High Speed Aerodynamics

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Module No. # 01
Lecture No. # 11
Waves and Supersonic Flow

A fluid propagates in the form of wave and we have discussed the properties of one dimensional wave motion. Now, if a body moves through a fluid, which is at rest, then that body disturbs the fluid and this disturbance, in general, is not small. Hence, this disturbance in this fluid propagates or transmits to the other parts of the body, and also to the other parts of the fluid through wave propagation. The wave motion is compatible with the motion of the body. This wave motions that determines the pressure on the body, as well as the complete flow field around the body, and usually when the flow is subsonic, the wave motion is usually not considered. It is not essential and it is not even convenient.

However, when the flow is subsonic supersonic, then this wave motion can be used to construct the flow, in a much more convenient way. Particularly, if we consider a motion is steady, then we can study the motion from a reference system, where the body is at rest and the fluid flows over it. Now, when the flow is supersonic that is a relative wind is supersonic, the waves cannot propagate ahead of the immediate vicinity of the body.

As a result, this wave system also travels with the body, and with respect to the reference system, this wave system appears to be stationary and that is the wave system that moves with the body with respect to that reference system, the wave system appears to be stationary. Because of this limited upstream influence, in case of a supersonic flow, where the waves cannot propagate ahead of the body. This principle, which is known as the Limited Upstream Influence principle, allows the flow to be analyzed or constructed in a step by step manner.

However, to consider flow about a body, we need one more tool. We have seen that in one dimensional flow that is a shock wave and that shock wave is essentially normal.

However, when the general motion is two or three dimensional, which is of course of much more geometrical complex. This stationary shock wave; it can either be normal to the flow direction or can also be oblique to the flow direction.

We need to obtain the relationship between various parameters, across an oblique shock. Now, these relations across an oblique shock can be obtained directly by considering the equation of motion in two dimensions or in three dimensions, if necessary. However, they can also be obtained in a much simpler manner, by transforming the normal shock relations to oblique shock relations. (No volume between: 04:32-04:51)

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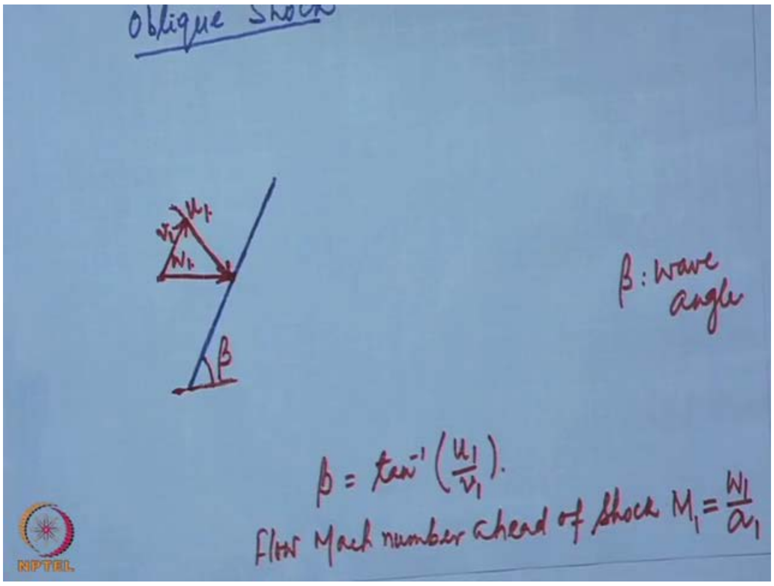

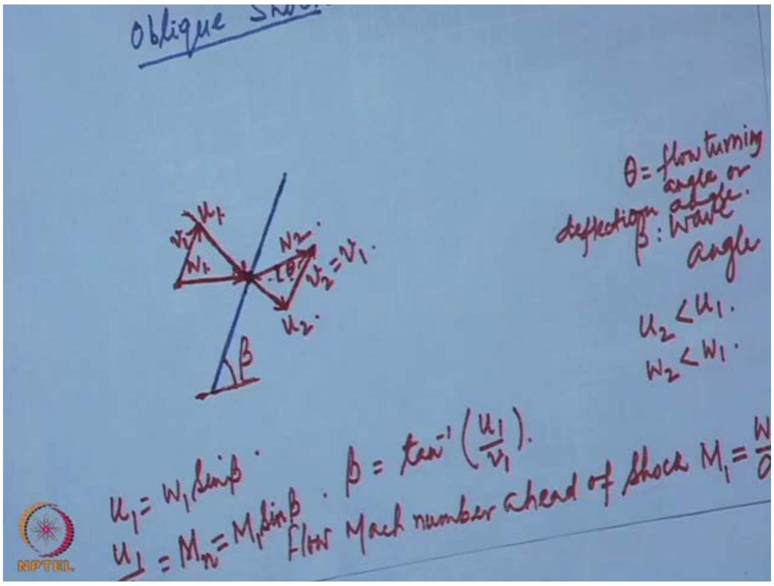

Let us consider an oblique shock and consider that the flow velocity ahead of this oblique shock is W 1, the flow velocity ahead of the oblique shock is say W 1. Now, we can resolve this velocity into two components, where one is normal to a shock. Let us say this component is u 1, and the other component is parallel to the shock. We call this to be v 1. This angle, the oblique shock angle with respect to the horizontal, we call this the wave angle beta. The beta, we call the wave angle and what we can see that this wave angle beta is tan inverse u 1 by v 1.

