High Speed Aerodynamics

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Module No. # 01

Lecture No. # 16

Flow through Ducts and Channels

In this lecture and few subsequent lectures, we will consider in viscid compressible flow in ducts and channels. Since, we will be considering the flow to be in viscid, there will be no viscous layer or boundary layer growth near the wall and we will assume that the condition at each cross section is uniform.

So, actually that represents the average condition over a particular cross section and with this, we will consider this flow to be one-dimensional. This simplified analysis of flow through ducts and channels are basically very simplified approach for solution of a number of practical aerodynamical problems. They are flow through diffusers, flow through nozzles, flow in a wind tunnel and also flow in combustion chamber, flow through pipes and many other such situations.

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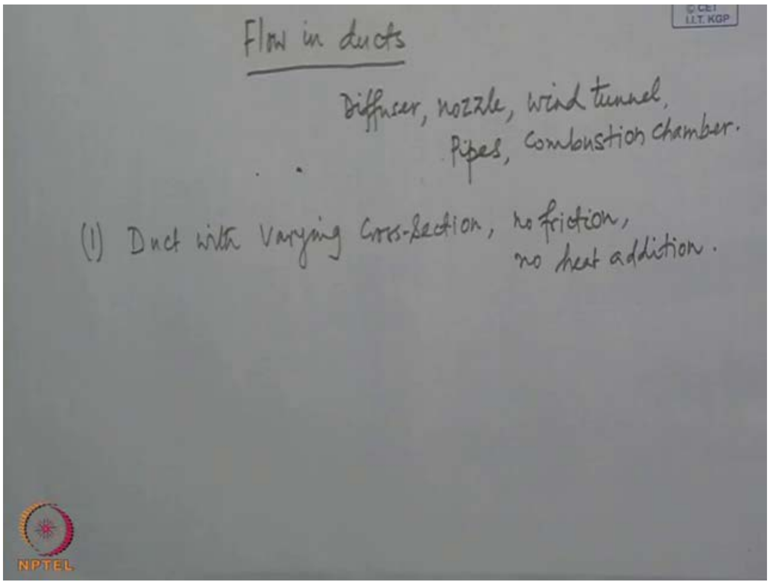

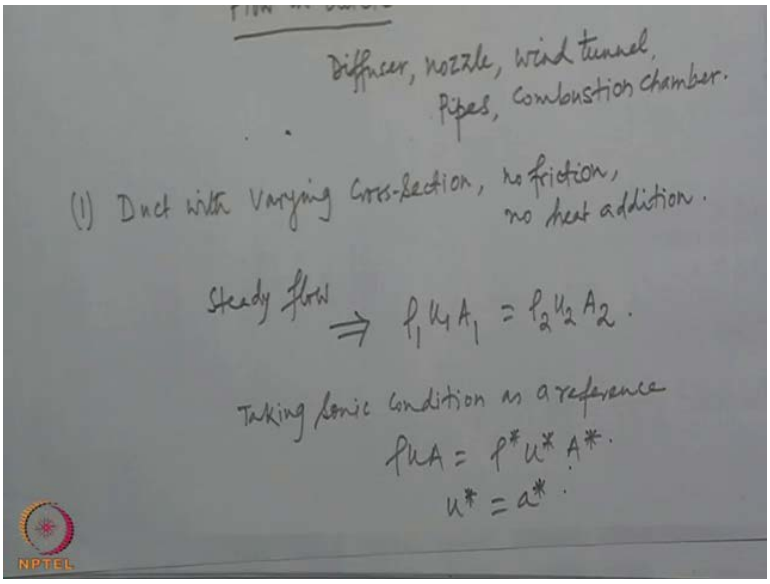

The first condition that we will use is that the tube is of varying cross section, that is the flow in duct problem, the flow in ducts. So, there are examples of these flows as diffuser nozzle, wind tunnel pipes, combustion chamber and many other such situations, where the flow can be approximated as flow through ducts or else channels.

Now, the first problem that we will be considering is duct with varying area, duct with varying cross sectional area, but no friction and no heat transfer. Earlier, we have derived the area of velocity relationship in a high speed flow and we have seen that in the subsonic part as area decreases, flow velocity increases. However, the rate of increase incompressible flow is first or then, in incompressible flow where area and velocity obey that to your inverse proportional law. Then, you have seen that the flow can reach sonic. If there is a throat in the duct and supersonic flow accelerates in a diverging duct contrary subsonic flow and this as we have discussed earlier can happen because in a supersonic flow, the density increased is so fast that to compensate for the same amount of mass flow, both area and velocity need to increase.

