**High Speed Aerodynamics**

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**Module No. # 01**

**Lecture No. # 17**

**Flow in ducts**

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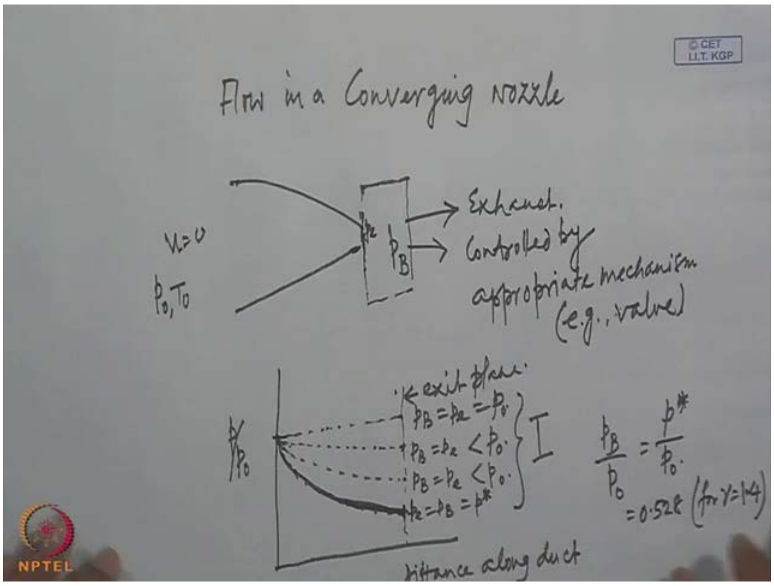


So, we will continue our discussion on flow in a converging nozzle. So, flow in a converging nozzle and what we stated that let us have a converging nozzle which is connected to a reservoir (( )) flow velocity 0 and pressure and temperature are of course the total pressure and total temperature. At the other end, we can control this pressure by some valve mechanism or if the pressure is p B controlled by appropriate mechanism, say as an example valve and then, the flow is going out through the exhaust.

The pressure at the exit plane will denote as p e, that is pressure on the exit plane and then, we have discussed how this flow will develop as p B is decreased gradually. You have seen that distance along duct say this is the end of the duct exit plane. When p B equal to p 0, p B equal to p e equal to p 0, then we will reach, there will be no flow because in viscid flow, the pressure gradient is the only driving mechanism and since, there is no pressure gradient, there will be no flow.

Now, if the pressure is gradually decreased, there will be a flow and pressure will continuously fall to the value of p B at the exit plane. If it is still reduced, the flow velocity will increase, mass flow rate will increase and these are all p B equal to p e, but that will be less than p 0. If we continue to decrease this p B, then we will reach to a condition of p e equal to p B equal to p star which we have derived earlier in our one-dimensional flow analysis at which the flow velocity in the exit plane reaches sonic value.

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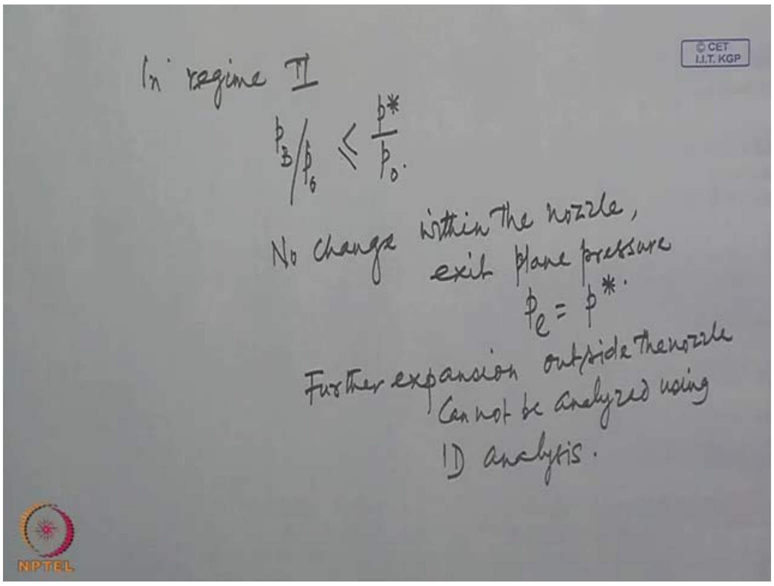


