High Speed Aerodynamics

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Module No. # 01

Lecture No. # 15

Shock Expansion Theory

In the last class, we have mentioned that the oblique shock and expansion theory can be used to construct many important aerodynamical problems, particularly for the geometries which have straight segments. The most important aerodynamical problem is of course, the flow past an airfoil and wing. Since, the airfoil and wings are very thin, it is customary to assume the wing and airfoil to be flat plate. Flat plate airfoils are very widely analyzed and always taken as the first problem in aerodynamic of wings and airfoils.

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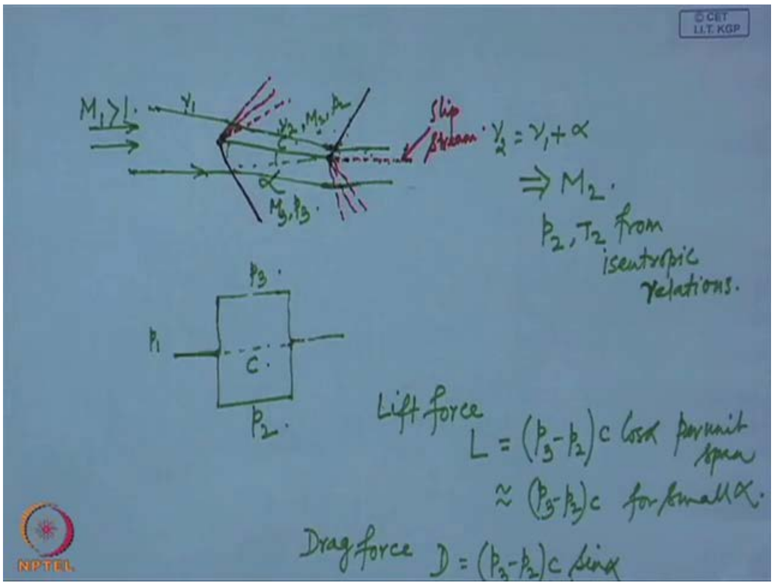

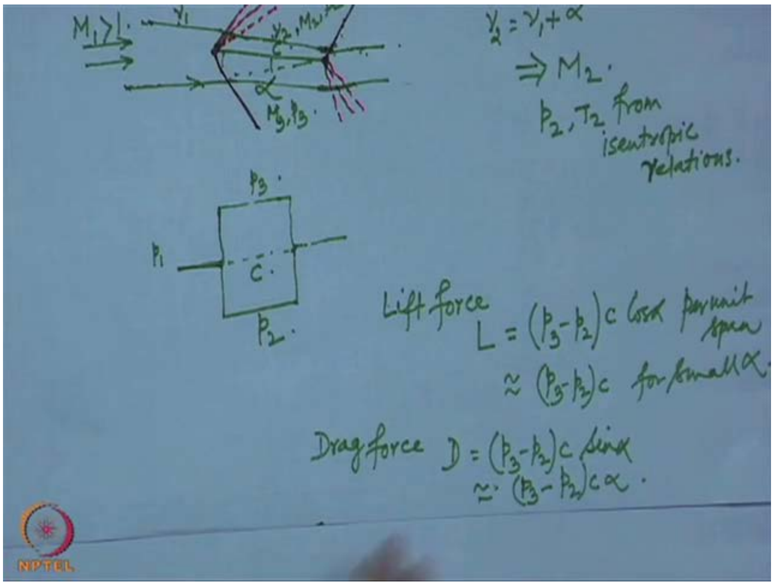

Let us consider a supersonic flow. At first, a flat plate, say at an incidence of alpha. Let us consider a flat plate of chord C and at an angle of attack alpha. The chord is C. Let us consider a supersonic stream is approaching this airfoil. We will consider this angle of attack is such that even after the disturbance you created on this free stream, the flow remains supersonic always. Now, since there is no upstream influence in supersonic stream, the streamlines ahead of this plate remain straight and the upper surface flow is independent of the lower surface flow.

Now, the flow on the upper surface turns and since it is an expansive turn; it turns through a centered expansion fan, and the Mach number increases, pressure decreases downstream or on the surface of the plate. The plate being straight there is no further change in the properties. So, if we consider a particular streamline ahead of these, which is straight, now turns and become parallel to the flat plate surface. Now, coming to the trailing edge, this flow needs to turn, so that it is parallel to the free stream and; obviously, these turn then in opposite direction and will be achieved through a shock and the streamline will now become parallel to the free stream.

