High Speed Aerodynamics

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Module No. #01

Lecture No. #14

Waves and supersonic flow (contd.)

Now, in the previous lecture we have discussed about interaction of waves of finite amplitude, where waves are of the same family. We have seen that for shock interactions of same family, the shocks pass through. However, while passing through, they bend and if the two shocks are of different strength, and then a slip stream is originated at the point of intersection. On the two sides of the slip stream, the flow properties are different, except the pressure and flow direction. The pressure and flow direction are same, but all other flow properties such as velocity, temperature, density, are all different.

The entropy is also different on the two sides. So, this slip stream actually differentiates between two different flow region having different flow properties and entropy. Since, the tangential velocity is different on the two sides of this streamline or the slips stream, this slips stream can be thought of as a line with discontinuity in tangential velocity, or all most like a vortex state, vortex line. Next, we will consider interaction of shocks or finite waves of same family.

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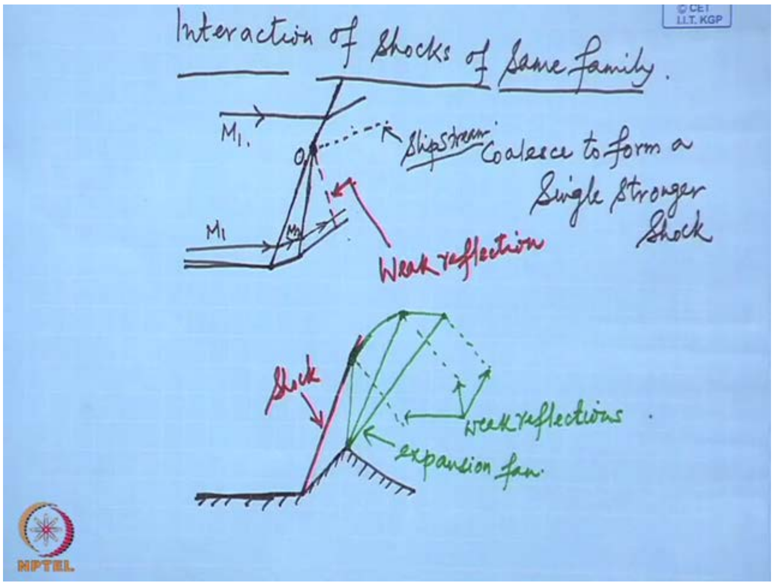


Interaction shocks of same family (no audio between: 02:03-02:20) and that is where two shocks are interacting, and that is both of them are from the same family that is both of them are either left running or right running.

See this sort of situation we have all ready encountered, when we discussed about compression by turning through successive corners. We have all ready mentioned that we have two successive corners. Let us see two corners with different amount of turn. So, the flow turns here, through a shock is this (Refer Slide Time: 03:05) and becomes parallel to this part of the flow, this part of the wall. Again, it turns through another shock and becomes parallel to this part, the downstream part of the wall.

Now, if the upstream mach number is M 1, then after the first shock, the mach number decreases and these two shocks have different wave angle and usually they converge. So, at certain distance, they will meet. Now, in this case the shocks being of the same family they do not pass through, rather they coalesce to form a single stronger shock; they coalesce to form a single stronger shock. So, they coalesce to form a single stronger shock and then march together.

Now, you see this; if we consider two particular streamlines. Let call this intersection point to be o and we consider two stream lines, one below this intersection point that is passing through two shocks and another stream line that is passing through the single shock. Now, these two stream lines or all streamlines, which are on one side of this intersection point and other set, which are on the side of the streamline, they will experience different entropy changes. Consequently, there will be difference in the entropy of the flow, which is passing through this part of the shock and which is passing through these two shocks and a slip stream will be formed.

A slip stream will be formed (no audio between: 06:05-06:21) where the entropy on the two sides are different as well as the velocity, density and temperature. However, this pressure on the two side must be equal and often it is necessary that another shock Sorry, another wave, of course, much weaker will come back, (no audio between: 06:47-06:55) a weak reflection (no audio between: 06:57-07:13) and this weak reflection is necessitated to make the pressure on the two side equal; to make the pressure on the two sides of the slip stream equal.