So, corresponding to p B by p 0 equal to p star by p 0, which we know that equal to 0.528 for perfect gas with gamma equal to 1.4 when gamma equal to 1.4. If this p B by p 0 value is brought down to 0.528, then the flow velocity at the exit plane will reach sonic and that is the maximum speed that can be achieved in a converging duct. Further, reduction in the pressure that is further reduction in the back pressure will not affect the flow in the nozzle that is within the flow. This part, this pressure distribution, it will let us call this belongs to flow regime one or you have seen that back pressure must be equal to exit pressure or if exit pressure cannot be different from the back pressure as long as this flow is subsonic and once the flow speed at the exit plane becomes sonic, the pressure at the exit plane become p star and further reduction in back pressure is not going to affect the flow within the nozzle. That is the further expansion from p star to p B will take place outside the nozzle in regime two. In the regime flow regime two which you can show here pressure below this, that is p B by p 0 less than p star by p 0. No change within the nozzle exit plane pressure p e remain fixed at p star and further, expansion outside the nozzle and this cannot be analysed using one d analysis, using one-dimensional analysis. This part of the flow we cannot analyse.

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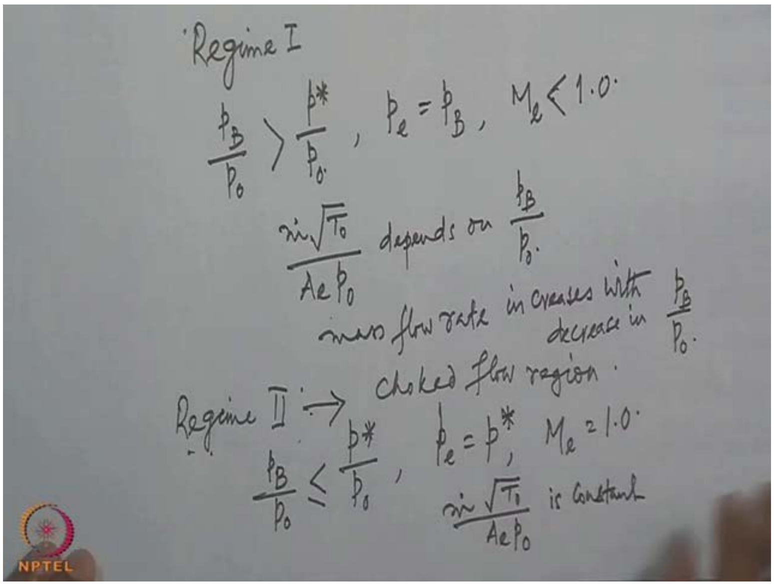


So, in nozzle flow, we can make in two regimes. In regime one, we have p B by p 0 greater than p star by p 0, p e p B exit mach number less than 1.0 and as we decrease the pressure from p 0 to p star, the mass flow rate continuously increases. That is that mass flow rate m naught T 0 by A e p 0 depends on p B by p 0. Eventually, this mass flow rate increases with decrease in p B by p 0.

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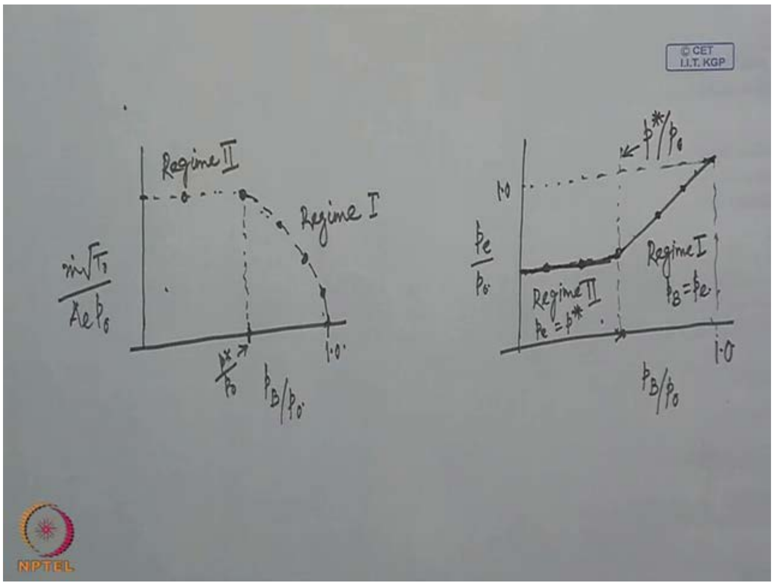