Now, here also, we will study the same problem, but with little more details. Now, considering this flow to be steady, considering a steady flow and then, this gives from the mass conservation law that at two different cross sections. Now, if we consider sonic condition as a reference, taking sonic conditions as a reference, we have rho ua and at sonic conditions we have, at sonic condition we have this.

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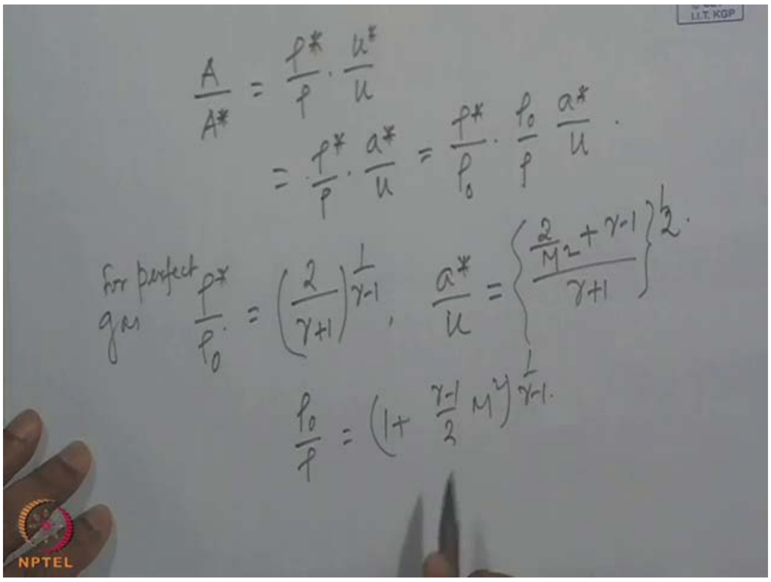

Now, this area A star is the throat area and if the flow is purely subsonic flow, then this is basically fictitious area. However, if it is a subsonic supersonic flow, then this is the actual throat area. Now, we can write this relation as A star by u and this can be modified using the stagnation conditions rho star by rho 0 into rho 0 by rho and A star by u.

Now, we have already seen that rho star by rho 0 for a perfect gas and A star by u which is inverse of the speed ratio, n star is given by 2 by M square plus gamma minus 1 divided by gamma plus 1 and the isentropic relationship rho 1 by rho is 1 plus gamma minus 1 by 2 M square to the power 1 by gamma minus 1.

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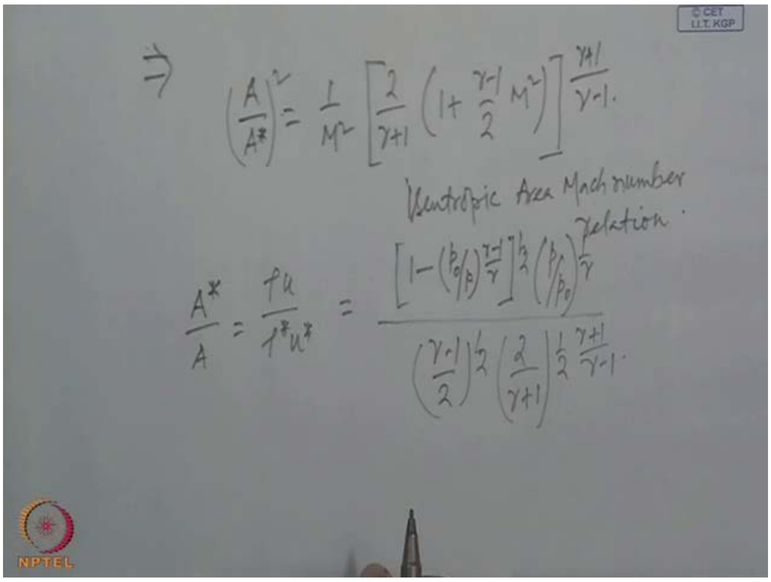

Now, substituting these three relations here, we get the isentropic area mach number relationship as 2 by gamma plus 1 into 1 plus gamma minus 1 by 2 M square to the power gamma plus 1 by gamma minus 1. So, this is what is the isentropic area mach number relationship. So, you see that is a definite area ratio for a particular mach number in one-dimensional flow through duct. This relation can also be expressed in terms of pressure and which is given by 1 minus p 0 by p to the power gamma minus 1 to the power half into p by p 0 to the power 1 by gamma by gamma minus 1 by 2 to the power half into 2 by gamma plus 1 to the power half into gamma plus 1 by gamma minus 1.