Now, this reflection can be either a compression or an expansion depending on the particular configuration of these shock and mach number. Anyway, whether it is a shock or expansion, it is much weaker then these primary waves or primary shocks. Usually, this second shock is much weaker than the first one, then this is a compression wave; however, if their strength are comparable this might be an expansion wave as well.

So, we can say that this part of the second shock is merging with the first shock and a small part is reflected back towards the wall. This is what usually happens when two shocks of same family interact that is they usually merge to form a single stronger shock. However, a small part of it can reflect back as a much weaker wave, which can be either compression or expansion, depending on the relative strength of these two interacting shocks. These slip stream (Refer Slide Time: 09:19) develops at the point of intersection which separates the flow in two different regions having different entropy and different density, temperature and flow velocity. Since, this slip stream represents a jump in tangential velocity; this is essentially a vertex line.

Now, when there are a number of shocks of same family and they coalesce together as it happens when there is a smooth turn or number of turns, then there are many such reflected waves. Even these reflected waves when it hits the wall again can reflect and consequently there is a series of reflection wave and reflected wave and their interaction. There are large number of slip stream from each of these intersection points making this entire region full of vorticity or flow freely is becomes rotational and also a continuously varying entropy field.

Let us now consider when a shock interacts with an expansion fan. When a shock interacts with an expansion fan of the same family that is both are moving in the same direction, let us say. (No audio between: 11:01-11:10) This is our shock develops from here. Now, we know at this corner as the flow turns in this corner, as the flow turns in through this corner, there will be an expansion fan, there will be an expansion fan, there will be an expansion fan (no audio between: 11:42-11:58) and both the waves that is a shock wave and the expansion waves are of the same family and they will interact. What happens in this case is that the shock is attenuated or the shock strength decreases at each case. Since, the shock strength decreases here; it has a different shock wave angle and it will move like this (Refer Slide Time: 12:28). Further interaction with this, it will again experience a reduction in strength or attenuation and it will further bend and will have a bench shock.

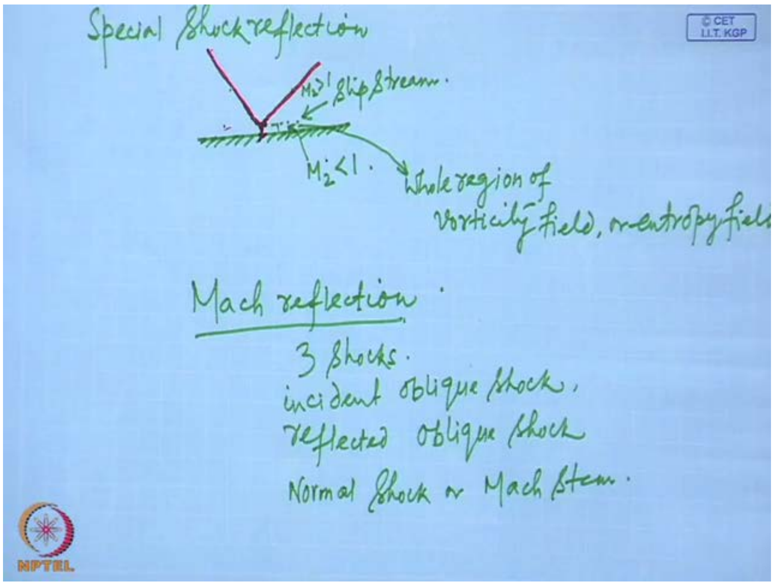
Now, in each case there is usually associated weak reflection ((no audio 12:54 to 13:24)) and ((no audio 13:25 to 14:00)); however, in each of these cases, you can see that the flow, which crossing the shock is having some change in entropy. Subsequently, some of these reflections, they can also be compression type; however, there will be a marginal change in entropy. Consequently, there will be a multiples shock slip stream here and we will have a whole region of vorticity downstream of this interaction.

So, we discuss mainly two type of shock interaction that is interaction of shocks of same family and the other is interaction of waves of different family, which we discussed last class. Interaction of same family, which we discuss today and we have seen mostly that in case of shock, these two shocks, they coalesce together with a weak reflection, which can either, be compression or expansion and there we usually will have slip stream from the point of interaction.