In regime two, we have already stated that p B by p 0 is less than p star by p 0, p e equal to p star M e and m dot root T 0, that is the mass flow rate is constant. So, what we see that the regime two is basically choked flow regime two is choked flow regime. Now, you can show how this mass flow rate variation of mass flow rate with back pressure on p B by p 0 is one. There is no flow. So, mass flow rate is 0. This is the value for p star by p 0 and so this part belongs to regime two, this part belongs to regime one. In regime one as the pressure ratio decreases, p B by p 0 decreases, the mass flow increases. So, these are corresponding to those different curves that we have shown there. So, this is what we call regime two, the choked flow regime and this is the unchoked regime.

We can also see that a relationship between the exit pressure and the back pressure, let us say this correspond to p star by p 0. On p B by p 0 is say 1.1 in regime. One the back pressure and exit plane pressure, they are same. So, these are however once the back pressure reaches that p star value of p star, then further reduction in back pressure is not going to effect the exit plane pressure and the exit plane pressure remain constant and this regime two, the choked flow regime and the unchoked flow regime ((no audio 16:14 to 16:45)).

Now, given a duct and the initial conditions and from the specified value of back pressure, we can calculate all the flow quantities using those isentropic area pressure relation, area mach number relation and other isentropic flow relation.

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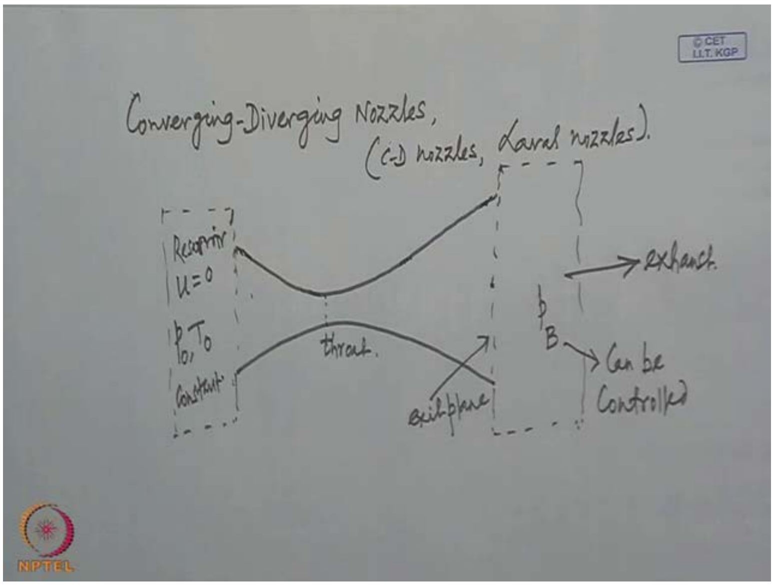


Now, let us discuss what is called the converging diverging nozzle. Converging diverging nozzles are also quiet often called CD nozzles. They are also known as level nozzles. Once again let us consider that this converging diverging nozzle is connected to a reservoir, where the flow velocity, there is no flow and p 0 T 0, they are held constant and these p 0 T 0 are held constant. This is the exit plane and let say this is in the downstream again connected to a mechanism through which we can control back pressure which can be controlled. This is the throat.