On the lower surface the reverse happens. The flow undergoes again a turn of angle of attack alpha, but through a shock, and at the trailing edge again it turns by an centered expansion and a lower surface streamline can be (no audio between: 04:55-05:11) Now, knowing the amount of turn alpha and knowing the Prandtl- Meyer function nu upstream corresponding to M 1 and alpha, we can see what is the Prandtl-Meyer function here (Refer Slide Time: 05:25).

So, by knowing nu 1, we can find nu 2 here, where nu 2 is simply nu 1 plus alpha. Alpha is the magnitude of the angle of attack. Now, knowing nu 2, this of course, gives us the mach number M 2, and using isentropic relation, p 2, T 2 from isentropic relation (No audio between: 06:06- 06:27) and again knowing the flow turning angle alpha here, and the mach number of ahead of these shock M 2, using the M theta beta relation or the M theta beta chart, we can find the wave angle corresponding to the shock. Once the beta for this shock is known, we can calculate the pressure, temperature, density; all other properties at downstream. The pressure of course, should be the upstream pressure.

Similarly, on the lower surface, knowing M 1 and the flow turning angle theta, we can find out the wave angle beta and the downstream mach number here. Let us call that to be M 3 and p 3. The pressure will be higher here, and again knowing M 3 and the turning angle, we can also calculate the downstream flow properties. Now, looking to the pressure distribution over the airfoil, we see that ahead of the flat plate, the pressure is p 1 and then on the upper surface pressure is reduced to certain value p 2, while on the lower surface; the pressure is increased to value p 3.

So, what we see is that there is higher pressure on the lower surface and lower pressure on the upper surface. However, the flat plate being straight over this, the pressure is uniform over the entire plate. Consequently, there is a lift force acting on this airfoil. The lift force L is now given by p 3 minus p 2 into c cos alpha, of course, per unit span, and this can be approximately written as p 3 minus p 2 into c, for small alpha. We can also see that there is a drag force given by p 3 minus p 2 into c sin alpha, which again is, approximately p 3 minus p 2 into c alpha, for small angle of attack.

Coming back to this flow problem again, you see that the lowest upper surface flow, first encounters an expansion wave, expansion fan, through which this mach number increases and the shock is at upstream mach number of M 2, which is higher than M 1. While on the lower surface, the flow encounters the shock first at a mach number M 1 and isentropic expansion fan at the trailing edge, which increases the mach number again back to M 1.

Now, the upper surface flow and then lower surface flow experiences the shock at different mach number. The upper surface shock being at higher mach number and consequently for same amount of turning, this will be a stronger shock. So, on the upper surface we have a stronger shock compared to the shock on the lower surface, as a consequence, the entropy change experienced by the upper surface flow and the lower surface flow are not equal. Hence, there is a difference in the entropy level on the upper and lower surface flow and consequently we will have a slip stream.

As you have seen, in earlier cases that on the two sides of the slip stream other than the pressure and flow direction, the other flow parameters can have different value. The entropy values are obviously different as you have seen. Similarly, the density and the velocity, they will also have different values or as if this works as a tangential vorticity, which creates a difference in tangential velocity across it.

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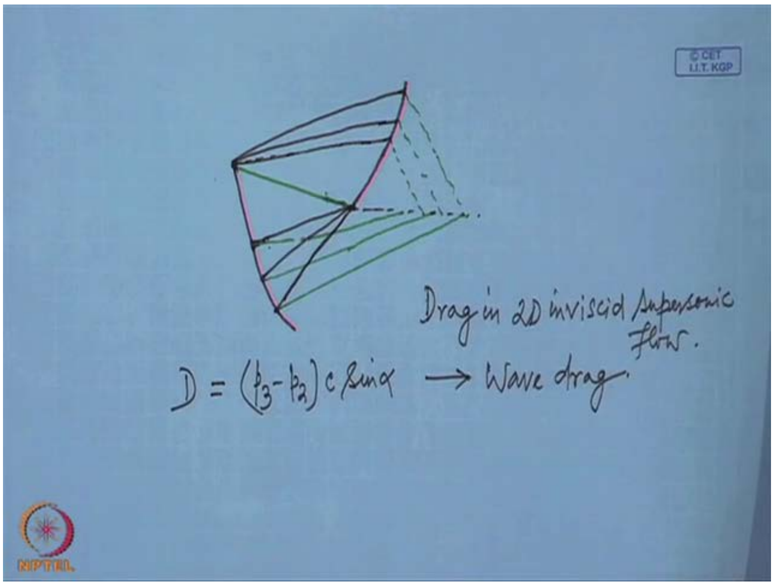