In case of an interaction between shock and expansion fan of same family, the shock experiences in attenuation of its strength at each interaction. Consequently, the shock becomes a curved shock and at each interaction it bends because of its strength reduced and we have a curved shock due to these interactions and at each point of interaction, there will be usually a weak reflection. In many cases, particularly if the interactions are weak, these reflections can be neglected; however, when the interaction is quite strong, these reflections cannot be neglected and there will be a multiple slip line and downstream there will be a whole region of vorticity or an entropy field.

Now, we will consider some other cases of interest. One such is that in case of a viscous flow or flow near a wall what happen to the shock.

(Refer Slide Time: 17:00)



Special Shock reflection

Whole region of vorticity field, or entropy fiel

Mach reflection.

3 shocks.
incident oblique shock,
reflected oblique shock
Normal shock or Mach stem.

Let us say that we have a solid wall (no audio between: 17:03-17:14) and a shock is coming and incident. Now, we know that near the wall there will be usually a boundary layer and flow velocity will continuously decrease towards the wall and there at the wall the flow velocity being zero. Now, within the boundary layer, there will be a small region of the flow where the flow is subsonic and the let us say the part of the flow such that the part of the boundary layer is supersonic. Consequently, the shock will be able to penetrate that supersonic part and will not be able to heat the wall where it is subsonic or where the flow velocity is zero and also little where the flow subsonic.

However, since the flow mach number upstream of the shock is continuously changing or decreasing in the subsonic part in the supersonic part within the boundary layer, the shock strength or shock wave angle will also continuously increased. It will behave all most like a normal shock and the configuration will be something like this (Refer Slide Time: 18:46). That is of course; it is shown here as touching the wall, but actually in reality, it will not reach the wall, but little above the wall until the region, where the flow is subsonic supersonic.

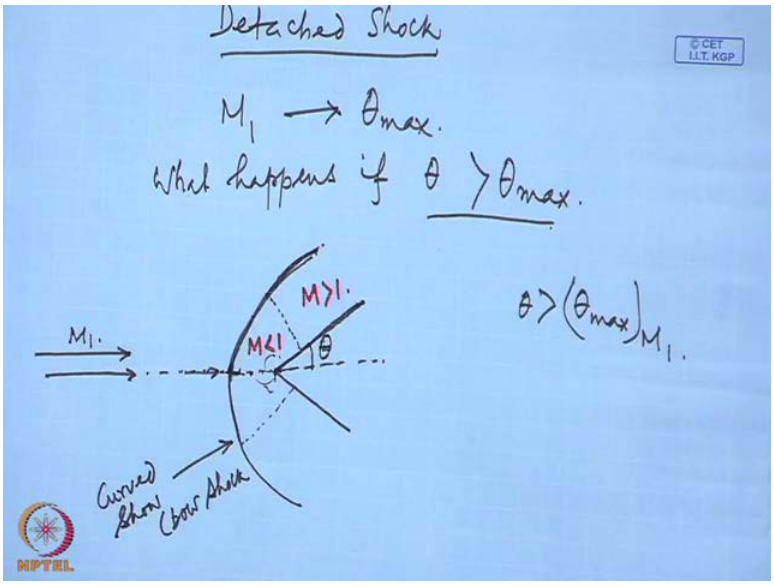
And within that boundary layer, there will be a small stream, which is all most like a normal shock. (No audio between: 19:04-19:19) In this region, this will be the slip stream and below that slipstream there is a continuous region of rotational flow or flow with vorticity. This whole region is of rotational and having entropy field. This part where M 2 is greater than one and in this part, M 2 is less than one; whole region of vorticity field or entropy field.

So, this is a special type of shock reflection, and usually this is known as mach reflection. So, we see this mach reflection is characterized by three shocks; the incident shock, the reflected shock and there is a small lag of normal shock called the mach stem. The series is characterized by three shocks; an incident oblique shock, reflected oblique shock and normal shock lag or called mach stem.

(no audio between: 22:10-22:19)

However, there are many such many other situations, where much more complex shock reflection and shock interaction takes place and we will not go into that. We will now consider another flow situation where the wave angel is greater than the theta max for the particular mach number.

(Refer Slide Time: 23:07)



We have all ready seen that for a given mach number M 1, there is a maximum possible value of theta max. That is for every M 1, (no audio between: 23:07-23:18) there is a corresponding theta max.