The mach number ahead of the, flow mach number ahead of the body, Sorry, ahead of the shock. The flow Mach number ahead of shock, (No volume between: 07:26-07:51) where a 1 is the speed of sound ahead of the shock. Now, since the normal component of upstream velocity that is W 1, normal component is u 1 and u 1 equal to W 1 sin beta. Hence, u 1 by a 1, which we can call the Mach number, based on the normal component. The Mach number based on the normal component is M 1 sin beta. Now, considering these two components of the flow velocity, we see the component v 1 or the tangential component is parallel to the shock and it does not cross the shock.

Hence, is not affected by the shock; however, the normal component u 1, which crosses the shock undergo the change as given by the shock relations that is normal shock relations, because this shock is normal to the flow velocity u 1 and as we have seen earlier across the shock the normal component of the velocity reduces. So, this u 2 is smaller than u 1 and will be given by normal shock relation.

The component v 1 does not change and remains the same that is v 2 equal to v 1, and the resultant velocity is W 2. So, we see that through the oblique shock, this W 2, which has a magnitude smaller than W 1, which is W 2 is also smaller than W 1. Not only that, the direction of W 2 is different from the direction of W 1. That is the flow when crosses an oblique shock or passes through an oblique shock, it changes its direction, or it undergoes a rotation, or turn this angle theta, which is the angle between W 1 and W 2 is called the flow turning angle or deflection angle (no audio between: 11:30-11:43).

Also, we can see that W 2 or the flow velocity, after the shock turns towards the shock. This is usually called the positive turn and that is where the flow is turning towards the shock.

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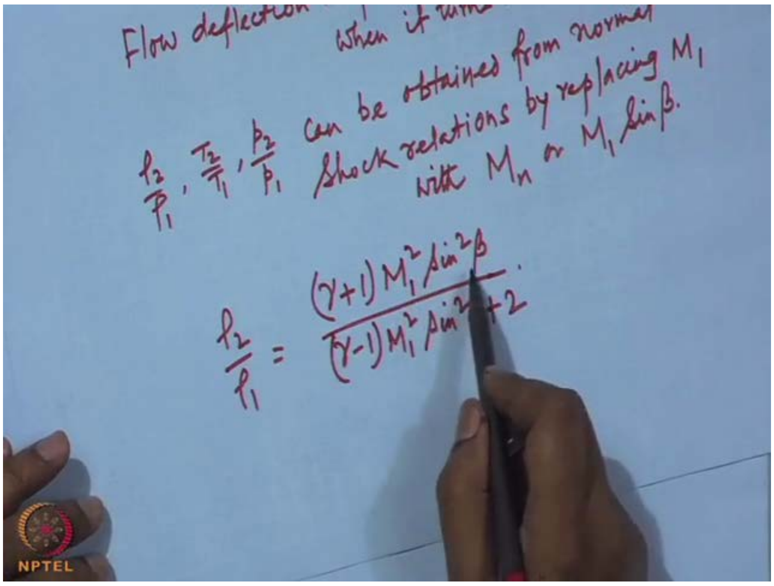

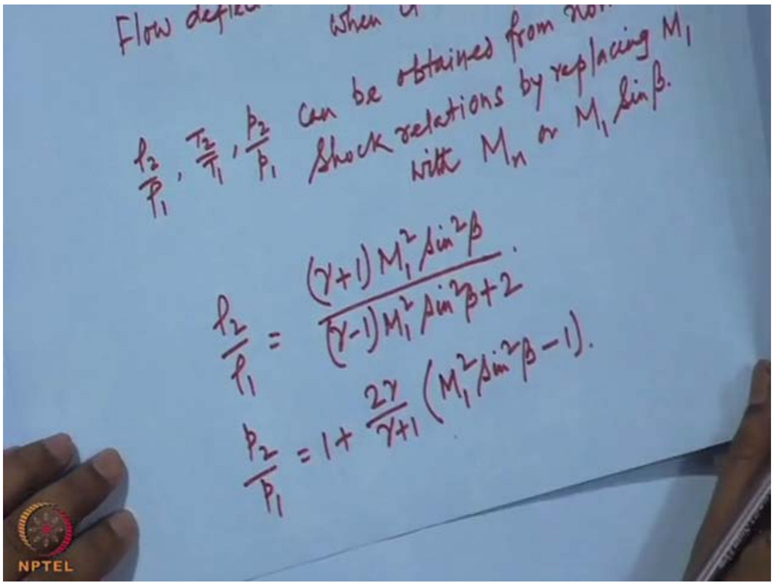

As it here, in this case, the deflection is positive, that is flow deflection is positive when the flow turns towards shock. Flow deflection is positive when it turns towards the shock. (No audio between: 12:41-13:09)