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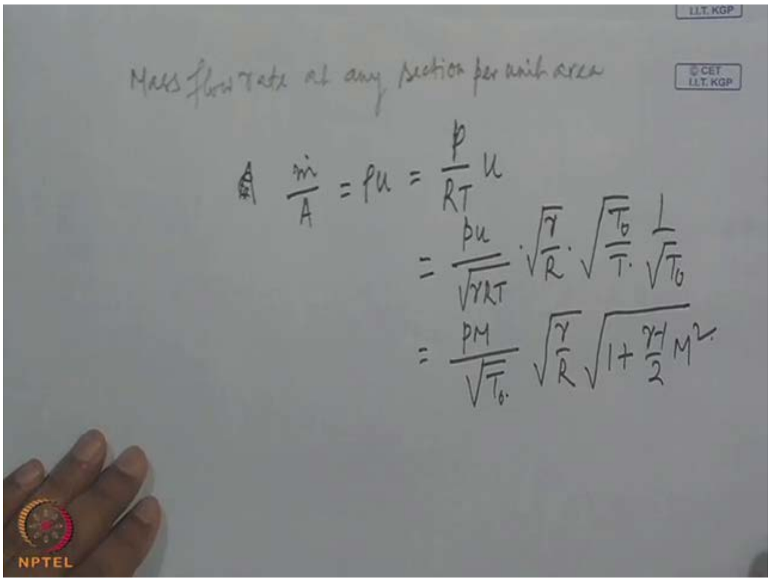

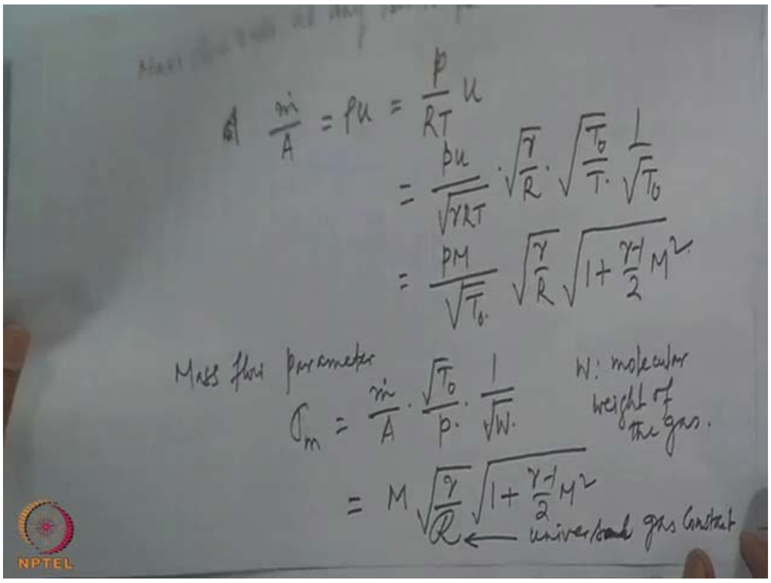

The mass flow rate at any section can be expressed as for unit area at that station is M star by A rho u, which can be written as p by R T into u which further can be written as p u by gamma R T into root gamma by R into root over R 0 by T into 1 by root T 0.

Now, this u by gamma R T is the speed of sound. So, u by A is mach number. So, this becomes p into mach number divided by root T 0 into root over gamma by R and using the isentropic relation T 0 by T is 1 plus gamma minus 1 by 2 M square.

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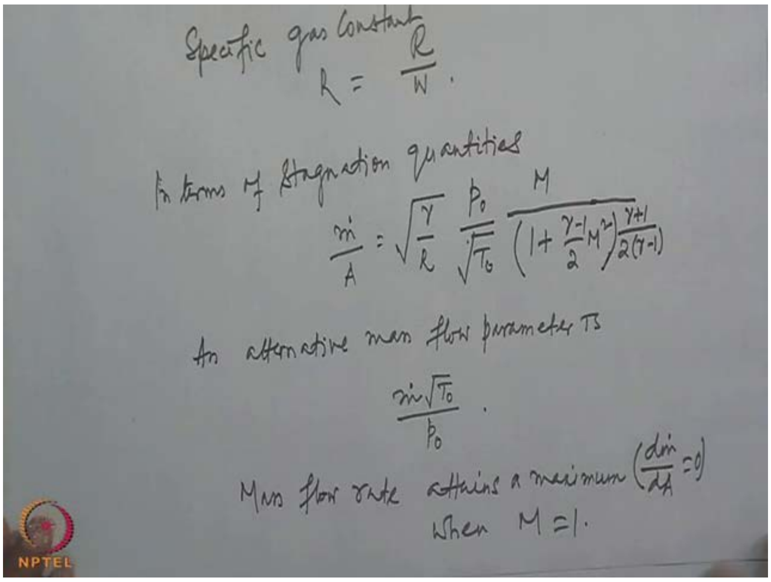

