High Speed Aerodynamics

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Module No. #01

Lecture No. #12

Waves and Supersonic Flow (Contd.)

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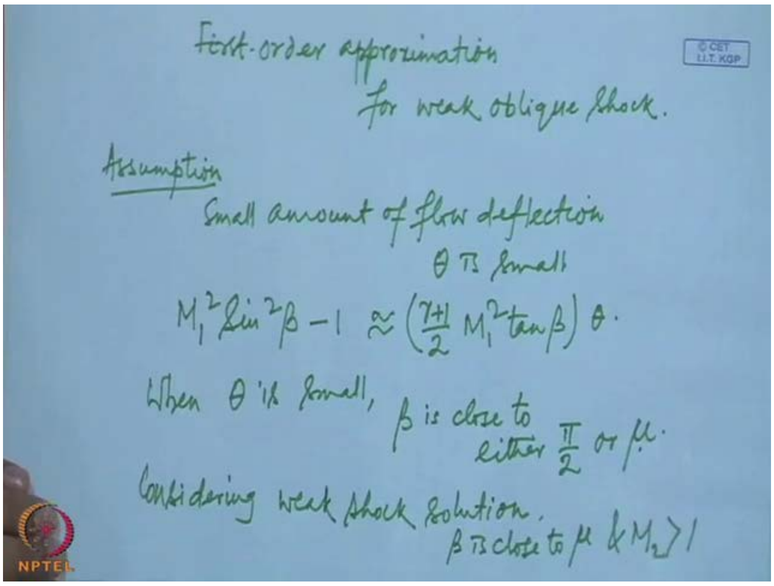

In the last lecture, we have derived oblique shock relations. We have seen that flow turns as it passes through an oblique shock and the turning angle is related to the wave angle and the upstream mach number. We have derived the well known theta beta M relationship. Now, this theta beta M relationship is exact relation; however, we see that this exact relation is quite difficult to handle for further analysis, because in most real problem, the flow turning angle theta will be known to us, as well as the upstream mach number and the wave angle will be the usual unknown.

We have seen that the wave angle in terms of upstream mach number and flow turning angle is given by an implicit equation; implicit trignometrical equation and it is not very useful to handle analytically. For that purpose, for our ease of use, we will try or look for a first order approximation of this relationship. For that will consider only weak oblique shock solutions and that is we will consider that part of the shock, where the solution is weak shock and the downstream mach number remains supersonic.

What we will try is we look for a first order approximation for a weak oblique shock and that is we will consider that the shock is weak and the flow turning angle is small. This is the assumption and that is small amount of turning or small amount of flow turning or theta is small. Now, we have that for small theta or I have already seen that for small theta, the shocks strength, as we have derived earlier that this is equal to gamma plus 1 by 2 M 1 square tan beta into theta .

So, when theta is small, the wave angle, if we look to the curve in theta beta curve, so when theta is small, beta is either close to mu or beta is close to either pi by 2 or mu. In the flow, there are two solutions. For small theta, one is very close to the normal shock where the flow wave angle is very close to pi by two or very close to the mach angle. Now, if weak some solution is considered or considering weak shock solution only, beta is close to mu and M to the supersonic that is the flow scale remain supersonic.

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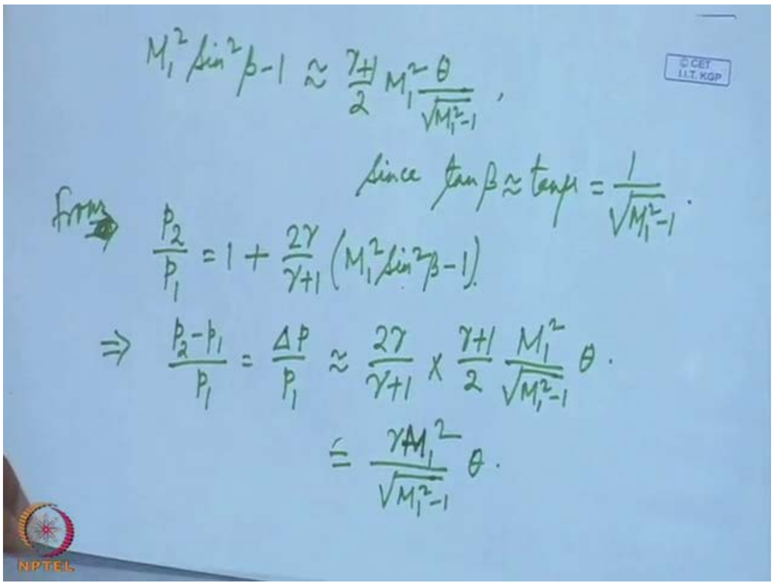