Now, this is the flow we are near the flat plate; however, if we try to look to the far field then you can see that these expansion fans may interact with the shock. That is as you have seen earlier that there might be shock expansion interaction, consequently the shock may become curve and there will be deflected surface or deflected flow pattern and the far field flow pattern is likely to be ((no audio 14:07 to 14:41)) and consequently the shock is likely to be curved, which shows the slip line and this there can reflect back ((no audio 15:10 to 16:17)) and again this reflector wave. ((no audio 16:18 to 16:36))

See that in the far field and particularly flow downstream is quite complex with having shock expansion interaction and then deflection and again interaction of those deflected waves and so on. However, in these cases, these reflections are will not be incident on the flat plate itself and consequently the flow over the flat plate or the in the immediate vicinity of the flat plate will remain undisturbed of these shock expansion interaction. The lift and drag for inviscid flow are quite accurate and exact.

One very important thing that we notice here is that in a supersonic flow, even where there is in two dimensional cases, there is a finite amount of drag force. You have seen that in subsonic two dimensional inviscid flows, there is no drag force; the drag force in inviscid flow can come only in three dimensions, which is lift induced drag. However, in case of supersonic flow we see that there is a drag in two dimensional inviscid flow as well.

This drag is known as the wave drag and this drag, which is in this case as p 3 minus p 2 into c sin alpha. This drag is called wave drag ((no audio 18:32 to 18:48)) inviscid supersonic flow. So, this is a special feature of supersonic flow that there is a drag even in two dimensional inviscid flows and this drag is called the wave drag. This is due to the presence of waves in supersonic flow this drag comes in and this will always be present in a supersonic flow.

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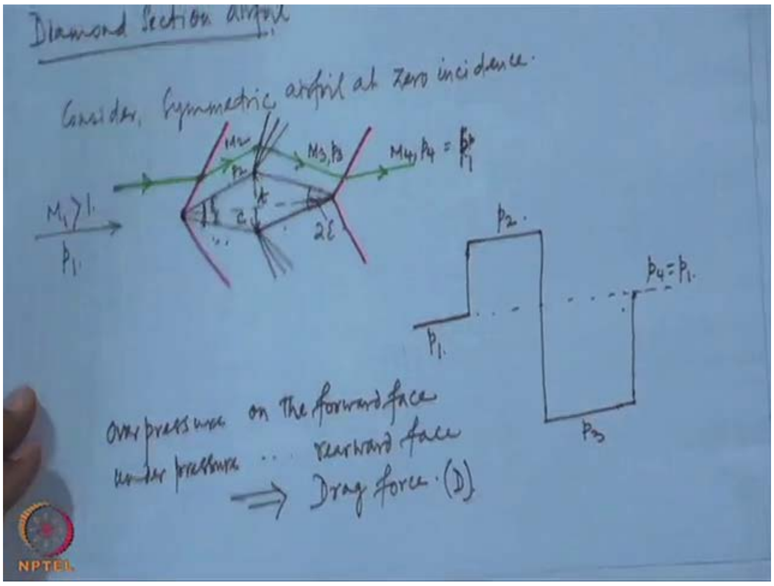

Now, let us consider another very important supersonic airfoil which is the diamond section airfoil. ((no audio 19:44 to 20:03)). Let us consider, we have a symmetric Consider a symmetric section of symmetric airfoil at zero incidence; however, the construction of the flow will be same even if the airfoil is not symmetric and the angle of attack is nonzero. Let us consider this angle to be epsilon and epsilon; that is total two epsilon angles and same here is also two epsilon. Let us consider again a supersonic free stream, now see that as you have discussed earlier that here the flow will turn and become parallel to the wall, through a shock and on the both surface this will be a shock in this case.