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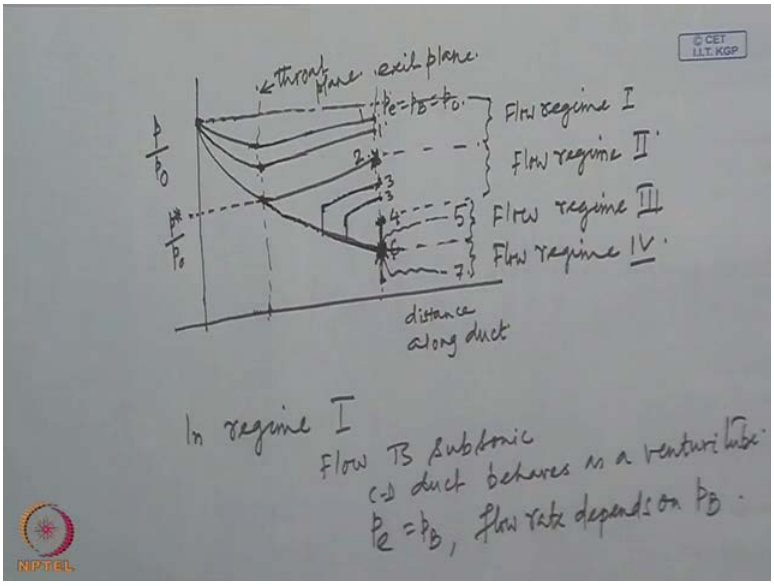

Now, once again considering this controlled pressure, let us say how the flow will behave within the nozzle as the back pressure is controlled. This is once again distance along duct. This is the exit plane when p by p 0 is same as p B by p 0, there is no flow. Now, if back pressure is decreased gradually, let us say that this is the location for the throat plane. If back pressure is slightly decreased or flow develops and within the converging part of the duct, the flow accelerates to a certain speed and then, in the diverging part that flow will decelerate and again pressure will increase and will reach to this value of p B. Still if it will decrease, flow velocity will increase, mass flow rate will increase and now at the throat plane, this is the value of the pressure that will be reached and again, the flow will decelerate in all these situations. The flow will remains subsonic and exit pressure will become same as the back pressure.

Let us say, now what you see here that the pressure at the throat is smaller than the pressure at the exit plane or back pressure. So, for certain value of back pressure, let us say for a certain value of back pressure, the pressure at the throat plane reaches the critical value. Let us say this is the critical value p star y. So, for this particular value of the pressure, let us say that we have reached what is the sonic pressure at the throat and flow velocity at the throat also becomes sonic. However, the pressure at the back is not yet sufficient (( )) low to cause further expansion in the diverging part. Consequently, the pressure in the flow in the diverging duct will again decelerate and will reach to this value p B.

However, if the pressure value is back pressure corresponds to say this level, then you see that there will be full expansion to the nozzle. Exit plane will denote all these curves by say, one this by two. Now, what happens if the back pressure is something between this and this, then the flow in the diverging part is not fully isentropic for any intermediate value of pressure say here. The flow initially will accelerate or expand in the diverging duct, reach to a certain supersonic speed and then, we will have a normal choke and then, it will further diffuse to the back pressure called both these curves as three.

At certain back pressure, we will denote it four, where this shock will stand at the exit plane and anything in between the normal shocks and any pressure between this point and between this level and this level. Then, there will be no more normal choke at the exit plane. However, oblique choke will develop from the leap of the nozzle and they interact in the jet downstream and through repeated interaction of between these oblique shocks and expansion waves in the jet downstream, the ultimate pressure will be established, so that this will be something like this. This we will call five and this level we will call six. At any pressure level below this full isentropic expansion, well that is denoted here by six.

We will have expansion fan originating from the leap at the exit plane of the nozzle and those expansion fan will interact and then, series of such expansion fan and oblique's of interaction, the final pressure or final expansion will be completed in the jet.

Now, we denote these as flow regime one ((no audio 29:14 to 29:46)), flow regime three and this will be flow regime four. So, in regime one, flow is purely fluid is subsonic. The converging duct, converging diverging duct behaves like a venture, behaves as conventional venturi that p e equal to p B and flow rate depends on p B which we are denoting by this curve or any curve within this regime.

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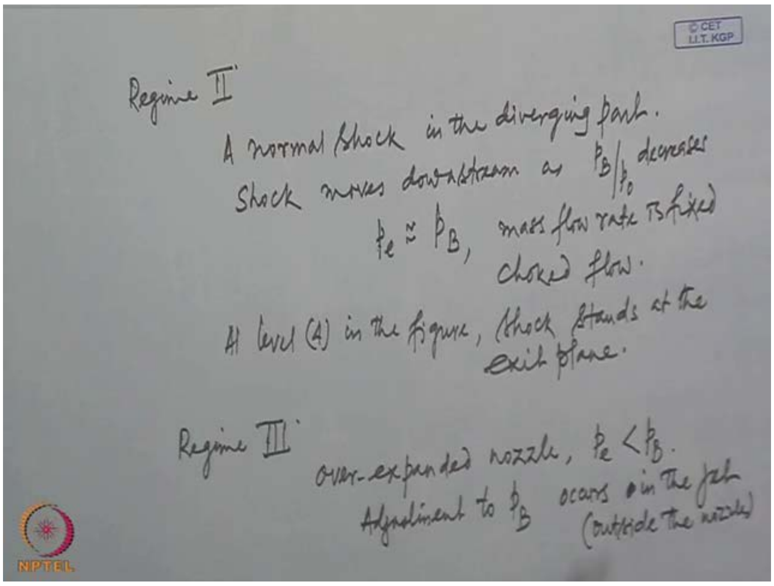