Then the approximation reduces to M 1 square sin square beta minus 1, becomes close to gamma plus 1 by 2 M 1 square, and tan beta is very close to (no audio between: 06:29 - 06:43) since we have tan beta is very close to tan mu, which is of course, one by root over M 1 square minus 1. Then we have p 2 by p 1 equal to 1 plus from 2 gamma by gamma plus 1, M 1 square, 2 gamma by gamma plus 1 and we substitute this relation here to give it gamma plus 1 by 1 M one ((no audio 08:11 to 08:40)) Gamma into M 1 square by root over M 1 square minus 1 into theta. So, this show that for a weak shock, the pressure rise across the oblique shock is proportional to the flow turning angle.

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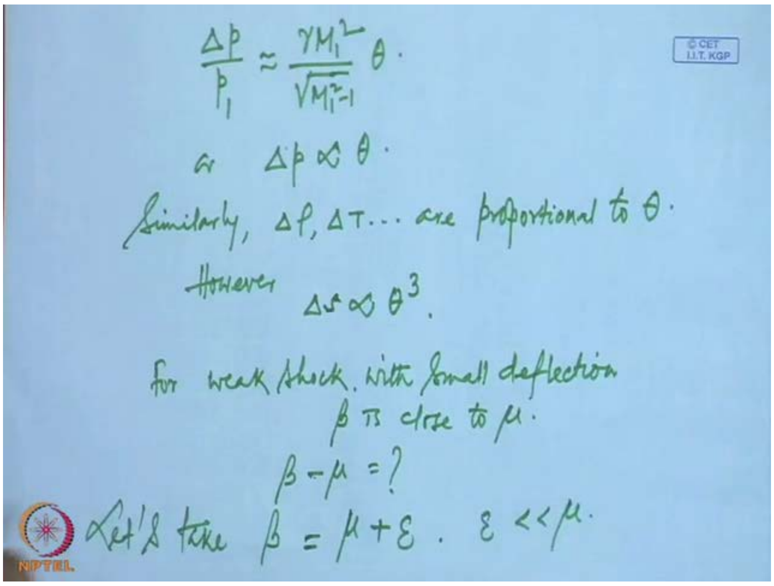

Now, delta p by p 1 to gamma M 1 square by root over M square minus 1 to theta or delta p proportional to theta. That is the change in pressure across a weak oblique shock is proportional to the flow turning angle. Similarly, other flow quantities, and similarly delta rho, delta T are proportional to theta. However, we are earlier seen that the change in entropy across a weak shock is proportional to the third order of the shock strength, which we have seen in case of normal shock and same applies for oblique shock also. Hence, the change in entropy delta s is proportional to theta cube. (Refer Slide Time: 11:10) You can see that when the flow turning is very small, then the change in entropy is almost negligibly small. We will also look to the difference between the flow turning angle beta and the mach angle mu. We have already seen that for weak shock with small deflection beta is close to mu; however, what is the difference between mu.