Now, if we compare this situation with a normal shock situation, what we see here is that with respect to this component of velocity u 1, the flow velocity, it is the shock is basically a normal shock. (Refer Slide Time: 13:22) So, we can treat this situation, as if a uniform component has been superimposed with the flow. Now, a superposition of a uniform flow velocity is not going to affect the static pressure and other static parameters like density and temperature. Hence, the relationship between static pressure, density and temperature, ahead and behind the shock, can be directly obtained from the normal shock; however, it must be remembered that the flow component that will come into picture is simply u 1 or the corresponding mach number that will come in the relation is M n that is M 1 sin beta.

Hence, we can get all the oblique shock relations particularly, just by simply changing M 1 to M n or M 1 sine beta. Consequently, that is rho 2 by rho 1, T 2 by T 1, p 2 by p 1 can be obtained from normal shock relation (no audio between: 15:00- 15:17) by replacing with (no audio between: 15:21- 15:37) M n or M 1 sin beta. We can now write rho 2 by rho 1 is equal to gamma plus 1, M 1 square will be changed to M 1 square sin square beta by M 1 square sin square beta plus 2. We can see clearly that these are the same relationship as was derived in case of normal shock, only that upstream mach number M 1 square is replaced by now upstream normal component of the mach number M 1 sin beta.

Similarly, pressure which is 1 plus two gamma by gamma plus 1 into that M 1 square again replaced by M 1 square sin square beta minus 1.

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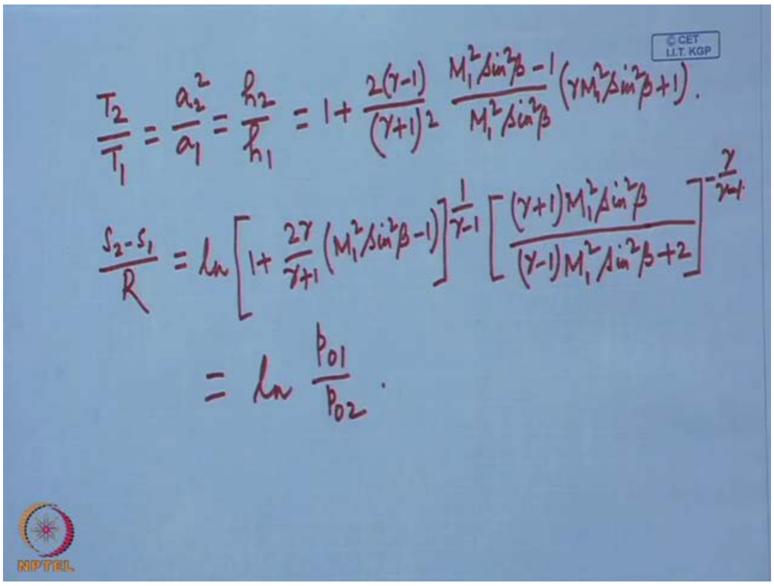