Now, what happen if theta (no audio between: 23:29-23:37) what happens if theta is greater than theta max? Let us consider a wedge and (no audio between: 23:52-24:10) in this case, say this theta is greater than theta max corresponding to M 1. Now, how will the flow behave? In this case, since we know that through an attached oblique shock, the flow can flow at mach number M 1, can turn maximum amount by theta max and to become parallel with the wall, but in this case this theta is greater than theta max. So, with an attached oblique shock, the flow cannot achieve this amount of turn to become parallel, as it must be to satisfy the boundary condition.

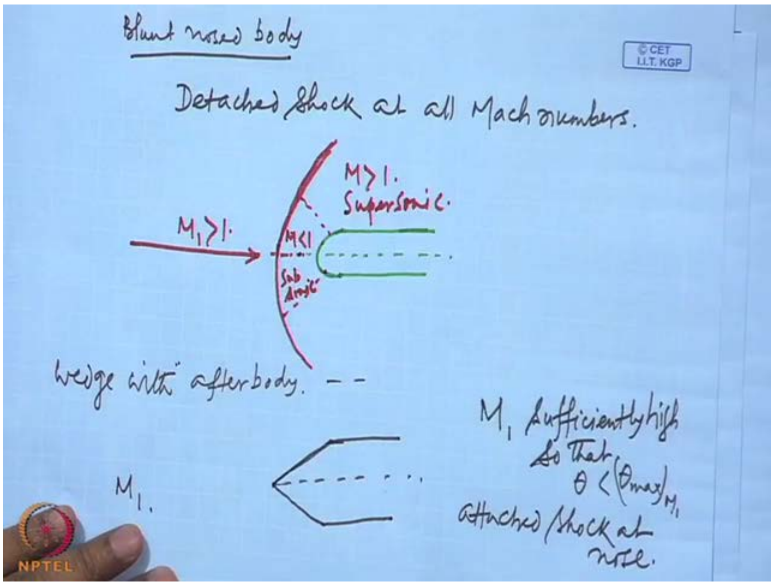
Consequently, what need to happen that the flow here in this region (Refer Slide Time: 25:02) has to be subsonic. That is flow near the nose region of the wedge has to be subsonic, where subsonic flow of course, can turn by any angle, and that means, this shock will no longer be attached to these wedge, but rather we will have a detached shock, and this flow will negotiate this turn by a curved (no audio between: 25:47-25:58) by a curved shock commonly called as bow shock. (no audio between: 26:13-26:34)

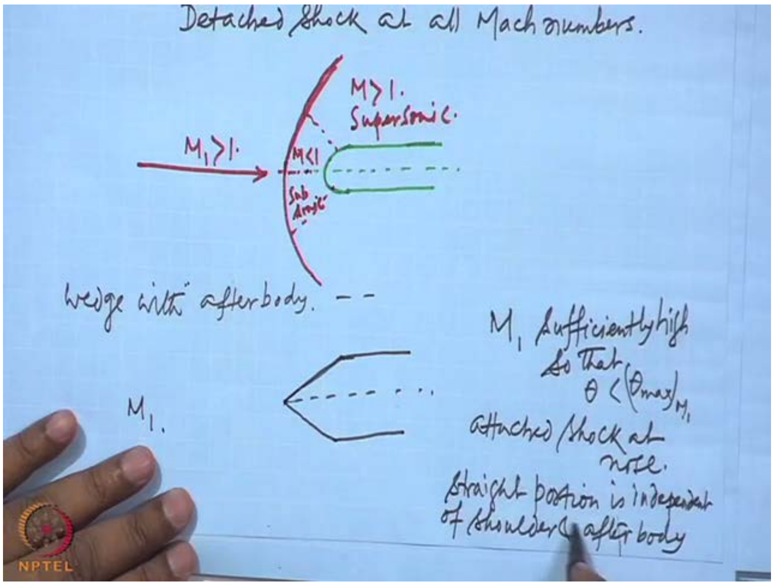
Now, the shapes of these curve shock, as well as the distance from this point to this point will depend on the geometry of the body and upstream mach number. On the central streamline that is on this, the shock is a normal shock and it does not turn, but it become subsonic. As we move away from this central shock line, the shock strength or wave angel gradually decreases from pi by 2 to smaller values, meaning that in this part of the shock, the shock angle is very close to pi by 2, resulting in the strong shock solution and downstream flow being subsonic.

