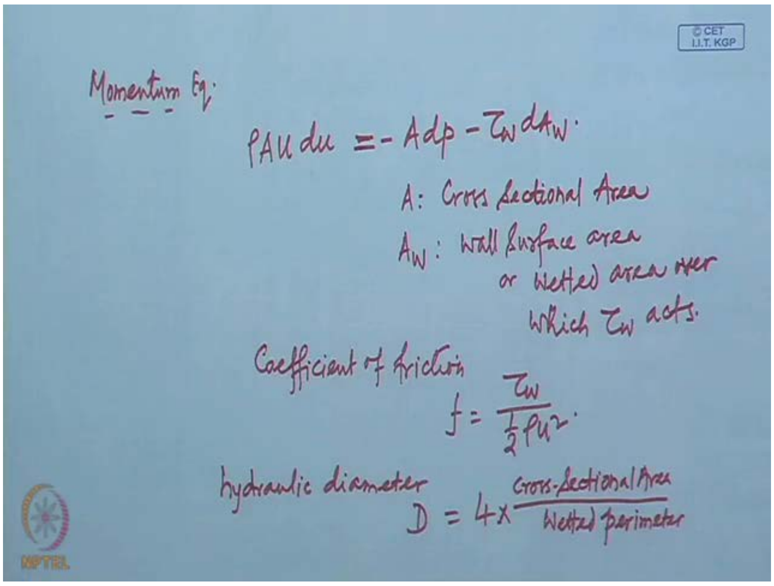

So, we already have four equations now we will write the momentum conservation momentum equation, but this time momentum equation with friction it keeps that the rho A U d u into minus A d P minus tau W. Now, this shear states that acts over all the entire circumferential area. So, this is will be multiplied by area, but not by the cross sectional area rather the surface area.

So, that A is cross sectional area associated with mass flow and A W is the wall surface area or more commonly known as wetted area over which tau W acts for a for a duct flow, a more useful form of this frictional force is expressed in terms of coefficient of friction. So, you define a coefficient of friction define a coefficient of friction friction f equal to tau W by half rho u square. It is also convenient to write a hydraulic diameter let us say D equal to four times cross sectional area by wetted Perimeter. And as can easily be seen that, if the duct is circular hydraulic diameter same as the geometric diameter.

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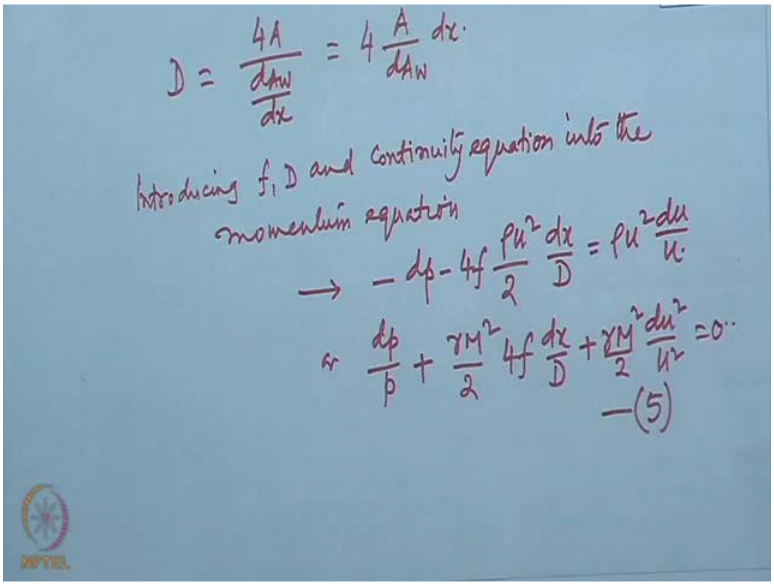

However, for a non circular tube the hydraulic diameter is more useful and mathematically that can be written as four times the cross sectional area by wetted parameter which is d A W by d x. And for our use, you can write this to be four A by d A W d x. Now, introducing this frictional coefficient of friction and hydraulic diameter and also the continuity equation into the momentum equation. So, introducing f D and continuity equation into the momentum equation we get in the differential form minus d P minus four f rho u square by two or d P by P plus gamma M square by two four f d x by D . So, yet this is our fifth equation also additional equation we can use. So, this is our let us say this we will denote as our fifth equation and coming back well this is our equation number one, this is equation two, this is equation three and this is four.