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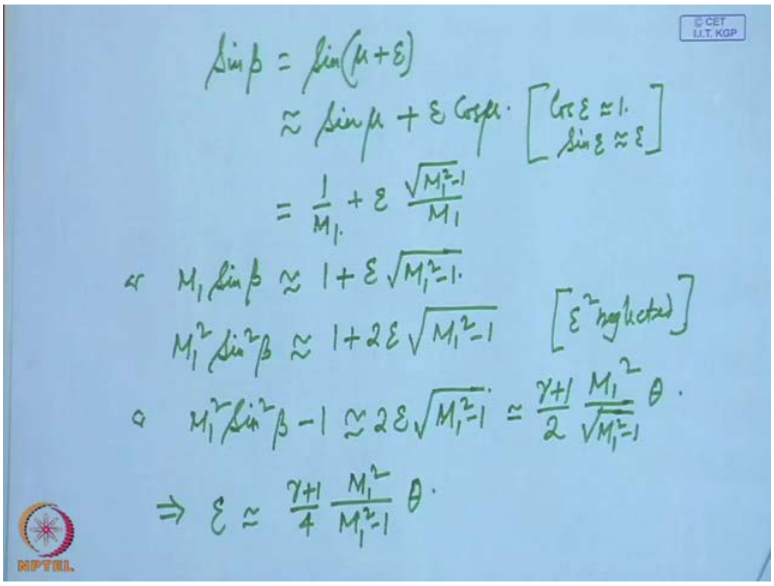

Let us say, where epsilon is quite small and is much smaller than mu, the mach angle, then we have sin beta equal to sin mu plus epsilon, which is nearly equal to sin mu cos epsilon and cos epsilon is taken as one, plus sin epsilon cos mu; sin epsilon is taken as epsilon and cos mu as same. (No audio between: 13:21-13:38) So, these are what is used comparable to the first order approximation and this makes sin mu as 1 by M 1, and cos mu is (no audio between: 14:08-14:29) or M 1 sin beta is approximately one plus epsilon root over M 1 square minus 1.

On squaring, we can write (no audio between: 14:49-15:06), where epsilon square neglected, which are comparable to first order approximation and this gives M 1 square sin square beta minus 1, which is approximately this quantity and which we have already seen to be gamma plus 1 by 2 of M 1 square by root over M 1 square minus 1 into theta. Hence, this gives epsilon and it is gamma plus 1 by 4 (no audio between: 16:29-16:43). So, see that for a deflection angle of theta, the difference between the wave angle beta and the mach angel mu is of the order of theta.

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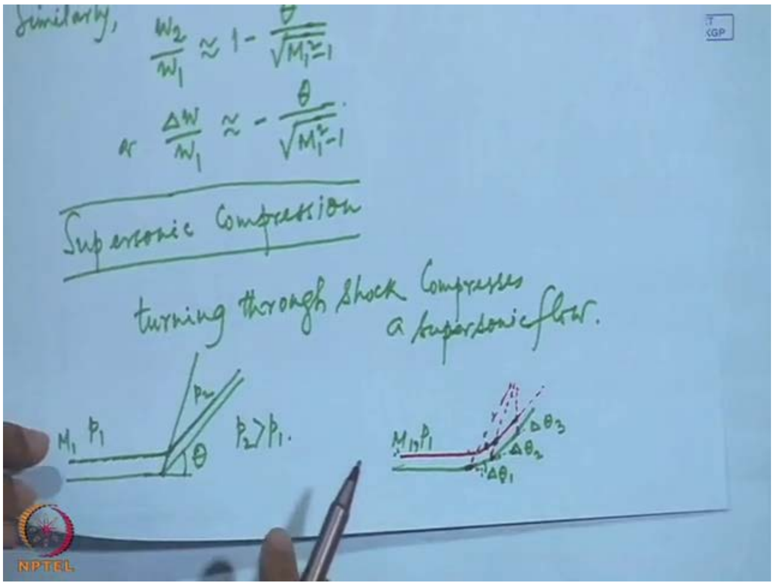

Similarly, to our first order of approximation, we can also see that the… Similarly, we can see that the change in velocity across an oblique shock is given as or we have instead of u by W 1 is minus theta by root over.... So, these are the relationship approximated for weak oblique shock for small flow turning angle. Now, we will discuss about the turning process or the supersonic compression using turning.

We have seen that across a shock, there is an increase fluid pressure and density. (No audio between: 19:02-19:26). That is when a flow turns through a shock wave or oblique shock; its density and pressure rises or the fluid is compressed, then this can be used to compress or supersonic flow. That is we can use a turn or a compressive turn to compress a supersonic flow, and this can is easily be achieved just by providing an angle. That is if we can deflect the supersonic flow by an angle theta, immediately the flow is compressed or the pressure is increased.