At condition two, we have seen that here the flow reaches the sonic value at the throat. However, the back pressure not being sufficiently low, further expansion is not possible. So, there is a very weak in a sense very weak almost 0 strength shock wave or mach wave at the throat and then, pressure decreases gradually to the level of back pressure. Now, between this level two and four which you are saying a regime two, the flow downstream of the throat accelerates to some supersonic flow and then, decelerates to a subsonic flow through a normal shock and for the downstream pressure wage will increase to reach the back pressure at the exit plane.

Now, the location of the shock will of course depend on the value of the back pressure. As you can see that as the back pressure decreases, the shock moves downstream. So, a normal shock that a normal shock in the diverging portion part, shock moves downstream as p B by p 0 decreases. So, here also the exit plane pressure at virtually identical to this back pressure. However, in this case, mass flow rate is that is any pressure below the level of pressure at two, the flow in the converging part remain unaffected and the flow is choked, that is there is no further change in mass flow rate since shock moves downstream as p B by p 0 decreases.

So, when the pressure back pressure reaches the level of four in the earlier figure, then the shock moves farthest, that is at level four. In the figure at level four, shock stands at the exit plane and at any pressure below that of level four, there will be no more normal shock anywhere in the nozzle. Now, what you see that if we can increase the exit plane, that is at the exit if we fit a duct of uniform perception, then that entire duct or at the uniform duct now behaves like the exit plane and the real exit is at the exit of the uniform duct. What happens then is that supersonic mach number exist everywhere in that uniform duct or constant area duct and the flow becomes subsonic at the end of the constant area duct and this constant area duct, then can be used as a supersonic wind tunnel. We see that the pressure level, back pressure level four is the maximum back pressure at which we can have a supersonic wind tunnel operate.

Now, see that in the regime three, where we have back pressure which is higher than the pressure level six which is the pressure value for fully isentropic expansion in the converging diverging duct, we have no normal shock anywhere in the duct, but oblique shock appears at the lips of the nozzle at the exit plane. What we see that the back pressure is higher than the pressure at the exit plane. We see we can say that the nozzle is now over expanded that is the expansion that we have allowed the value of p B that we have set the value at the pressure at the exit plane is smaller than that. So, in the nozzle, the flow has undergone an over expansion and consequently, these over expansion will be adjusted by subsequent compression in the jet downstream. So, in regime three, these are over expanded nozzle adjustment to p B occurs in the jet that is outside the nozzle.

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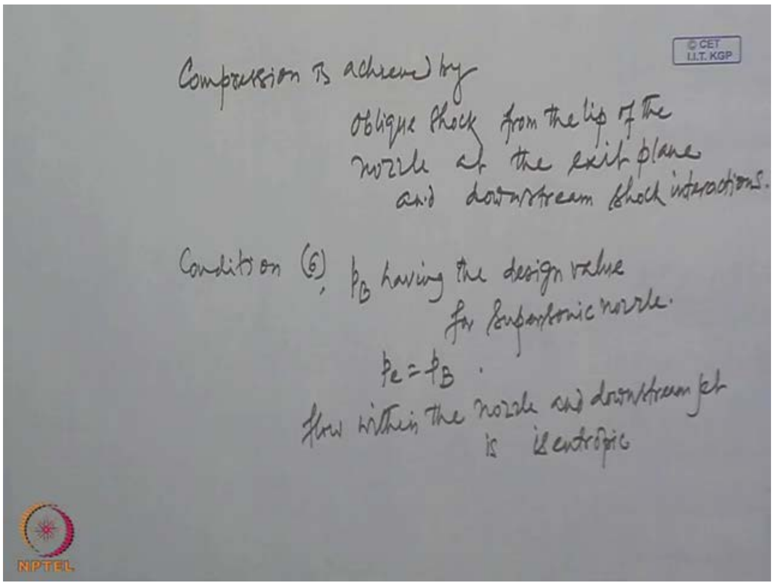