Up to certain distance, the shock is strong shock and the downstream flow is subsonic; however, beyond that the flow is...(Refer Slide Time: 27:48) So, this part is M 1, while this part is… What in a sense, we see that the shock satisfy the entire branch of solution. The complete shock solution is available here. As we move further, the wave angle decreases and shock finally, reaches to the mach angle; asymptotically it reaches to the mach angle or the weak post of zero strength shock. Thus the condition that we obtain along the detached shock wave, contain the whole range of oblique shock solution, for the given mach number. In such a configuration, the shock inclination corresponding to strong solution is found as well as the weak solution is found.

Now, in this part (Refer Slide Time: 29:05) where the flow is subsonic, we know this shock is no longer independent of the downstream conditions. A change in geometry that is our change in pressure in the subsonic portion affects the entire flow up to the shock. The shock will try to adjust itself to the new condition. This is what happens for waves with theta greater than theta max corresponding to M 1. Now, for a blunt nosed body that theta is greater than theta max corresponding to any mach number and consequently for a blunt nosed body the shock is detached at all mach numbers. (no audio between: 30:01-30:25). For a blunt nosed body.

(Refer Slide Time: 30:29)



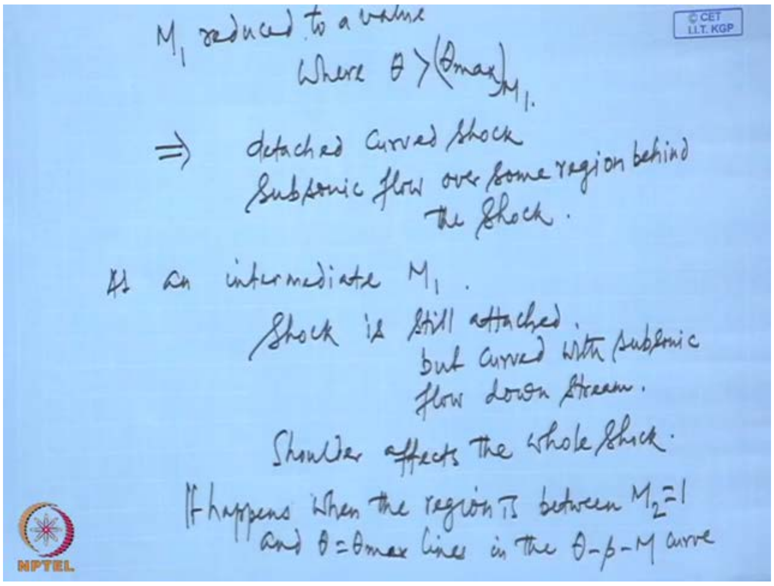


If we have a blunt nosed body (no audio between: 30:26-30:42) has a detached shock at all mach number. (No audio between: 30:47-31:03) Let us say, we consider a hemisphere cylinder type of body, (no audio between: 31:13-31:26) we have a detached shock at all mach number ((no audio 31:29 to 32:10)). So, we see that if we use a conventional air fall that are used for low speed or transonic speed aircrafts, which have we conventional curved leading edge, for a smooth flow, we will we always associated with a detached oblique shock in front it when they fly at supersonic speed. (No audio between: 32:38-32:56)

We will now consider once more that the flow phenomena qualitatively on a wedge with after body. So, on a wedge with after body, (no audio between: 33:13-33:24) what type of events that happens? (no audio between: 33:27-34:04) When M 1 is sufficiently high when M one is sufficiently high… When M 1 is sufficiently high, so that theta less than theta max corresponding to M 1. We have attached shock at the nose; attached shock at the nose, and we can see that the state portion is independent of the shoulder and after body.

The flow on the straight portion is independent of the shoulder and after body. Now, the shock angle increases as M 1 decreases; shock angle increases as M 1 decreases. So, at a certain reduced mach number the flow after the shock become subsonic. So, as M 1 reduced, attached shock at nose, and we can see that straight portion is independent of shoulder and after body.

(Refer Slide Time: 36:06)



M_1 reduced to a value

where $\theta > (\theta_{max})_{M_1}$.