However, whether it is a shock and expansion fan that will depend on the amount of turn the flow will undergo. So, in case this airfoil at an angle of attack and that alpha is such that this becomes an expansive turn, in that case, there will be an expansion fan here. So, subsequently of course, here the flow will turn again through an expansion fan, and then again it will turn here by an oblique shock. So, remember that we are considering the flows where even occur decrease in mach number through a shock, the flow still remains supersonic.

We are always considering only weak solutions. So, if we consider a streamline upstream of the mach numbers. Upstream of the airfoil, the streamline is straight at a mach number M 1 with pressure p 1. At the first shock it turns and become parallel to this straight segment. Through the expansion fan, the flow again smoothly turns and become parallel to this downstream surface, again it turns through the oblique shock and become parallel to the free stream and exactly identical happens at the lower surface.

Now, if the flow is symmetric or if the geometry perfectly symmetric, then the mach number at the corresponding segment on the upper and the lower surface are exactly identical. All the shocks occur at same mach number and the entropy change undergone by this flow on the upper surface, either on the lower surface, are identical. Hence, in the wedge, the flows have same entropy and there is no slip stream here. However, if there is symmetry in the geometry, which will cause symmetry in the flow, then there will be a slip stream from the trailing edge, which deduction will be determined by the total term that the flow undergoes the upper surface and on the lower surface.

Now, let us say the mach number here is M 2, and the pressure is p 2, and here it is M 3 and p 3, and here it is M 4 and p 4, which should be p 1. This p 4 should be p 1 (Refer Slide Time: 24:54). Now, the pressure distribution, if we plot on the airfoil surface of the upstream of the geometry, the pressure is uniform at p 1 at the shock pressure jumps to p 2 and on the surface pressure changes to pressure remain constant at p 2. On the downstream face the pressure is again uniform at p 3 at which is much lower, remember the turn here is the double of the turn here. ((no audio 27:03 to 27:37))

Now, if the geometry is symmetric and at zero angle of attack the flow is also symmetric on the upper and lower surface, as we have discussed, and consequently the pressure on this face is also equals p 2 and pressure on this face is also equals p 3. Consequently, there is no lift force in this case, the pressure distribution having upper and lower surface symmetry, no lift force, pressure distribution has;

However, since there is an over pressure on this forward face and lower pressure, under pressure on the downstream face or their rearward face; over pressure on the forward face, and under pressure on the rearward face, this results in a drag force. If this maximum thickness is denoted by t, and the chord by c, then the drag force can be written as drag force D.

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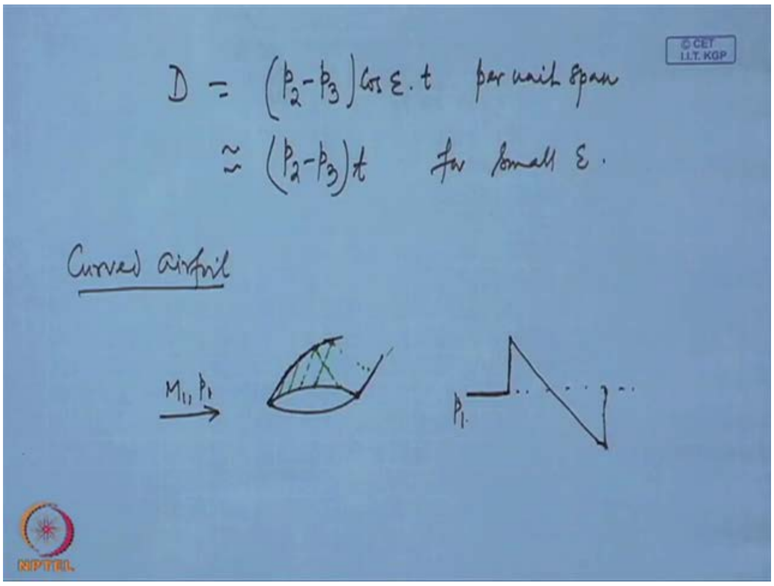

This drag force can be written as p 2 minus p 3 into a component of e in the drag direction into t per unit span and that is equal to p 2 minus p 3 into t for small epsilon. Now, let us consider a curved airfoil section. Consider a curved airfoil section for supersonic flow the airfoils are usually having sharp leading edge so that the shock remain attached and there is no detached bow shock and hence no large pressure loss. So, considering a curved airfoil and in this case there is a shock attached at the nose; however, subsequently there is a continuous expansion occurs along this surface and consequently in this case, these expansions are very close to this shock and they will interact and consequently the shock will become curved.