Now, this turn turning through (no audio between: 20:31-20:58) now, if we turn the flow where angle theta we can find out how much will be the pressure rise using the shock relation and similarly, how much will be the density and temperature rise. All these can be found out by the shock wave oblique shock relations and also you can determine how much the entropy range is across this oblique shock or compression due to turning.

However, if this angle theta is divided into several small angles, that is instead of providing one corner with theta, if we provide a number of corners with much smaller angle, but the total angel being this theta, then the compression will occur through successive shocks and each of these shocks will be weaker than the total shock that would have been there, if the flow a turn once though an angle theta. Now, when we achieve this turn, by a number of smaller corners, then the shock at each corner will divide the flow field near the wall into number of segments of uniform flow; however, away from the wall the shocks will tend to interact and the flow will be as if passing through a single shock.

Let us say that if you would have one turn of theta with the shock angle being beta ((no audio 23:29 to 24:21)). In each case, there will be a shock ((no audio 24:24 to 25:01)) and in this also, using the principle of limited upstream influence, we can construct this flow step by step coming to this first corner knowing delta knowing delta angel delta theta and M 1 here, M 1 and p 1 here, we can find out what will be the M mach number here and what will be the pressure here. Again, knowing the mach number here and the flow turning angel here, again we can find out what will be wave angel beta and we can get this part of the flow and so on we can proceed and get the complete streamline.

Now, after passing through this first shock, the mach number will decrease. We will assume that the mach number remain supersonic throughout; however, the mach number here that is downstream with the first shock will be smaller than the mach number upstream of the first shock and since the mach number ahead of the second shock is smaller than the mach number ahead of the first shock. So, if these two flow turning angels are same, then wave angel here will be different or higher from this wave angle. Consequently, these two shocks will be convergent as if that is little away from the solid wall they will tend to mark or intersect.

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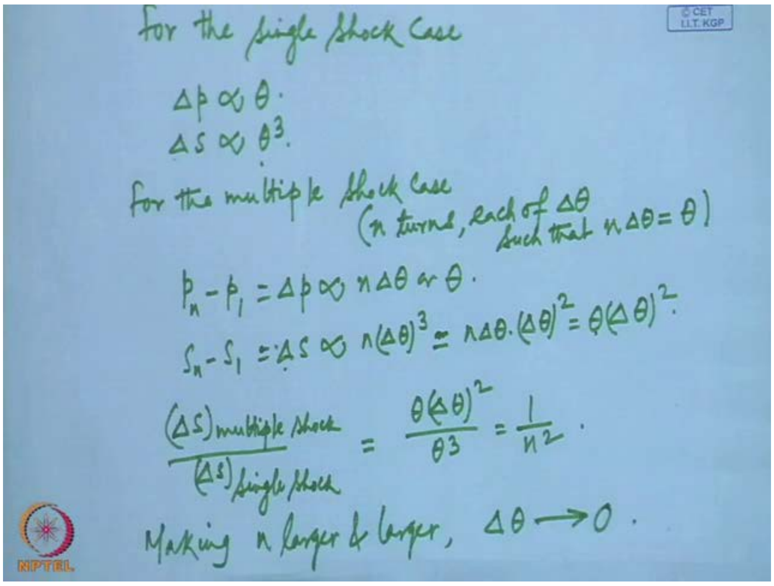

Now, for single shock, let us say for single shock case, assume still that weak shock and flow deflection angle is small, that is theta is still small, the pressure rise across the shock is proportional to theta and the entropy rise is proportional to theta cube. For the multiples shock case (no audio between: 27:51-28:03), let us say that there are n turns each of delta theta, each of delta theta, such that in delta theta equal to theta. Then let us say that the final pressure is p n minus p 1, that is again delta p, is proportional to n into delta theta, which is theta. Once again the pressure rise is the same as we as we had obtained by turning flow with a single shock; however, the entropy rise in this case will be ((no audio 29:17 to 30:16)). And this is clearly much smaller than this quantity.