$\Rightarrow$ detached curved shock

subsonic flow over some region behind the shock.

At an intermediate M_1.

Shock is still attached.

but curved with subsonic flow down stream.

Shoulder affects the whole shock.

It happens when the region is between $M_2 = 1$ and $\theta = \theta_{max}$ lines in the $\theta-\beta-M$ curve

Now, M 1 reduced and (no audio between: 36:05-36:23) M 1 is… reduced to a value where theta greater than theta max M 1. Then we have seen already that there will we have detached curved shock and the flow after the shock. Then we have detached curve shock, subsonic flow over some region behind the shock. (No audio between: 37:15-37:38).

Now, see while reducing this M 1, from the first value to this second value; obviously, we will come to a situation where M 1 is such that the shock has become curved, but still attached, (no audio between: 38:04-38:24) and the flow at an intermediate M 1. At an intermediate M 1, the shock is still attached, but curved (No audio between: 38:34-38:44) with subsonic flow downstream. (No audio between: 38:48-39:04) Then in that situation the shoulder will effects the (no audio between: 39:08-39:21) whole shock. (no audio between: 39:26-39:36) this is This is between the region, it happens, when the region is between M 2 equal to 1 and theta equal to theta max lines in the theta, beta, M curve.

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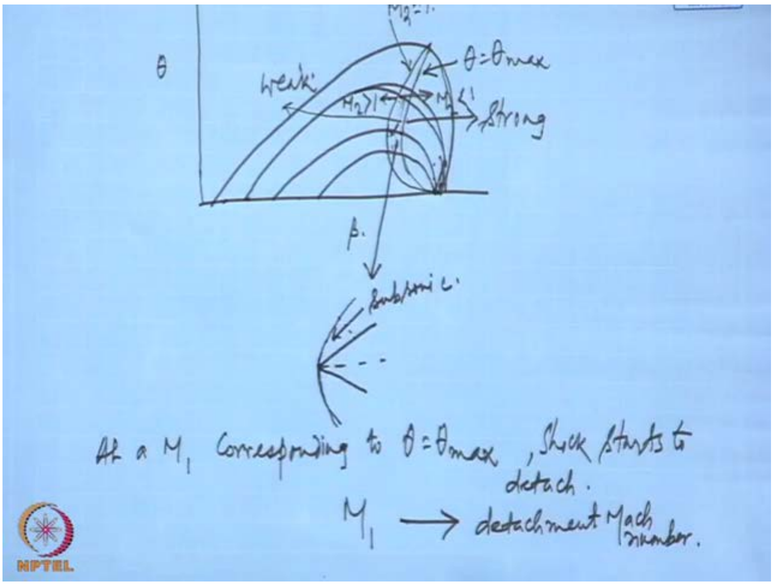

At a M_1 Corresponding to $\theta = \theta_{max}$, Shock Starts to detach.

$M_1 \longrightarrow$ detachment Mach number.

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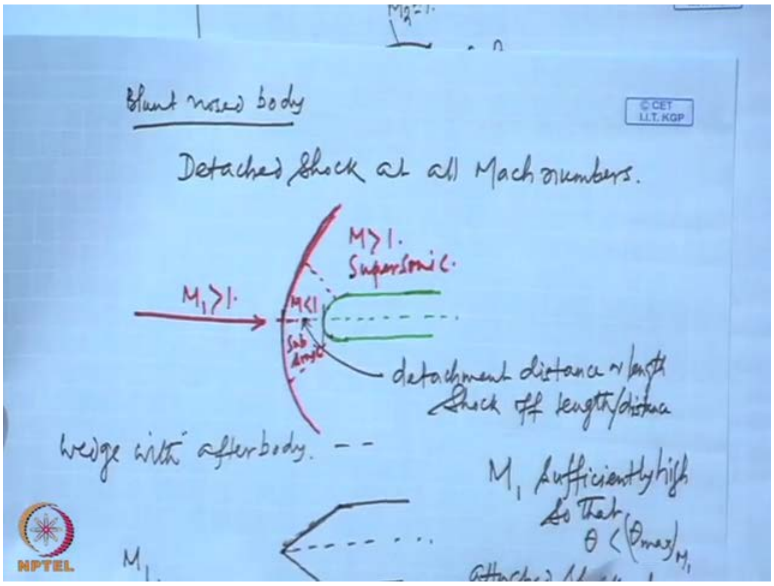
That is, if we remember that if we recall the theta, beta, M curve, which we have earlier drawn ((no audio 41:00 to 41:37)) if we remember that ((no audio 41:40 to 42:37)).