Now, in turbo machineries, quite often a mass flow parameter is defined which is a mass flow parameter sigma m is defined as m dot by A into root T 0 by p into 1 by square root of w. W is molecular weight. This mass flow parameter is quite often used in turbo machineries to express the performance of compressor and turbine and nozzle and using this definition here, we can see that this becomes mach number into root over gamma by R into 1 plus gamma minus 1 by 2 M square. R is now universal gas constant. This R is the universal gas constant and the relationship between the specific gas constant R is the universal gas constant by molecular weight.

In terms of stagnation quantities ((no audio 16:28 to 17:08)) 1 plus gamma minus 1 by 2 M square to the power gamma plus 1 by 2 into gamma minus 1 and this must be remembered that the flow that we are considering now is flow through duct of varying cross sectional area. However, there is friction or viscous effect and there are no heat additions or heat transfer. Since this is adiabatic frictionless flow, essentially isentropic and the flow entropy remains constant everywhere and consequently, the stagnation quantities p 0 and T 0 are also constant throughout and also, we have seen that earlier that in an isentropic flow, the sonic condition is constant, that is A star is also a constant throughout.

Now, this relation, see it clearly. It shows that the flow rate per unit area is proportional to the stagnation pressure and temperature directly proportional to the stagnation pressure and inversely proportional to the square root of stagnation temperature. An alternate mass flow parameter, an alternative mass flow parameter is ((no audio 18:44 to 19:14)). These relationships can be used to show that the mass flow rate has a maximum and this maximum is attained at mach number. One that is a mass flow rate attains a maximum that is when M is 1.

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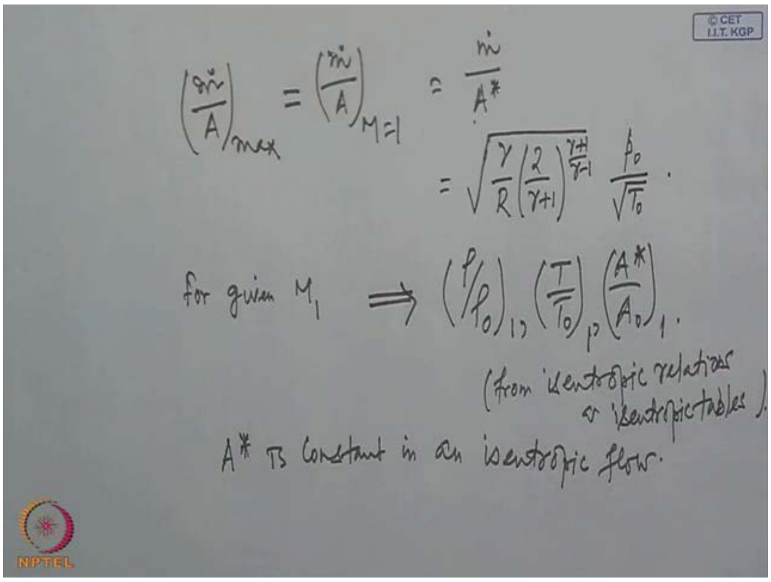

We can see that the maximum mass flow rate m dot by a maximum and this is ((no audio 20:37 to 21:11)). So, for a given gas, the maximum mass flow rate depends on the ratio p 0 by square root of T 0 and for a fixed p 0 and T 0 and of course, if the gas remains same and a fixed passage, then the maximum mass flow that can pass is relatively large. If the gas has high molecular weight and it will be smaller if for a gas with low molecular weight, that is if you have a fixed value of p 0 and root T 0 and the geometry is also fixed, passage is fixed. If the gas is having a higher molecular weight, then its mass flow rate will also be higher and since, that mass flow rate per unit area has maximum is related with a very interesting and important effect which is called choking. That is particular when the mass flow rate reaches the maximum, we call the flow is choked.

Now, for evaluating various values at any cross section at in the duct, so for given M 1, we can very easily find rho by rho 0 to 1 T by T 0 at 1 and A star by A 0. At cross section, one these can be obtained using the isentropic relationship or from isentropic chart, from isentropic relations or isentropic charts, isentropic tables.