So, when we have multiple shock, but each shock or each turning is much smaller angle then our total entropy rise is also much smaller compared to the single shock turn. If we take the ratio, this is delta s for multiple shock or multiple turn, shock by delta s of single shock, then we (no audio between: 31:13-31:33) have… Let us take an example and let us say that we turn the flow by a single shock through 10 degree and that is we have only one deflection corner of 10 degree. We achieved the same compression by providing 10 number of smaller turn, each of one degree.

What happened? The pressure rise in both the cases are same; however, when we compare the entropy rise, you see in the second case that is when we are achieving the same amount of compression by 10 one degree turns, the entropy rises 100^{th} ,that is hundred; a factor of hundred of the entropy rise that we have would have obtained if it were only one ten degree turn, that is compression by a single ten degree turn, we will have hundred times more entropy rise than compression by ten one degree turn. However, in both cases near the wall, the pressure rise, density rise, velocity rise all sorry velocity decrease all will be of the same order.

Now, if we continue this process of subdivision, we can make this delta theta to be infinitesimally small, and making n larger and larger, we have delta theta approaches zero.

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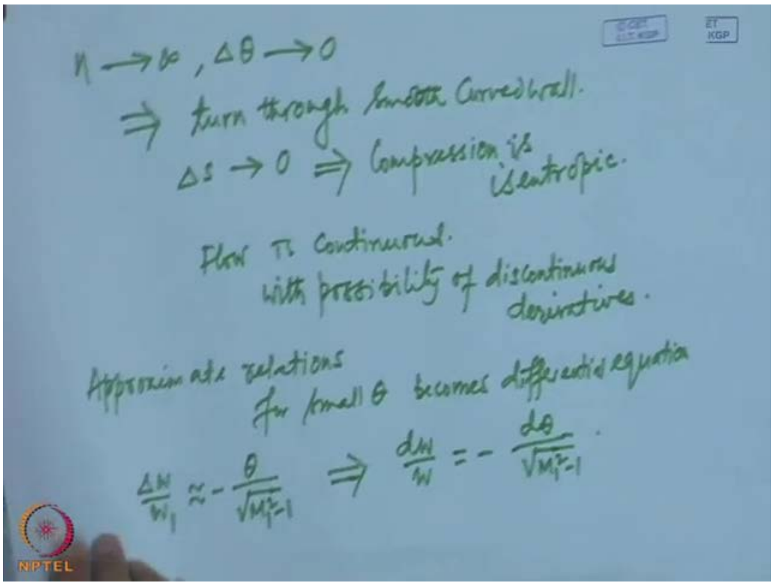

What we achieve is basically a turn through a smooth curved wall, that is n tending to infinity and delta theta tending to zero, is basically a turn smooth curved wall and then the change in entropy tends to also zero, that is compression is almost isentropic. (No audio between: 34:51-35:15) Now, in this case, when the shocks are becoming vanishingly weak that is when delta theta tends to zero, the shocks become vanishingly weak and basically these shocks are then mach lines, and then each segment of the uniform flow between two mach lines. Then each segment of that uniform flow becomes narrow and narrow and finally, they coincide with the mach line.

In that situation flow inclination and mach number are constant on each mach line. Thus when we limit or when we approach to this smooth flow, the velocities and flow inclination are continuous. That is the velocities and flow inclinations are continuous.

However, the derivative of the flow parameters may still be discontinuous derivatives. (No audio between: 36:31-37:12) discontinuous derivatives.

Now, the approximate expression that we have already derived for small flow deflection, they now become differential equation; approximate relations or relations for small theta becomes differential equation and in particular that is delta W by W 1 (no audio between: 38:13-38:23). It is to be noted that even though we have not written a delta theta, the theta which is the turning angle, is always represents a difference in direction. Consequently, this is always in the differential form or difference form and for a smooth turn this relation can be written as (no audio between: 38:49-39:16).