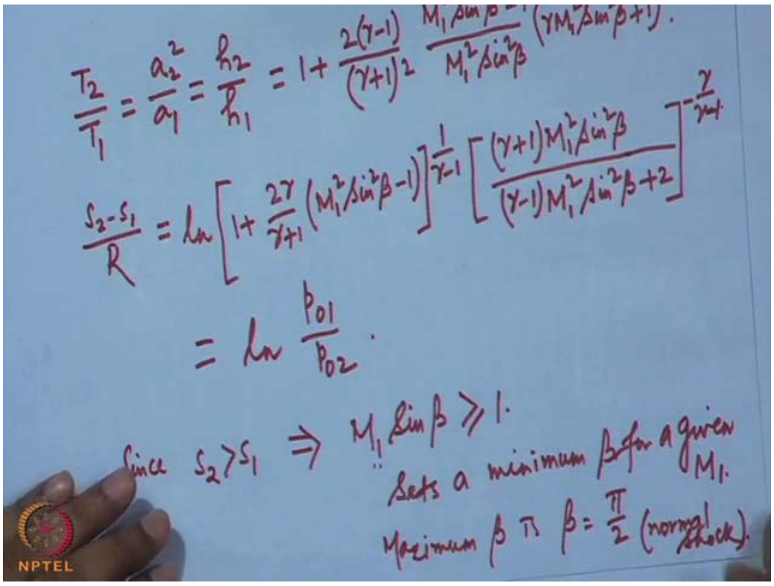

Similarly, the temperature ratio, which is also equal to the square of the ratio of the speed of sound and also the ratio of the static enthalpies is again one plus two into gamma minus one by (no audio between: 17:30-17:49) M 1 sin square beta. See, that in all these variations for a normal shock, the parameter M 1 is now replaced by M 1 sin beta. The entropy change is again given by log of 1 plus 2 gamma by gamma plus 1 into M 1 square sin square beta, minus 1 to the power 1 by gamma minus 1, into gamma plus 1, M 1 square sin square beta by gamma minus 1 into M 1 square sin square beta plus 2 to the power minus gamma by gamma minus 1.

This also gives the stagnation pressure and once again as in case of a normal shock, since this process is adiabatic, the total temperature remains the same behind and ahead of the shock. Once again, we see that the ratio depends only on the normal component of the velocity or normal component of the mach number and of course, on the gas itself. Once again that since s 2 must be greater than s 1; we find that this requires that M sin beta must be greater than 1. Now, since s 2 is greater than s 1 and this imply that M 1 sin beta has to be greater than or equal to 1.

Of course, this sets a limit or a minimum value for the wave inclination for a given free stream mach number M 1 or given upstream mach number M 1 there is minimum value of beta which is possible. The maximum is of course, when beta is pi by 2 or when the shock is normal. So, this sets a minimum beta for a given M 1. Maximum beta is that is a normal shock.

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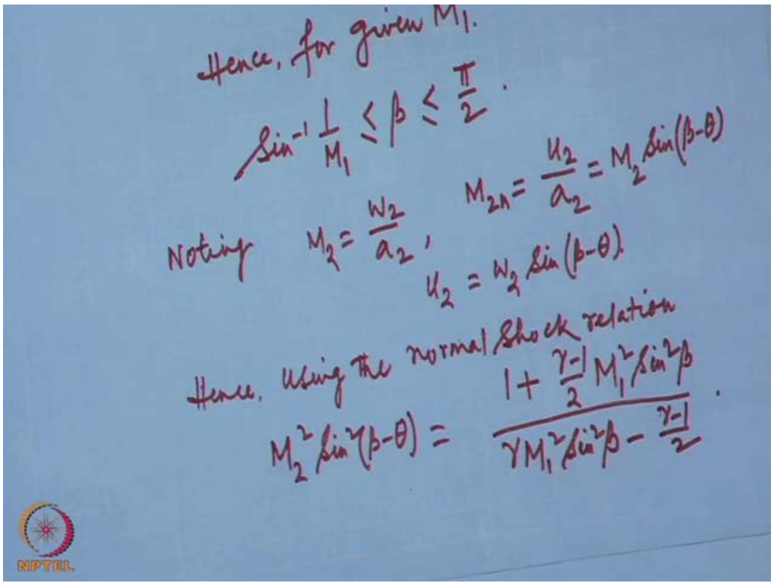

Hence, for a given upstream mach number M 1, the beta lies within the range of sin inverse one by M 1 less than beta or equal to beta less than equal to pi by two. For each wave angle beta, there is a corresponding deflecting angle theta. Now, to find the mach number downstream of the shock, we can obtain this by noting that M 2 equal to W 2 by a 2, M 2 n is u 2 by a 2 and u 2 is simply is M 2 sin beta minus theta and that is because u 2 equal to. (No audio between: 23:54-24:09).

Once again, writing the relationship between M 1 and M 2 for normal shock, and replacing upstream mach number M 1 by the normal component of M 1, which is M 1 sin beta and the downstream mach number M 2 by M 2 sin beta minus theta, we have M 2 square sin square beta minus theta is 1 plus gamma minus 1 by 2 M 1, here M 1 is replaced by M 1 square sin square beta, M 1 square sin square beta minus gamma minus 1 by 2. And this gives us all the necessary Rankine Hugoniot relationship for oblique shock.

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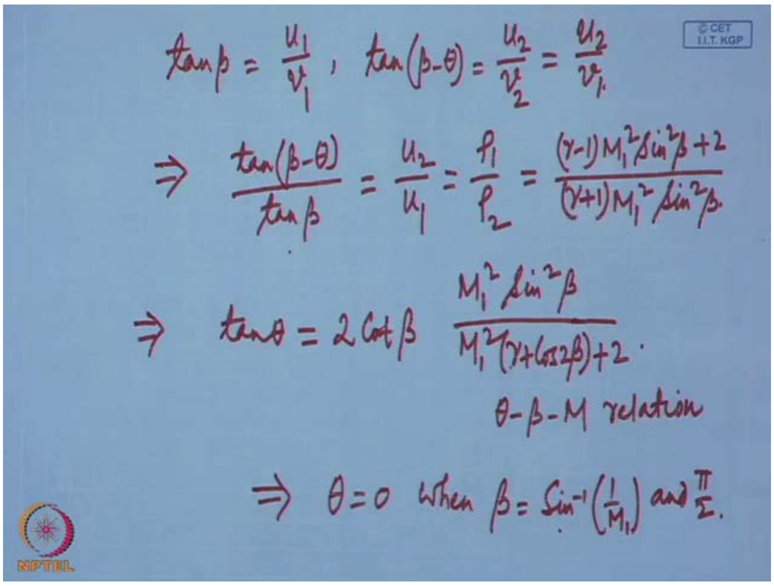