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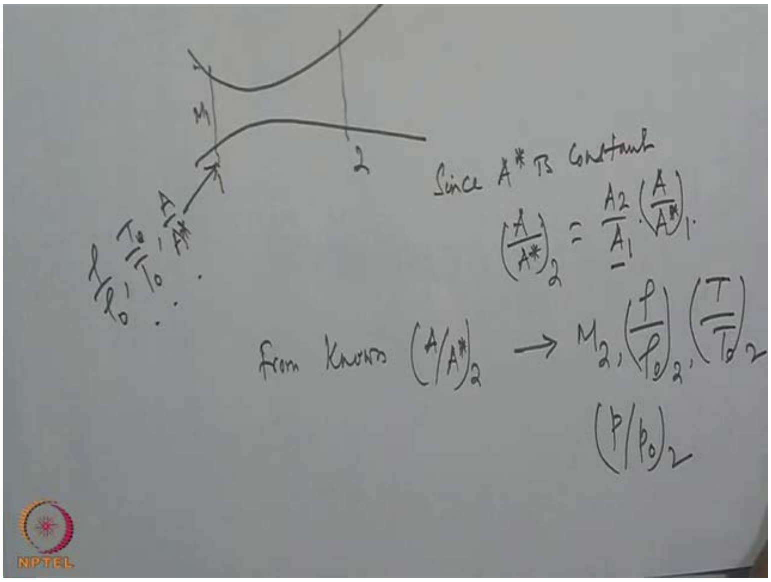

Now, in an isentropic flow A star is constant. So, let us say, considering let us say this is station one and this is station two and we know M 1 here and this gives us all those parameters rho by rho 0 T by, sorry T by T 0 A by A star and any other. Now, since A star is constant, we find that A by A star at 2 can be found from A 2 by A 1 into A by A star at 1. Since, A 2 by A 1 will usually be known, so we find A by A star. Now, from known A by A star at 2 gives us the other parameters M 2 rho by rho 0 at 2, T by T 0 at 2 p by p 0 2 and the flow being isentropic p 0 T 0 rho 0. They are all constant, that is the stagnation quantities are constant and hence, all the properties at station two can be found.

So, the flow can be completely obtained if we know the area ratio between different sections or the area ratio through the duct, that is area ratio in terms of the throat area or the sonic area and mach number at any station. Subsequently, all other properties at any station can be evaluated by using these simple isentropic relationships or isentropic tables.

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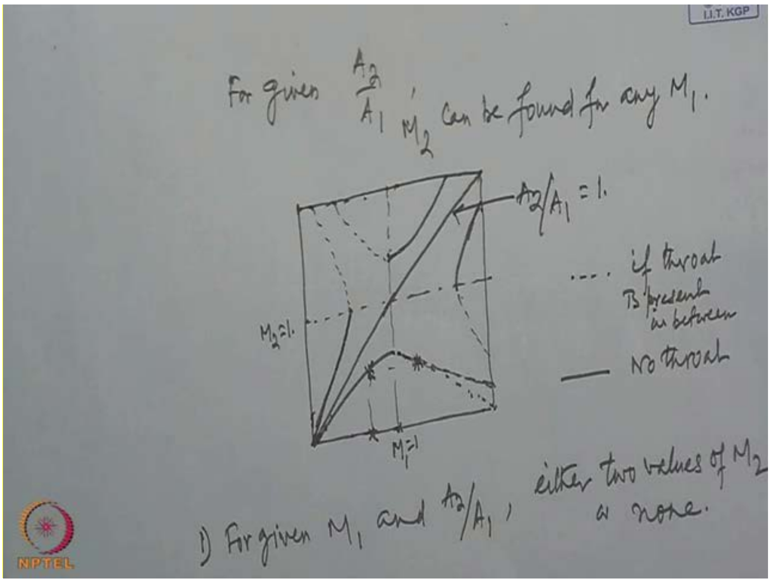

Now, for a given area ratio for given A 2 by A 1 M 2 can be found for any M 1 and the solution if we plot is look something like this, if we plot the solutions ((no audio 28:51 to 30:58)) what you see here that for a given initial mach number M 1 and given A 2 by A 1, either there are two solutions for final M 2 or none, that is for given M 1 and A 2 by A 1, either two values of M 2 or none.