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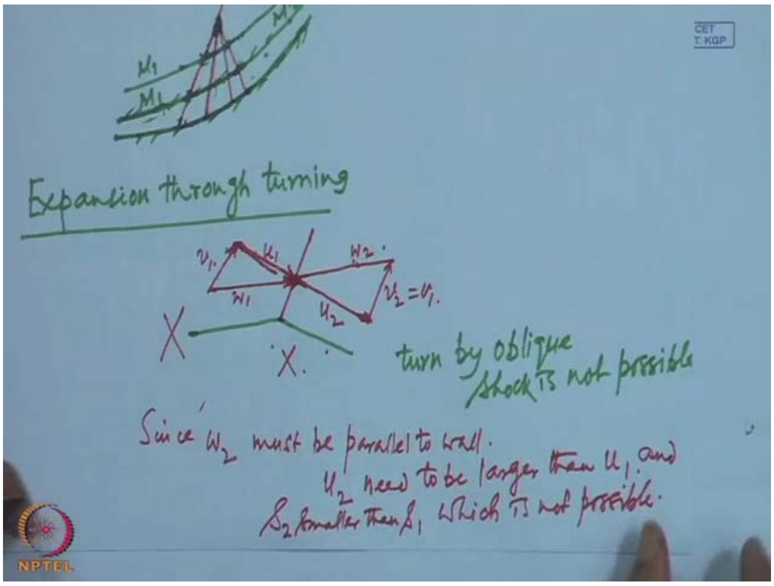

Now, let us see this configuration, for close to the wall, (no audio between: 39:28-39:38) where there is no sharp corner, but the wall is turning smoothly. Similarly, let us say there will be very weak shock from each point and those shocks are almost the mach lines. However, because of the flow convergence because of the convergence of these mach lines, since the mach number is continuously changing and the change is decreased so that mu is increasing and we can show some of these mach lines are ((no audio 40:27 to 40:58))

And we consider one or two streamlines close to the wall; of course, in the inviscid flow the streamlines will be parallel to the wall. Let us say that the mach number here is M 1 and the mach number here is M 2, of course, its holds good for both the streamlines.

Now, we can see this change in from mach number M 1 to mach number M 2 and it occurs over, let us say that this distance for this particular streamline; however, the same change occurs over this distance for this particular streamline, and since these mach lines are convergent, this distance is considerably larger than this distance. That is on this streamline, the changes are occurring over a smaller distance. Consequently, the gradients of velocity temperature and all other flow parameters on these particular streamlines are much higher than the on these streamlines. At these points of intersection, where all these mach lines are meeting, it simply implies that this change is occurring over almost a zero distance and the gradient is infinitely high.

Now, since this cannot occur, that is where it cannot occur in the region where mach lines converge and these very high gradient occurs and actually what happens that the flow, which was very nearly isentropic in this part, will no longer remain isentropic in this part. The diffusive mechanism will come into action and this will be, that is all these mach lines will merge together to form a simple oblique shock. With respect to an observer, looking far away from this corner, it will appear to a single shock with same strength that would have been there for it, this turning over by a single shock through an angle theta.

This convergence of these mach lines in a supersonic stream is again a typical non-linear effect and a decreasing mach number or increasing flow inclination both of them tend to make these mach lines steeper and this mach line converges. However, for any of these streamlines can be replaced by a wall and that is for these or these (Refer Slide Time: 45:12) and flow near the wall can be treated as isentropic.

Now, again let us say that if we have met this wall, this is also a wall and replace these streamline is also in wall, if we replace these streamline is also by wall, then between these two wall, there will be on isentropic compression and because the gradients will always remain smaller and the compression process will remain isentropic without having a single strong shock stronger shock.

Now, we consider the case of expansion that is expansion by turning. (no audio between: 46:30-46:46) This appears that if we have a corner like this that is for a concave corner then the flow will turn and it will expand. In this case, turn by an oblique shock is not possible (no audio between: 47:32-47:44) and we can see that if we had a shock here, let us say if we had shock here, then W 1 and once again we could have resolve these in terms of (no audio between: 48:23-48:42). Now, there will be no change in flow direction of this normal component, and this component v 1 will remain unaltered, and will be seeing that (no audio between: 49:03-49:15)… now, these of course, cannot happen, because it simply implies that if we need to have since W t must be parallel to this wall so this cannot happen. Since, W two must be parallel to wall ((no audio 49:47 to 50:17))..