Let us see that we also have tan beta equal to u 1 by v 1 and tan of beta minus theta is u 2 by v 2 and v 2 is of course, v 1. So, this is u 2 by v 1 and consequently, we get tan beta minus theta by tan beta equal to u 2 by u 1 or equal to rho 1 by rho 2. This rho 1 by rho 2, we have already written as gamma minus 1 M 1 square sin square beta plus 2 by gamma plus 1 M 1 square sin square beta.

Now, using this trigonometric relationship tan beta minus theta by tan beta equal to gamma minus 1 M 1 square sin square beta, plus 2 by gamma plus 1 M 1 square sin square beta, can be manipulated, means the trigonometric relationship can be manipulated to show that tan theta equal to 2 cot beta M 1 square sin square beta by M 1 square into gamma plus cos 2 beta plus 2, which is a very well known and very widely used relationship and usually known as theta beta M relations. This is called the theta beta M relation and it expresses the flow deflection angle in terms of the wave angle and upstream mach number and of course, the gas property specific heat ratio. However, we see here that theta can be explicitly determined if M 1 and beta are known.

However, if M 1 and theta are known, beta can only be solved from the simplicity relationship. We also see that theta equal to zero and this shows that when theta equal to zero, when beta equal to sin inverse 1 by M 1 or and pi by two. That is at the lower limit of beta and the upper limit of beta, in both cases, the flow deflection is zero. Of course, we have seen for normal shock that there is no deflection in the flow. The flows retains its earlier direction and also see that beta is the lowest value that is sin inverse 1 by M 1, then also there is no deflection in the flow.

Now, this theta beta M relation also shows that within this range of beta, that is beta lying between sin inverse 1 by M 1 and pi by 2, the theta is positive, and since theta is zero at both, at the lowest value as well as the highest value of beta, so basically that theta must have a maximum within this range of beta. For each value of M 1, there is a maximum value of possible theta. It shows that if theta is less than theta max then for each value of theta and M there will be two corresponding value of beta or two corresponding solutions.

Now, out of these two solutions, the larger value of beta gives a strong shock and for the lower value of beta, the solution is called the weak shock, and usually for a strong shock the downstream flow is subsonic. However, for a weak shock, the flow is usually supersonic downstream; however, there is a very small range of value of theta, which is slightly smaller than theta max. Even for a weak shock solution, the downstream mach number can be subsonic.

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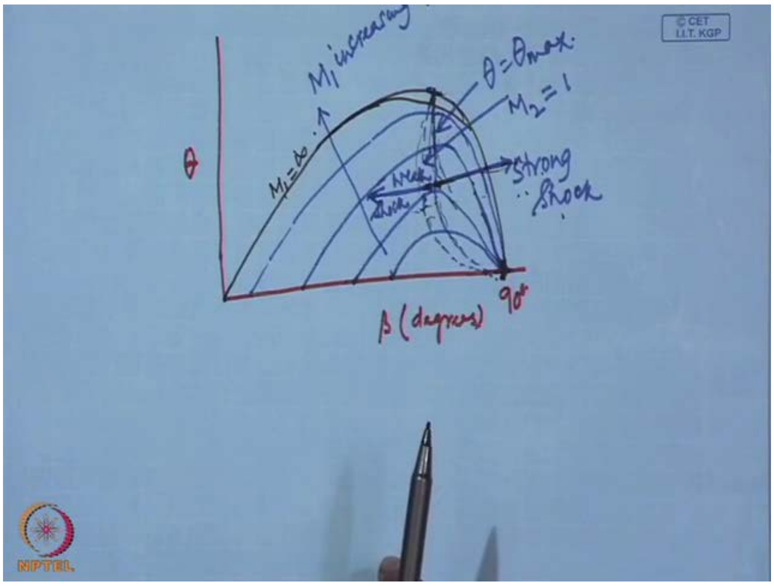