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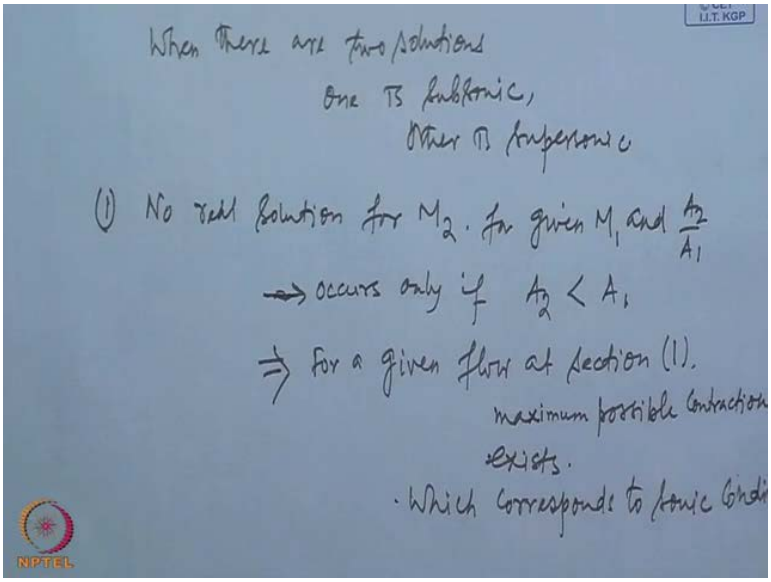

Whenever there are two values of M 2, we can see that one is subsonic, the other is supersonic. Let us say that this is a particular value of M 1 and correspondingly, for given this curve is for given area ratio, we can have either this M 2 or this M 2. One is subsonic; the other is supersonic which of these will occur of course, when there are two solutions. One is subsonic, the other supersonic.

Now, which one of these two solutions will really occurs that of course depends on whether there is a throat between one and two or not. If there is no throat, then the supersonic solution cannot occur. The supersonic solution can occur only if there is a throat in between and that is what we have denoted (Refer Slide Time: 28:24) here by these dotted curves, that is dotted curves will occur.

Only if there is a throat in between, if throat is present between and this will occur when there is no throat. Let us take an example that M 1 is subsonic and the passage is converging and then, M 2 must be subsonic, but if the passage is converging and diverging and has a throat between one and two, then the flow at section two may be supersonic provided the pressure difference across the duct is sufficient.

However, if the pressure difference across the duct is not sufficient, then the isentropic flow at station two will be likely to be subsonic, that is if there is a throat in between. When there is no throat, M 2 can only be subsonic. When there is a throat in between, M 2 can be either subsonic or supersonic depending upon the pressure imposed at the inlet and at the outlet. If the downstream flow that is downstream of the throat, the flow is subsonic, then the duct acts as eventually where both in the converging and diverging part of the flow is subsonic.

However, when the pressure difference between inlet and outlet is sufficient and flow in the diverging section is supersonic at all session, then the duct acts as a nozzle. Also, we have said that there is a possibility that no solution exist for a given M 1 and A 2 by A 1, that is the mathematical solution imaginary. This can only occur if A 2 is smaller than A 1. The second situation that is no solution, no real solution for M 2, the mathematically solution is imaginary for given M 1 and A 2 by A 1. This occurs only if A 2 is smaller than A 1, that is this signifies that for a given flow at station one, there is a maximum contraction which is possible.

That is for a given flow at section one, there is a maximum of contraction maximum possible, contraction exists and if the area is even smaller than that, then there is no solution for that section, for the given inlet and outlet conditions and this maximum contraction correspondence to this which corresponds to sonic velocity which corresponds to sonic condition, that is that maximum contraction corresponds to sonic condition, that is if we have a specified condition at section one, then the mass flow is fixed and then there is a minimum constructional area required to pass that flow. If area is smaller than that, then mass flow cannot pass through that cross section. These phenomena is called choking, that is for a mixed fixed mass flow a minimum cross sectional area is required to pass the flow corresponding speed is sonic and the phenomenon is called choking.

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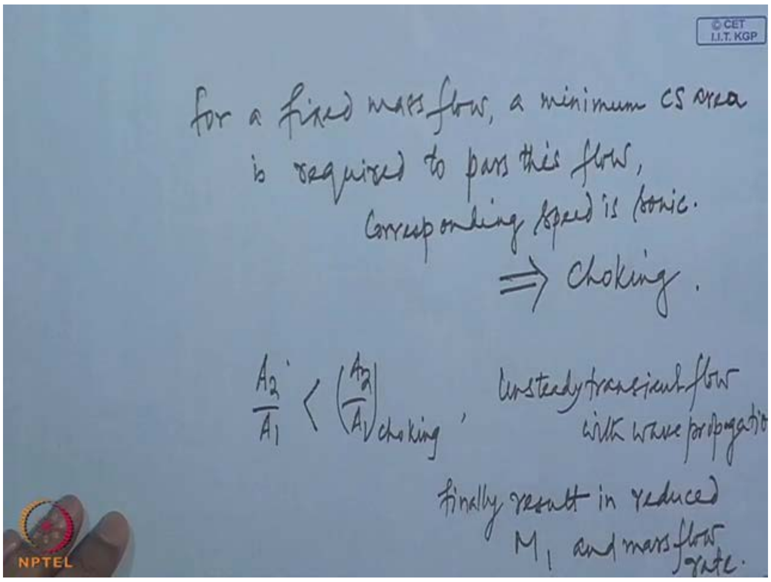