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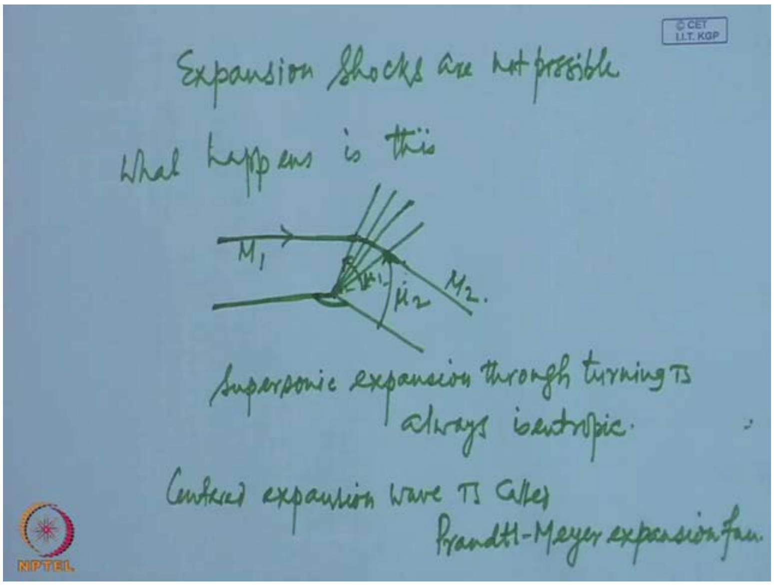

And consequently this expansion cannot be achieved through an oblique shock or expansion shocks are not possible. This relation of course, we have seen earlier also. Expansion shocks are not possible because what happens that if you see this ((no audio 50:59 to 51:31)). What happens is this what happens is this that is (no audio between: 51:40-52:06) and expansion fan occurs here. (no audio between: 52:20-52:32)

If the front has an angle mu 1 and similarly, these has an angle mu 2, so that is and as we have seen that through expansion fan, the flow turns through each of these characteristics, but smoothly to become parallel to the downstream wall. This we have already seen that the characteristics patterns or avoidance. Again, the non-linear mechanism that makes the compression waves steeper and steeper, it makes the expansion wave flatter and these characteristics lines become divergent. Consequently, the process always remains isentropic, since the gradients decreases. So, expansion even have achieved through a corner, supersonic expansion even are achieved through a corner or sharp corner, is still isentropic expansion. Supersonic expansion by turning or through turning is always isentropic and we see that flow up to the corner is uniform at mach number M 1 and consequently the leading mach wave, a straight line at the mach angle mu 1. Similarly, the flow downstream is again uniform at mach number M 2 and the terminating mach line is at an angle mu 2 with respect to the downstream all and this centered expansion wave, the centered expansion wave is called Prandtl-Meyer expansion. ((no audio 55:40 to 56:11)).

What we see here that in a supersonic flow we can achieve compression and expansions simply by turning it. A compressive turn or turn through a concave angle or flow through flow round a concave turn is (no audio between: 56:45-56:56) flow through a concave turn is an expansion turn, while flow through a convex corner is a compressive turn and when there is a sharp corner, a sharp compressive corner, the flow turns by through a shock and is compressed.

However, if the turn meets through a successive number of large numbers of smaller angels, the turning approaches almost isentropic nature and in the limit of a smooth turn, smooths compressive turn, the compression is also isentropic. For a expansion expansive turn or flow through a concave turn is always isentropic, even if the expansion corner is sharp, the turns still remain the isentropic, as well as turns through a smooth wall, expansive turns a smooth wall it also be isentropic.

So, what you see that expansive turn always isentropic irrespective of whether the turning or corner is sharp or smooth; however, compressive turn can be non-isentropic or isentropic. When the turn is through a smooth wall it is nearly isentropic turn, but here also one must remember that near the wall only the turning is isentropic, but as you move away from the wall, because of the non-linear stiffing effect, the mach lines are convergent and little away from the wall these nearly isentropic mach waves become a non-isentropic shock and the turn little away from the wall is always through an oblique shock and non-isentropic.