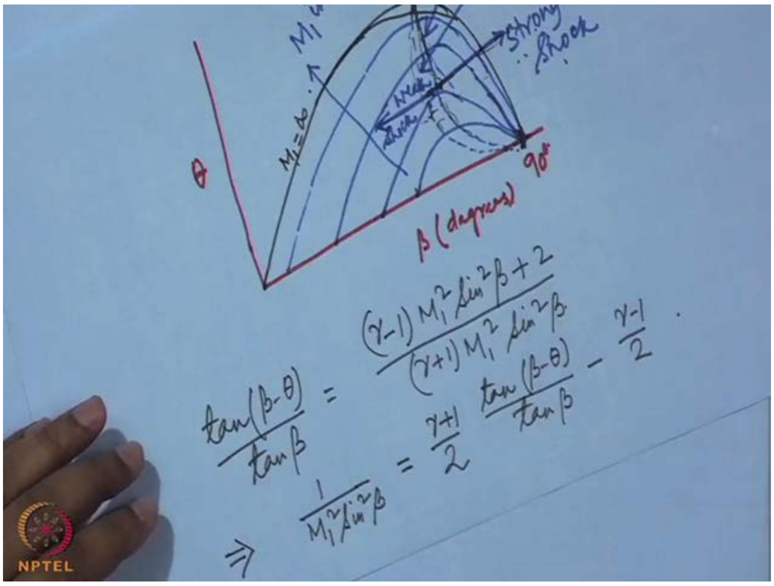

We can plot this M theta beta relationship, as (no audio between: 32:44 – 33:02) say in degrees and theta is also say in degrees. Let us say this is a ninety degree corresponding to the normal shock. For different values of M 1, we will have different curve. If we consider upstream mach number to be infinite like this. This is for M 1 equal to infinity, where the lower limit is zero. For all other values, there will be ((no audio 34:17 to 35:08)) and this is that increasing direction for… You see as far as the value of M 1 decreases the value of the lower limit of beta that is sin inverse one by M 1 increases.

Now, if we join all the theta max values for each of this mach number; let us say that this is what is the Sorry, say theta equal to theta max line and this is the line for M 2 equal to 1 ((no audio 36:17 to 36:55)) Now, if we look to this curve, you can see that higher curve for higher mach numbers envelopes the curves for the lower mach number. If we have the solutions corresponding to a value for which beta is higher of the two then that solution is called a strong shock solutions that is to the right of the line corresponding to theta equal to theta max.

Similarly, this is the line corresponding to downstream mach number one and that is for all these part of the values; the downstream mach number will be less than one. For all values to this side the downstream mach number will be more than one. That is the downstream flow remains supersonic if the solution corresponds to any point within this part of the curves. Similarly, for this part of the solutions the downstream mach number is subsonic.

What we see that is for all strong shocks, the downstream mach number is less than one. But, for weak shock for only those values where theta is slightly less than theta max. The weak shock can also have downstream flow as subsonic. So, this of very useful relationship and the curves are also very useful theta beta M curve and so the theta beta M relations. Now, going back to the theta beta relation, in this form, that is tan beta minus theta by tan beta, equal to gamma minus 1 M 1 square sin square beta plus 2 by gamma plus 1 into M 1 square sin square beta; this relation can be expressed also in the form by one by M 1 square sin square beta gamma plus 1 by 2 tan beta minus theta by tan beta minus gamma minus 1 by 2.

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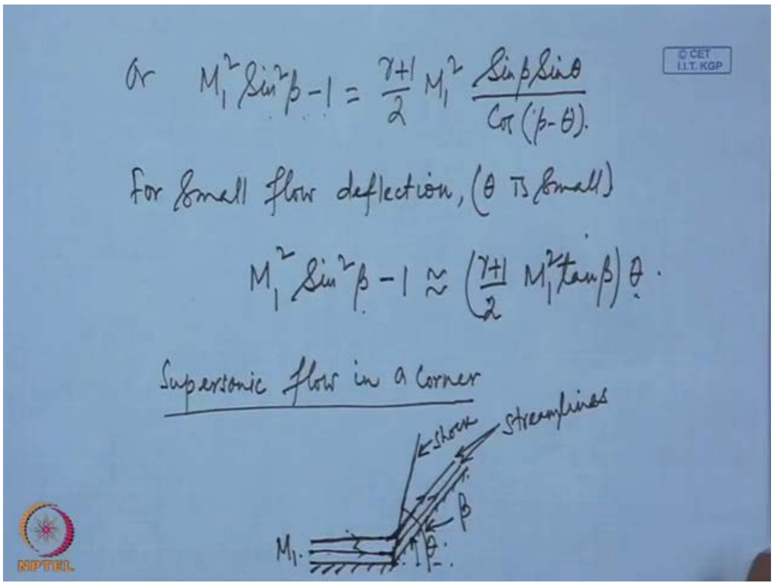

This can also be written as M 1 square sin square beta minus 1 equal to gamma plus 1 by 2 of M 1 square sin beta sin theta by cos beta minus theta. We should also note that the term M 1 square sin square beta minus 1, takes the role of M 1 square minus 1 in the normal shock relation. It can also be called as the strength of the oblique shock, as M 1 square minus 1 is strength of the normal shock. Similarly, M 1 square sin square beta minus 1 can be taken as the strength of the oblique shock. So, this is the way that oblique shock strength can be expressed.