At each interaction, the shock strength will decrease and due to this attenuation the wave angle will also change and the shock will continuously bend. The pressure distribution here, we can see that at there is no upstream influence as in the case of supersonic flow. So, the pressure up to the leading edge remains at p 1 and at the leading edge there is a increase in pressure due to the shock; however, the pressure continuously falls and again it increases to the free stream pressure.

The shock at the trailing edge will also be curved because these interaction of these expansion fan and shock, expansion waves and shocks, there will a reflection and these reflections will definitely hit or intersect, interact with the trailing edge shock. Not only that, even these reflected waves may interact from the airfoil surface and through these interactions, there will be continuous change in entropy and also of vorticity and the flow will be rotational even though this is in inviscid flow.

Now, in this case, of course, because of this continuous change in pressure, it is not straightforward to apply this shock expansion theory and using this exact shock expansion results, you cannot compute the properties on this surface of these airfoils. To do this, we need to simplify the shock expansion theory or results of the shock expansion theory.

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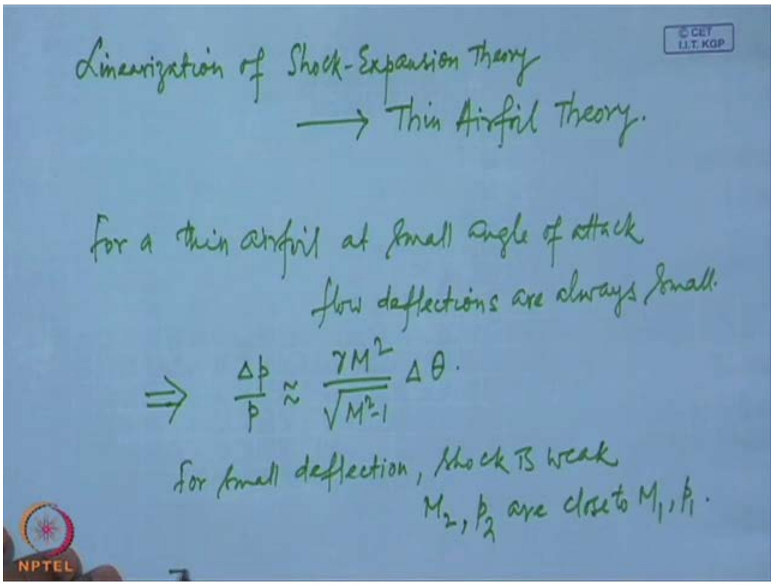

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So, we will and which gives us a supersonic thin airfoil theory. As you have seen that if we apply this shock expansion theory we get exact result; however, this shock expansion theory is not very convenient for a curved airfoil section and that is the shock expansion theory result cannot be expressed in a concise analytical form. Hence, it is not suitable for various application particular having curved surfaces.

Now, we can linearize this shock expansion theory for a thin airfoil, because when you have a thin airfoil theory at small angle of attack; the flow deflections are always small. Consequently, we can use that basic pressure rise relationship, approximate pressure rise relationship. (No audio between: 37:52-38:06) Now, when the turn is very small and the pressure rise is also small, the flow deflection is also small and the difference in pressure ahead and behind the wave is also not large. Consequently, p 2 and M 2 are, that is the pressure and the mach number behind the wave is not much different from p 1 and M 1 that is pressure and mach number ahead of the wave. For small deflection, shock is weak and M 2, p 2 are close to M 1, p 1.