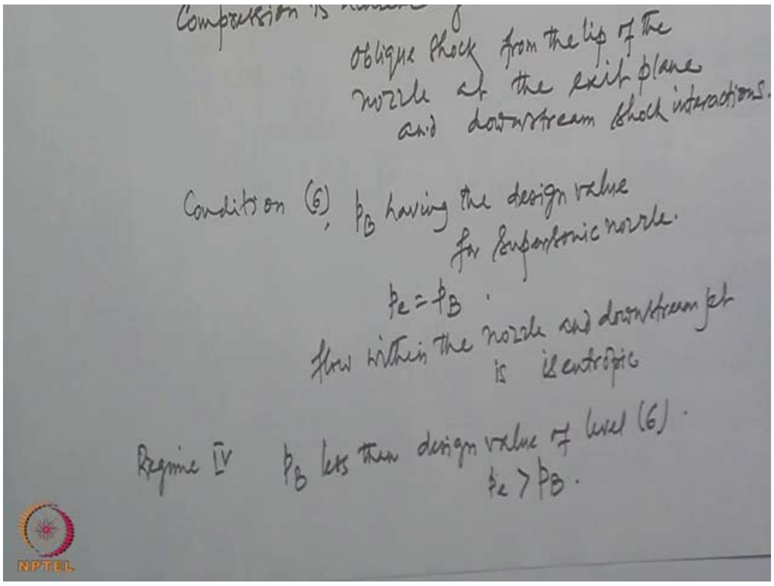


This compression is achieved by oblique shock from the lip of the nozzle at the exit plane and downstream interactions which what in a sense happens as that from the lip, the oblique shock generates and they reflect from the jet boundary which in a sense the constant boundary, but not a solid boundary and from these reflections and their interactions, a complex flow pattern develops in the jet. Finally, the pressure which is the value of back pressure for downstream, the pressure level six is basically the downstream or the design condition for supersonic nozzle where the exit pressure is identical with the back pressure.

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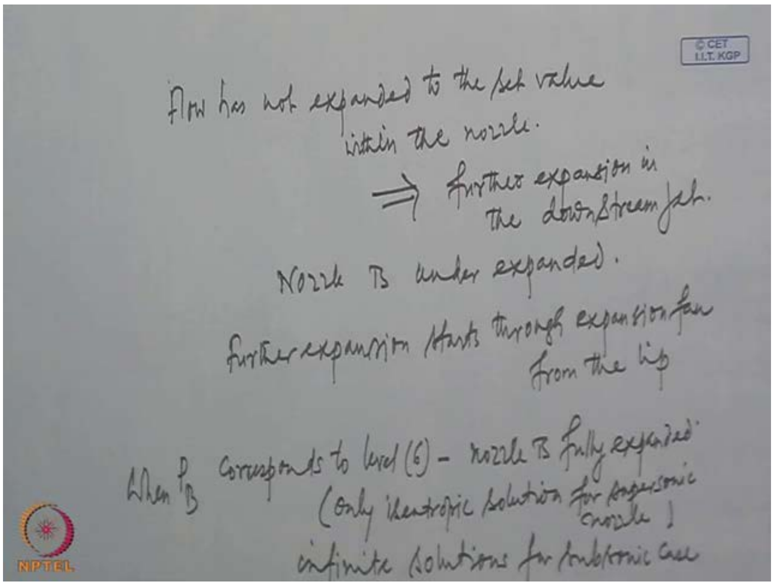


So, a condition six, p B having the design value for supersonic nozzle and in this case, p e equal to p B and there is the flow everywhere in the nozzle and also in the jet off side is isentropic flow within the nozzle and downstream jet is isentropic. There are no shock, neither normal nor oblique nor any expansion wave nor any expansion (( )) the perfect isentropic expansion in the duct. Now, in regime four, p B less than design value of level six in this case, p e is greater than p B, that is further expansion is possible. So, the flow has not been able to expand to the full capacity, flow has not expanded to the set value in the nozzle.