Now, we can make us approximation that for small values of theta, we have for small flow deflection that is theta is small, we can approximate this relation M 1 square sin square beta minus 1, is approximately gamma plus 1 by 2 of M 1 square tan beta into theta. This is for small deflection angle the oblique shock strength is proportional to the flow deflection angle. Of course, the flow deflection angle is expressed in radian.

So, these are the basic relationship that are required for oblique shocks and with knowing about these oblique shock, we will now first try to construct our first example of supersonic flow that is supersonic flow over a wedge. Now, for this, we will use the basic concept of inviscid flow and that is any streamline in inviscid flow can be replaced by solid boundary. Incidentally and immediately, we see that then the oblique shock provides the solution of a supersonic flow or inviscid supersonic flow in a corner.

Let us say that is if we have a supersonic (No audio between: 44:20-44:36) flow in a corner. Let us say that this is what we have a (( )) and this is the corner. (No audio between: 44:46-45:03). We know that the inviscid flow boundary condition enforces that must be parallel to the solid wall. So, if a flow, which is parallel to this and this is a straight line of the flow, then here also it must be parallel to the body or parallel to the solid wall.

So, these are the streamlines (No audio between: 45:42-45:52), these are the streamlines (No audio between: 45:55-46:09) and a corner of angle theta turns to the flow, by an angle theta. What we have seen that earlier is that is an exactly an oblique shock does. It turns a flow by an angle theta for a given supersonic stream of mach number M 1, will be turned by angle theta, provided there is a oblique shock (No audio between: 46:37-46:48). So, this is the oblique shock of given angle beta.

Once we know this M 1 and theta; from the theta, beta, M relations or theta, beta, M curves, we can find out what is the wave angle beta. However, we should note once again that the theta beta M relation, though gives theta explicitly when M and beta are given; however, it does not give beta explicitly rather beta needs to be solved implicitly. A question of course comes that out of the two solutions, which solutions should be taken? That is also simple; if theta is less than the theta max, we should take lower value of beta and that is the weak shock solutions. To be precise, the nature usually prefers the weak shock solution.

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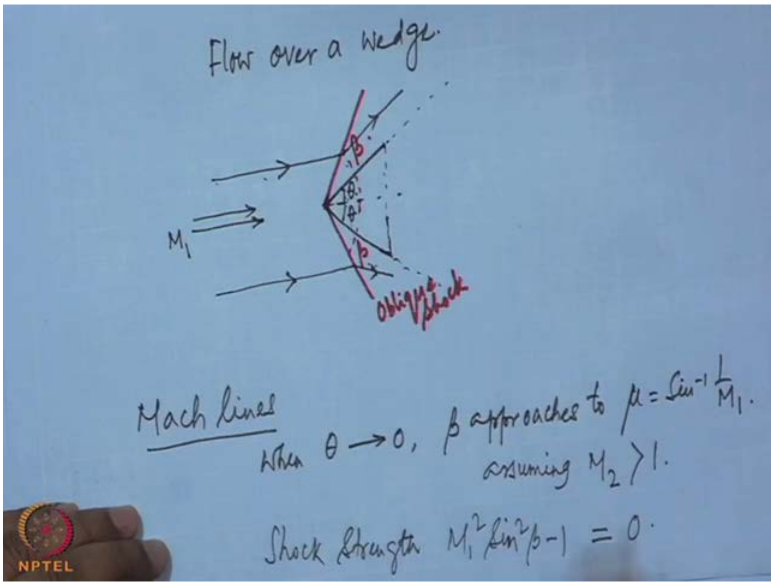

Now, this can easily be used to obtain the flow over a wedge. So, if we have a symmetric wedge and if we consider flow over a symmetric wedge, let us say flow over a wedge and of course, only supersonic flow. See, if the wedge is symmetric, ((no audio 48:43 to 49:13)) let us say that both are theta and the flow will be obtained by two oblique shocks at angle beta (No audio between: 49:33-49:58).

The flow on each side of the wedge is determined only by the inclination of the surface on that side. So, it is not essential that the wedge have to be symmetric. If these two angles are different that is this theta 1 and theta 2 are different, the two angles will be different. However, this part of the flow will be simply governed by this inclination and this side of the flow will be simply governed by this inclination. A parallel streamline here will remain parallel again (No audio between: 50:32-50:49). For the time being, we will not consider about this corner. What will happen in that corner? Later on, we may discuss about the turning at this corner, but for the time being we will consider that these wedge are infinite.