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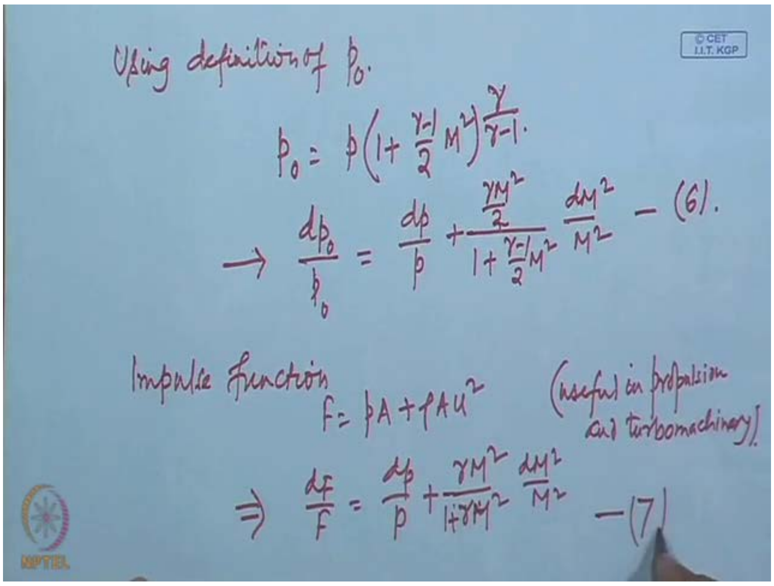

Using definition of P 0 that is P 0 equal to P in to one plus gamma minus one by two M square by gamma by gamma minus one. We have d P 0 by P 0 equal to d P by P plus gamma M square by two plus one by gamma minus one by two M square which will be our equation six. In many cases, an impulse function is used and which is a very useful particularly turbo machineries and Propulsion devices and which can be defined as P A plus rho A u square and turbo machinery applications and this results in d F by F equal to d P by P plus gamma M square by one plus gamma M square d M square by M square and this we will denote as our seventh equation. So, you see that you have now seven equations in seven unknowns and instead of treating them as differential equation, we can even treat them as algebraic equations or then the unknowns or the differentials.

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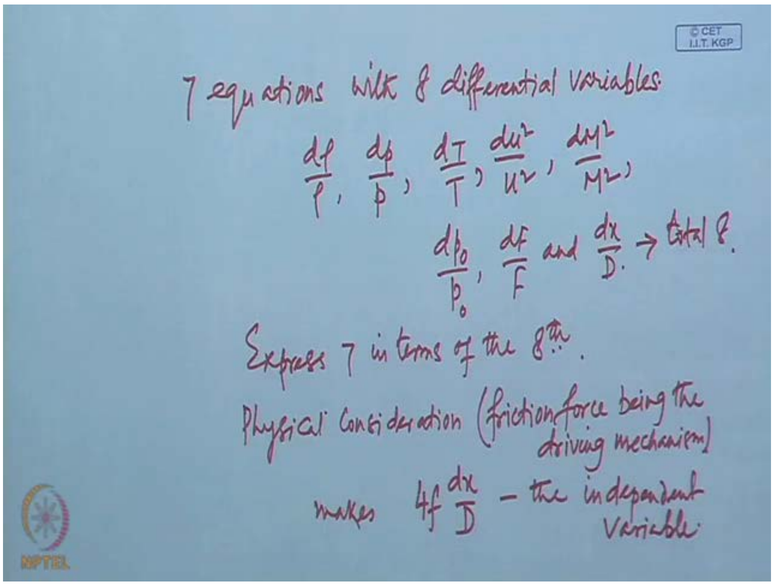

So, we have here seven equations with eight differential variables. So, we have seven equations with eight differential variables, the eight differential variables are d rho by rho, d P by P, d T by T, d u square by u square, d M square by M square, d P 0 by P 0, d F by F and d x by D . So, what we have all these seven equations we treat them as algebraic equation not as differential equations and treat these as the variables. So, we have seven equations for eight unknown variables and we can express seven in terms of the eighth.

So, we express seven in terms of the eighth, now for eighth we can have any one as our choice. However, physically this flow is the driving mechanism; in this flow is the friction and hence the term representing viscous friction can be taken as the independent. So, physical consideration that is driving mechanism being the friction force, being driving mechanism you can see that that four f d x by D can be chosen as the independent ((no audio from 55:38 to 56:09)). This also justifies because this contains the geometric parameter d x by D and obviously more independent than the others.

So, we will now solve these seven equations for the seven unknown variable. Of course, all these variables will be expressed in terms of four f d x by D and we will do that in the next class ((no audio 56:47 to 57:40)).