So, this implies further expansion downstream in the downstream jet and to call this nozzle to be under expanded, this further expansion starts through expansion fan from the lip and once again these expansion fan, they reflect and interact. Finally, the pressure is adjusted to the back pressure for downstream from the exit plane (Refer Slide Time: 20:37).

Now, once again looking back to the pressure distance curve, we can see that there are infinite number of isentropic solution possible for the subsonic part or that is within the regime one. However, there is only one fully isentropic solution for the supersonic wave here of the converging diverging nozzle which is this. So, only for this particular pressure value, this converging diverging duct will act or will work as a fully expanded of full expansion of supersonic stream, full isentropic expansion of supersonic stream.

So, when the back pressure is set at that level of pressure six, we call the nozzle is fully expanded. So, when the pressure when p B corresponds to level six, nozzle is fully expanded and this is the only isentropic solution for supersonic nozzle. There is no change in entropy, either within the nozzle or even in the jet downstream, but infinite solution for subsonic case, infinite number of solutions, infinite number of solutions for subsonic case, this happens because in a supersonic flow the pressure ratio depends fully on the area ratio while in a subsonic flow that is not.

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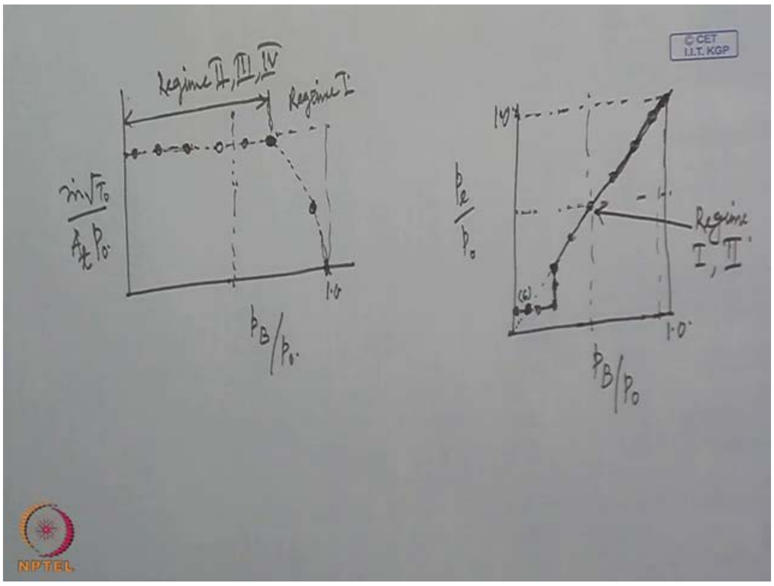

Let us also consider the mass flow rate in (( )) nozzle with p B by p 0 at by p 0 at p B by p 0, there is no flow. So, mass flow zero rate of mass flow, then as p B by p 0 falls mass flow increases when p B by p 0 reaches to the level of 0. This is ((no audio 51:56: 52:26)).

What you see that in this case, the choked condition is achieved when the pressure at the throat become the p star sonic pressure while the back pressure is still less than that, so p B by b 0. So, choked flow condition is achieved before back pressure falls to p star. So, you call, so this is regime one and in all regimes two, three, four and fully expanded nozzle four, the mass flow rate remain constant.

Similarly, we can also see how the pressure ratio behaves ((no audio 53:48 to 54:44)) in regime one. In regime one, pressure at the exit plane remains same as the back pressure and even at in regime two. When there is a normal shock within the nozzle, again pressure reaches to exit pressure. Let us say this is the pressure level corresponding to condition four after that exit pressure is not same as back pressure. Initially the exit pressure is higher than the back pressure and then, again so flow regime one and two that is where the exit pressure is subsonic and either the entire nozzle is subsonic or part of the diverging part is supersonic. However, at the exit, the flow is subsonic and the pressure is same as p e by p e equal to p B. Beyond in regime three and regime four, the back pressure is again not same.

However, again at six this will be same. So, this can be taken as level six. So, this is a typical variation of back pressure and exit pressure relationship. We will continue further discussion on converging diverging nozzle and its possible use as a supersonic wind tunnel in our next lecture.