So, if the solution or the theta lies in between these region, that is little less than theta max, but very close to be it, in that region. It happens that the shock is still attached to the nose, but it is curved and downstream flow that is flow over the straight portion of the wedge here, straight portion of the wedge (Refer Slide Time: 43:29) here is subsonic, a part of it is still subsonic and that is we have (no audio between: 43:24-43:52).

(Refer Slide Time: 45:52)



Blunt nosed body

Detached Shock at all Mach numbers.

detachment distance ~ length
Shock off length/distance

wedge with afterbody. - -

M_1 Sufficiently high So That $\theta < (\theta_{max})_{M_1}$

Attached

This particular mach number at which this first happens can be thought of critical mach number for this particular case. (no audio between: 44:03-44:26) At M 1, corresponding to theta equal to theta max, corresponding to theta equal to theta max the shock waves starts to detach, shock starts to detach and this particular M 1 is then called the detachment mach number. detachment mach number and at As M 1 decrease further, the detached shock moved upstream of the nose, and the separation distance of the shock from the is called detachment distance or shock of distance. This distance is called detachment distance or length or also called shock of length or distance.

So, what we see that in case of a wedge with after body. Let us say for a given wedge angle, particularly when the mach number is considerably high, so that the wedge angle is less than the theta max value corresponding to that mach number. we have an attached oblique shock at the nose and the flow over the straight portion is independent of the shoulder and the body.

Now, as we decrease the upstream mach number that is M 1, the shock angle continuously decreases and at a certain reduced mach number, where the flow after the shock becomes subsonic and the shoulder flow over the shoulder, now effects the whole shock. Consequently, the shock may be curved, but still remaining attached. This is we have shown in theta beta M curve and that is in where this can happen that is in the region where the wave angle is just marginally less than maximum value of theta max.

The region lies between M 2 equal to 1 and theta equal to theta max. This situation can happen. Further reduction in mach number corresponding to theta max, the shock waves start to detach, this is called the detachment mach number and when mach number is further decreased, the detach shock moves upstream of the nose. The particular shock of distance will dependent on the geometry as well as the mach number. Once again, since, there is a considerable subsonic region behind the shock or over the straight portion of the over the shoulder of the body that completely the affects the upstream solution and the flow is no longer or the shock is no longer independent of downstream conditions.

For a blunt nosed body, we have seen that at any mach number, at any upstream mach number, the flow turning required is larger than the theta max and a supersonic turning through an oblique shock is not possible. The flow becomes always super subsonic ahead of the blunt nose and a detached oblique curved shock always stands ahead of the blunt-nose that is for detached shock we will have. For a blunt-nose body, we will have detached shock at all mach numbers. These very simple analyses of isentropic waves and their interactions can help us to analyze many practical two dimensional supersonic flow problems, particularly for all those geometries, which have straight line segments.

Of course, when the mach number is considerably high and the flow turning required are not large and the flow never becomes subsonic, because we see that the step by step construction of flow is possible only for a supersonic flow alone. If there is a subsonic region, we cannot use that, because at subsonic flow is influenced by the entire boundary or all boundary conditions; the whole flow field is inter connected.

So, this piecewise construction of supersonic flow can be used to find certain solution of some practical problems, of course, for two dimensional and for geometry with straight segments. In aerodynamics, we are quite concerned with air foils and wings and we know that a flat plate is a very good approximation of an aircraft wings and the flat plate is a straight geometry, so we can analyze the flow over the flat plate using these simple theories that we have discussed about for shock and expansion.

Similarly, in supersonic flow or diamonds of the airfoil or is very widely used and which is also made up of straight line segment and once again we can analyze that flow; both qualitatively and quantitatively. Similarly, we can also try for any other geometry suitable for supersonic application and particularly, if the geometry is having straight line segment, the analysis will be quite straight forward and simple, and will now try to construct some of the solution and derive very important and useful results.

However, we will do that in our next lecture until that thanks.