Now, for a given area reduction, if you have subsonic flow, there is maximum initial mach number which can be maintained stably that if we have a fixed duct and with say minimum area at its exit, we know that the maximum mass flow that can pass through that area given, the area is fixed and that can pass only when the exit speed is sonic. This will need a fixed inlet flow velocity of course subsonic and for any other conditions, the flow in duct will not remain steady in a supersonic flow, a minimum initial mach number which can be maintained steadily. It also depends on this area reduction. In other way that when we fix the area ratio for a duct, then if we have a subsonic flow, then there is a fixed inlet mach number to maintain steady flow in the duct and if the flow through the duct, that is if the flow is duct is converging diverging and we have supersonic flow, then for that the initial mach number which can be maintained steadily is also fixed by this area ratio. Any of these limiting condition, the flow at section two is sonic and is said to be choked.

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Now, consider a subsonic, let us come back to this curve once again. Consider subsonic flow at station one. Now, if A 2 is equal to A 1, then at all conditions at two will be identical to one. That is a purely isentropic flow in a uniform duct is uniform. Now, a slight reduction in A 2 will produce some effects are two and will comprise an increase in M 2 and of course, a decrease in p 2 and T 2. Now, the slight reduction in A 2 is accompanied without any change in condition one, then it is necessary that the back pressure p 2 is reduced. If you further reduce A 2 and continue to reduce A 2 such that M 2 reaches unity.

Now, once this point is reached, then there is no way of reducing the area further without simultaneous change in the steady state condition at section one. Let us for example, if the pressure and temperature at one are held constant, then a reduction in A 2 by A 1 beyond its limiting value will result in a steady state M 1, different M 1 before earlier before what we had earlier and a simultaneous reduction in the mass flow rate. However, this change in the initial condition or the inlet condition will be achieved after a transient period of unsteady wave propagation, that is let us make it more clear that if we have a fixed duct, let us say consider that converging duct only for the time being and the duct is such that for a given inlet mach number and we have given inlet pressure and given outlet pressure. The area ratio is such that mach number at the exit, we call that exit to be station two is sonic, that is the flow has reached the chocking condition. The flow is now chocked.

Now, let say that somehow we reduce this area A 2 without changing the initial condition, that is A 2 by A 1 is further reduced. Now, we know that with this reduced A 2, the same amount of mass flow now cannot pass through this duct even at sonic condition. Now, what will happen? Then the flow will initially be unsteady. There will be wave propagation and finally, it will reach to a steady state, but with a reduced M 1, that is the reduced mach number at the inlet and a reduced mass flow rate, that is it will enforce a change in the inlet condition.

However, this new flow configuration can be maintained if the back pressure is also adjusted accordingly. So, A 2 by A 1 less than A 2 by A 1 for choking condition unsteady a transient unsteady flow with wave propagation and finally, result in a reduced M 1. So, we see that A 1 dimensional flow through duct, either converging or diverging is quite simple. Just by using these isentropic relations or isentropic tables and using the fact that in an isentropic flow, all the stagnation properties and sonic properties remain in altered flow condition at any cross section can be solved or found. Also, we have seen this important phenomenon of choking which corresponds to the maximum mass flow rate that is possible in a given duct with this.