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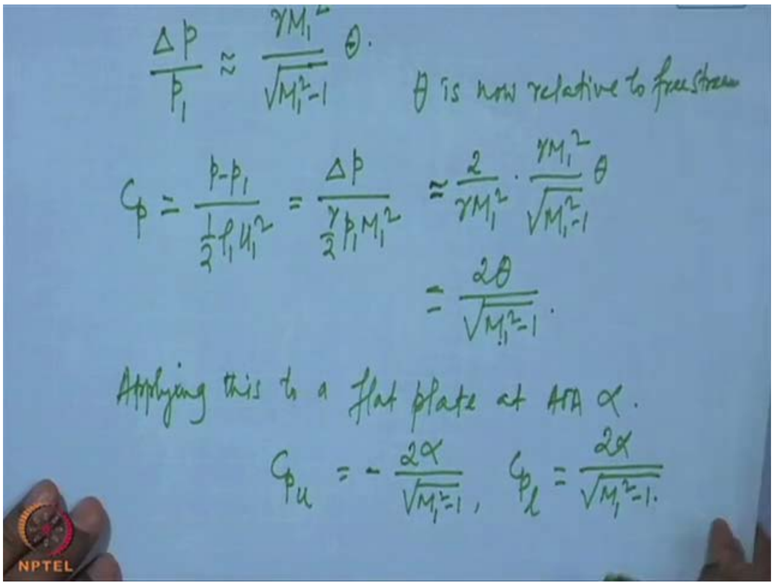

Using this approximation, we can even write delta p by p 1 is approximately equal to gamma M 1 square by ((no audio between: 39:52-40:06). Now, theta is now relative to free stream and now the pressure coefficient is defined as p minus p 1 by half rho 1 u 1 square and that is delta p by gamma y 2 p 1 M 1 square. ((no audio 41:03 to 41:42)) that is pressure coefficient at any point is expressed in terms of the slope of the local position.

So, this is in a sense local inclination theory, which says that the pressure coefficient at any point on a thin airfoil is simply given by the free stream mach number and the local flow inclination. Now, considering, if we applying this to a flat plate, at angle of attack alpha, the pressure coefficient on the upper surface is minus 2 alpha by ((no audio 42:58 to 43:11)) and pressure coefficient on the lower surface is ((no audio 43:15 to 43:35))

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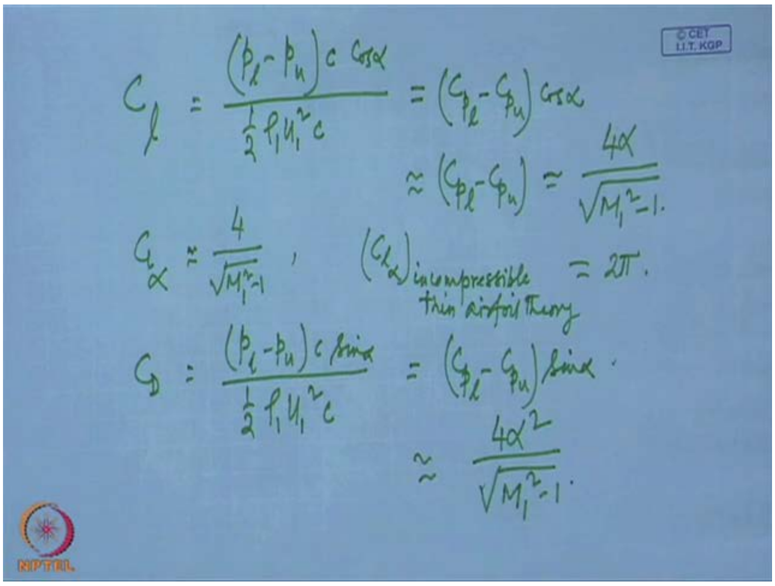
c 

Consequently, the lift curve, lift coefficient C l is p l minus p u into c cos alpha by ((no audio 43:54 to 44:39)). So, for flat plate at small angle of attack alpha, we can see that lift coefficient according to this linearized theory or thin airfoil theory in a supersonic flow is 4 alpha by root M 1 square minus 1, or the lift curve slope is ((no audio 44:57 to 45:13)), while the same result for incompressible thin airfoil theory; incompressible thin airfoil theory is 2 pi. We also see the drag coefficient ((no audio 45:46 to 46:34)). So, that the wave drag coefficient from thin airfoil theory; we can clearly also see that the aerodynamics enter in this case is at the mid chord wing.