So, that a parallel stream simply turns by these wedge angle or wedge half angle and remains parallel to the wedge surface. (No audio between: 51:21-51:32) Now, we will come to one more situation. Let us say that the flow downstream remain supersonic; the wave angle beta decreases with decrease in the wedge angle theta. The wave angle beta decreases with decrease in the wedge angle theta. Now, when theta decreases to zero that beta decreases to the limiting value mu. So, we will consider what are called as mach lines. When theta approaches zero, beta approaches to mu equal to sin inverse one by M 1. Of course, assuming M two still remains greater than one. Now, in this situation shock strength M 1 square sin square beta minus 1 is zero and so are all the jump relations.

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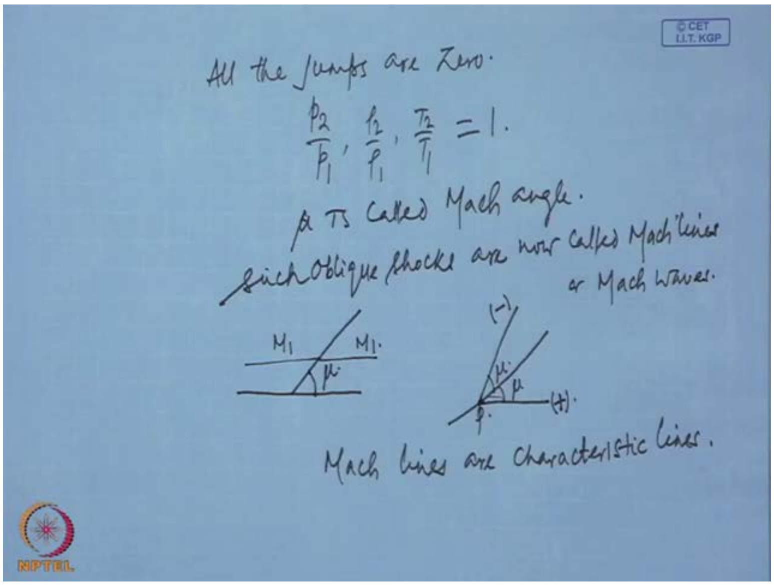

All the jumps are zero and that is p 2 by p 1, rho 2 by rho 1, T 2 by T 1; they are all equal to 1. In this situation, practically there is no discontinuity anywhere. The flow is continuous and we can say that there is nothing about, nothing unique about this particular oblique shock. That is the limiting condition of oblique shock; no discontinuity is created, there are no jumps in any of the flow quantities and the flow is continuous. Hence, there is no nothing unique about the point where this wave originates. We can take it any point in the flow, and this angle mu, then becomes simply a characteristics angle associated with upstream mach number M 1. This M 1 is called Sorry; this mu is called the mach angle. Here, mu is called mach angle. The oblique shock, in this case becomes or oblique shocks are now called mach lines or characteristics lines. Mach waves, they are nothing but the characteristic lines and considering any point on any particular streamline, we can have a mach line to this line as well as to this side. These are then called left running mach lines. (No audio between: 56:43-57:04)

Now, you see the flow is non uniform then the mach number changes throughout the flow and consequently the mach angle mu also changes and these mach lines become curved. So, at any point in a two dimensional flow, there are always two lines, which intersects a streamline at the angle mu. Similarly, in 3-D flow, the mach lines or the characteristics they define a conical surface with the vertex at p.

So, in a two dimensional supersonic flow is always associated with two families of mach lines. They are denoted by these lines plus or minus. The plus set runs to the right of the streamlines and called the right running characteristics and the minus set runs to the left of the streamline and called the left running characteristics. These are analogous to the characteristics that we have discussed earlier in our discussion in one dimensional wave. They are same characteristics line as in the x-T plane. Like those characteristics in the x T plane, these mach lines also have distinguish direction and that is the direction of the flow or the direction of increasing time. This is related to the fact that there is no upstream influence in supersonic flow.

So, to summarize, we have obtained the oblique shock relations from the normal shock relations themselves, just by replacing the normal component of mach number in place of mach number that is M 1 is in replaced by M 1 sin beta and M 2 is replaced by M two sin beta minus theta, where beta is the wave angle and theta is the flow deflection angle. We have seen that in an oblique shock, the wave angle is subjected to a fixed range, with minimum value being sin inverse 1 by M 1 the maximum value is of course, the normal shock angle pi by 2.

We have also seen the possibilities of weak solution and strong solutions and also we have seen that there is a maximum deflection possible for a given mach number. We have also seen that when the wave angle becomes sin inverse 1 by M 1 then the shock is practically of zero strength and there is no jump across the shock and each oblique shocks are then called simply the mach waves or the characteristics, which are analogous to the characteristics in one dimensional wave motion in x T plane.