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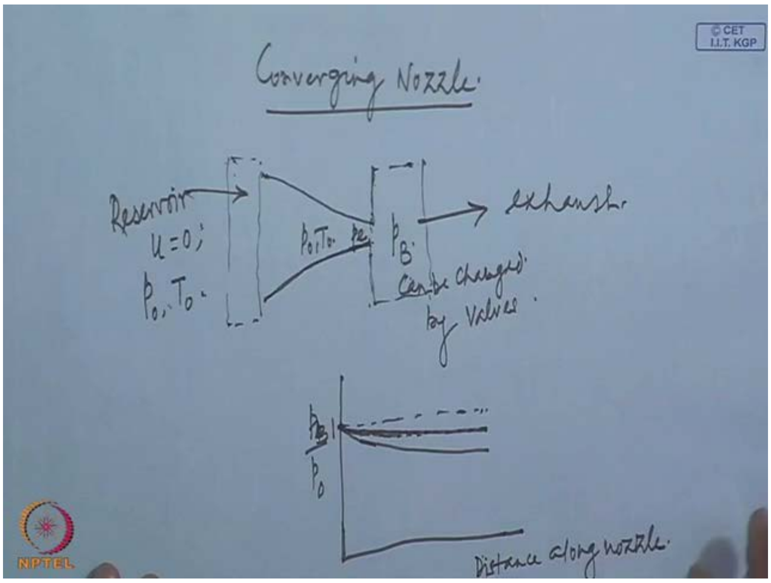

Now, let us consider what a converging nozzle is. Let us see more closely the flow behavior in a converging nozzle. Let us see we have a converging nozzle connected to a reservoir. Let us say this is connected to reservoir with speed, practically 0 speed, practically 0 and consequently the pressure and temperature at the reservoir can be considered as the stagnation pressure and stagnation temperature will consider that the flow is isentropic as we have done in our analysis.

So, this p 0 and T 0, they remain constant. So, same p 0 and T 0 throughout and let us say at the exit, it attains a pressure of p e exit and let us say, this is connected to some mechanism which can be changed by valves and others, that is let us say this goes to the exit is to a certain chamber and they need finally goes to exist.

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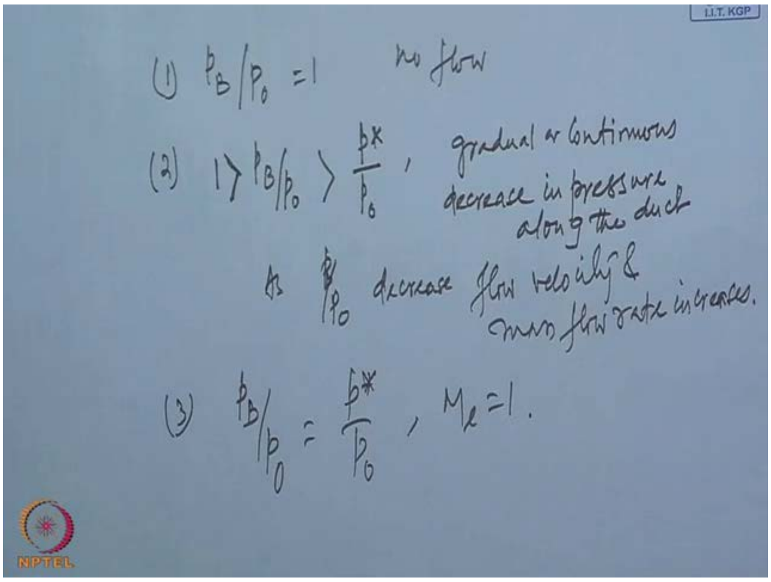

Now, what will be here is we have seen that the value of p 0 T 0 will remain constant. The p b will change. Let us say that when p b is same as p 0 and T 0 p 0 of course, there will be no flow ((no audio 53:44 to 54:17)).

If p e by p 0, then there is no flow. However, if p is slightly reduced, then when p e or p b is slightly reduced, there is a flow with pressure constantly decreasing through the nozzle. Since, the exit flow is subsonic, this p e must be p B, p e must be equal to p B. However, due to certain viscous effects and all, there might be a slight variation which is practically negligible. See what happen is that e p is, if p e is substantially larger than p B, then there will be an expansion. After leaving expansion of the flow, after leaving the nozzle required an area increase, but such an area increase in subsonic speeds causes the stream pressure to raise further. Now, since the back pressure is the pressure to which the stream ultimately reaches or the ultimate pressure of the stream, then p e cannot be much larger than or considerably larger than p B. Similarly, p e also cannot be substantially less than p B.

So, further p B is slightly reduced. There is a gentle flow through the nozzle with continuous decrease in pressure along the duct. If p B is reduced further, the qualitative behavior remains the same, that is still the flow speed increases, mass flow rate increases, but there is not much of qualitative change in the performance of the nozzle. However, when p B reaches to the, so 1 p B by p 0 equal to 1, no flow p B by p 0 less than 1, but it is continuous decrease in pressure along the duct and as p B by p 0 decreases flow velocity and mass flow rate increases and when this condition is reached, then the mach number at the exit plane reaches one. Further, reduction in pressure ratio will not produce any change in the flow conditions within the nozzle. We will continue this in our next lecture.