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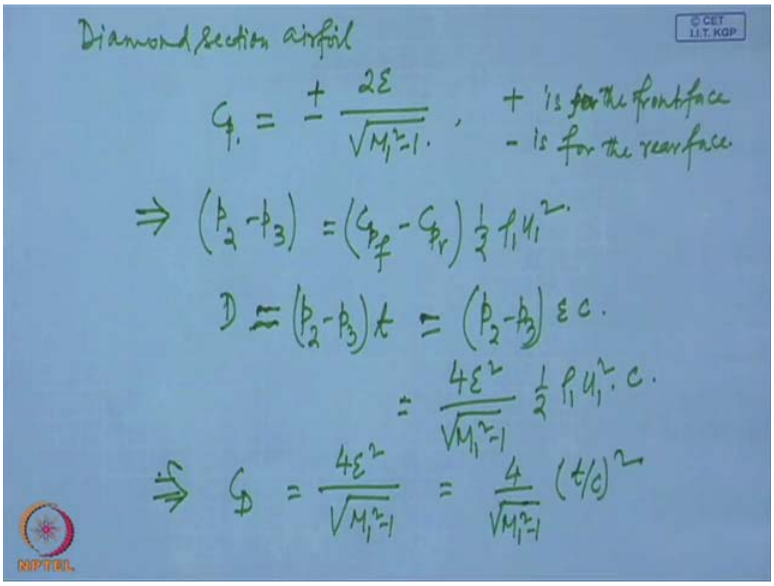

Similarly, if we consider the diamond section airfoil for the diamond section airfoil; this C p is given as plus minus 2 epsilon by root over M 1 square minus 1; plus is on the front face and similarly, minus is for the rear face, and this gives the pressure difference on the front face and the rear face and that had to be front face minus rear face into half. Using the formula that we have derived earlier the drag is approximately p 2 minus p 3 into t, which is p 2 minus p 3, and this t can be expressed in terms of epsilon into c. Substituting these pressure coefficients, this becomes 4 epsilon square by root over M 1 square minus 1 into half rho 1 u 1 square into c and this simply is C D equal to (No audio between 50:02-50:29).

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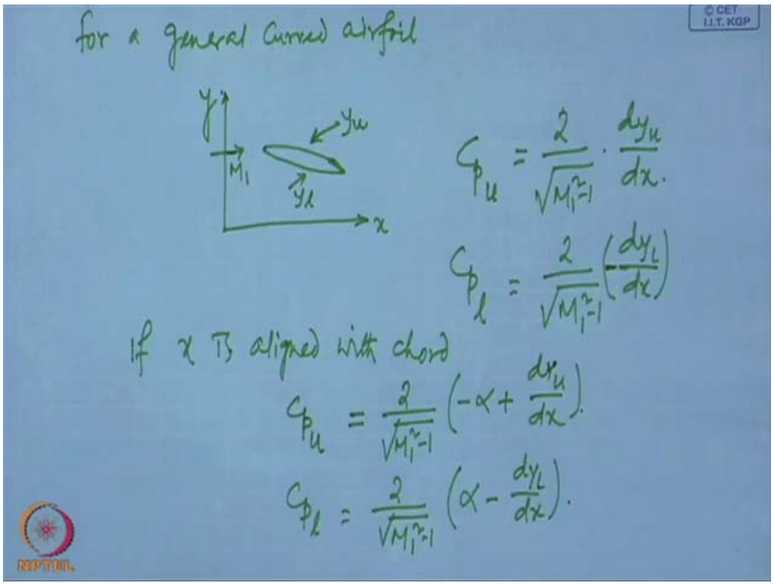

Now, for a curved general curved airfoil for a general curved airfoil ((no audio 50:34 to 50:48)) similarly, we can express pressure coefficient in terms of the local surface inclination and ((no audio 51:00 to 51:33)) in terms of the local surface inclinations, pressure coefficient on the upper surface is ((no audio 51:40 to 51:59)) and on the lower surface it is ((no audio 52:03 to 52:26)). If x axis is aligned with chord then C p u ((no audio 52:53 to 53:52)).

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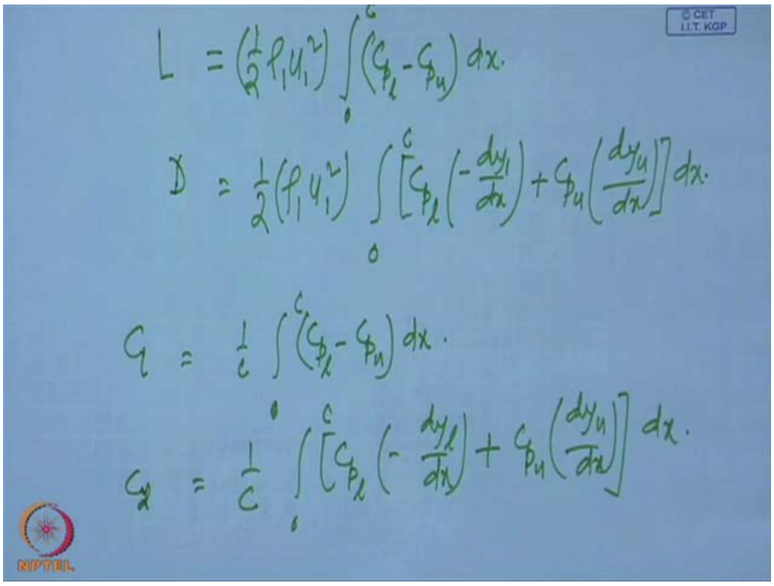

The lift force can be obtained as half rho 1 u 1 square into 0 to C, and drag force can be obtained as half rho 1 u 1 square into 0 to c ((no audio 54:26 to 54:57)) and similarly, the lift coefficient can be 1 by c, 0 to c, C p L minus C p u of d x and the wave drag coefficient similarly, will become 0 to c, C p L minus d y l d x plus p u d y u d x. In this case, we have assumed that the x. These all horizontal reactions not aligned with the chord and the appropriate relations it can be used if we align the x with the chord

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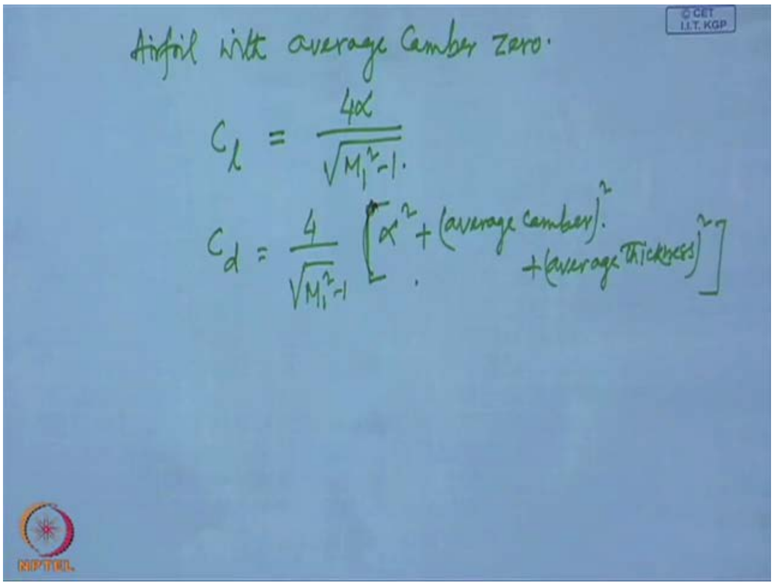

Since, the average camber, if the average camber over the entire airfoil is zero, the lift coefficient (no audio between 56:39-56:58) airfoil with average camber zero. In that case, this lift coefficient will again come to 4 alpha by root over M 1 square minus one; however, C d will become as 4 by root over M 1 square minus 1; alpha square plus square of average camber plus average thickness. So, to summarize we have first constructed flow problems for some simple two dimensional geometrical configuration, which have state geometry and we have seen that for these cases we can apply our exact shock expansion theory to find the forces acting on these airfoils.

However, since exact shock and expansion theory results are not amenable to concise analytical form, we have seen that these exact relationship are not straightforward or not conveniently used for geometry with curved section. For such situation, we have linearized or made approximation of thin airfoil or thin geometry. We set a thin and small angle of attack and we have approximated for shock expansion theory results and seen that the pressure coefficient at any point on geometry is a function of the local surface inclination. Using that theory, you have again computed the lift and the coefficient of some simple geometries; however, this must be remembered that these are based on the assumptions that the flow patterns remains simple at least on the surface and very close vicinity of the geometry, that is the reflected waves they do not; they are not incident upon the geometry and if the reflected wave is incident of the geometry, then of course, the simple relation simple analysis is invalid and more complex analysis or in general numerical simulation of the complete (( )) equations are essential.