### Wind Energy

### **Prof. Ashoke De**

### Department of Aerospace Engineering, IIT Kanpur

#### Lecture 07: Fluid Mechanics- Boundary Layer & Turbulence

Welcome back to this session where we'll continue our discussion on the external flows of the fluid mechanics. So, in this particular session, what we'll do, we'll look at the boundary layer. I mean, essentially the flow, external flows and the boundary layers. And once we discuss about those, then as I pointed out earlier we'll quickly touch upon the turbulence and the property of turbulence little bit before we wind up discussion or on feedback, so that then after that we'll move to the topic of wings and all these things. okay! now, well if you recall your dimensionless Navier-Stokes equation so that looks pretty much similar to that what is important to note here is that you end up getting this Reynolds number sitting here Okay. And, this is your momentum equation.

And, this is your continuity equation or mass conservation equation. So, obviously, I mean, one can solve these equations. Then you get a complete spectrum of the flow features, whether it's an internal flow or flow around external object. Whatever it is, you will get that.

# **Recall dimensionless N-S/Continuity Eqns.**

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \frac{1}{\text{Re}} \nabla^2 \mathbf{u} \qquad \nabla \cdot \mathbf{u} = 0$$

what is there in this equation, if we drop the viscous term? which is this term, then it becomes an kind of, it's an Euler equation, which is known as a potential flow equation. That means the flow doesn't have any viscous effect. Obviously, the potential flow solution predicts zero drag or lift for all objects, which is a phenomenon known as D'Alembert's paradox, because that is something not possible. That's why it is known as the paradox. Now, coming back to the flow over flat plate, So here is an infinitely thin flat plate.

### Consider flow over a flat plate



So you have this uniform flow. Okay. And then if you try to solve potential flow, what will happen? The flow over the plate would be flipping over, which is not as such possible. What is possible when the flow is passing over the flat plate, you can always have this kind of a boundary layer profile growth. And inside this small region, you will have viscosity-dominated situation.

And at the wall, there would be no slip condition to be satisfied. So that is where, obviously, when the flow passes over this flight, this region, these are always viscous-dominated. But beyond that region, when you come outside this, where the potential flow assumption may be there. the boundary layer what it is known as a very thin layer close to the solid surface where viscous effects are dominant. So, when the flow passes like this the uniform flow passes over a flat plate this is the growth of a this is known as boundary layer.

So, in this region we will have viscous effect which is going to be dominant and this idea was introduced by prandtl in and his co-workers, and this is a very, very important concept of finding fluid mechanics of the Navier-Stokes equation. This allows us to look at the flows close to solid body and allows us to characterize the behavior and things like that. As we said that the boundary layer is a thin layer, which is close to the solid

surface, where the viscous effects are dominant. So outside the boundary layer, flow limits largely inviscid and there. So this is a uniform flow.

Again, you can see the uniform flow, which is coming in. So this is where the boundary layer grows. And this region is viscous dominated, where at the outside, that one can have potential flow solutions. Obviously, this is the boundary layer thickness where Prandtl showed that the length is also quite less than the length along the streamline or along the flow direction. That we can see how that varies and things like that.



So, which is very common in the fundamental of the fluid mechanics and which has been taught in both undergraduate level and sometimes in the post graduate level, what you can see is the laminar boundary layer over a flat plate so if there is a flat plate we have a flow so the laminar bound so this is called the laminar boundary layer profile course and this is something probably close to turbulent boundary layer profile the difference is that You can see the boundary layer thickness. This case it is quite large, this case quite small and you see the velocity profile. There is a significant difference among that. Obviously, one case you have sharp gradient, other case less gradient. So once you have that kind of differences, it's going to lead to your skin friction coefficient and the stresses due to that.

Obviously, at a given x location if this is the direction the Rex leads to thinner boundary layer thickness so the boundary layer thickness at a thickness of delta where you can the u would be 99 percent of the free stream. So, the boundary layer thickness this thickness is called delta. So, the thickness is defined such that the just at the edge of the boundary layer the velocity is going to be almost 99 percent of the free stream velocity. So, if we continue our discussion as we have already seen so we can see that the boundary layer over a flat plate how this grows so you have a uniform flow coming in then this is the point over which the flow profile grows and along this is the direction x and this is the

delta x because here the boundary layer thickness is different than here the boundary thickness so delta is also a function of x because at different location obviously That depends on other parameters like Reynolds number and such things. So what boundary layer does, it just shifts the streamlines upward.



## Boundary Layer over a flat plate



So this is what the streamlines is supposed to be. So it shifts the streamline upward. And there is a displacement thickness, which is called delta star. So, one can find 0 to infinity, 1 minus u/U dy. So, from there, you can find out that thickness.

now, this flat plate boundary layer where we use the free stream velocity so h is the initial height so, it is quite larger than delta then what we can find out that at x equal to zero the so it's essentially doing the mass balancing so at any particular locations we have h plus d so h plus d if i do this is what i get and that's what i get the calculation for displacement thickness. So, that's i mean it's assuming a control volume like this and that control volume one can do the mass balance and if you do the mass balance this is what you get because conservation of mass or continuity equation is something is very very fundamental.



Now, continuing with that discussion, so if you have this flat plate and the boundary layer profile grows here, then I can find out obviously what it does is that boundary layer actually causes some momentum deficit unlike potential flow, so I can find out how much is there. So, once you do that, you can find out the momentum deficit also. These are some mathematics involved, but essentially you can have a control volume and just doing the integration would get you that.



So, you can go through this calculation. These are very simple calculations. So, you need to draw a control volume and then get the integration done at two different phases. So, once you do this algebra, you finally get this. So, you can also check in the textbooks and all these things.

So one can find out the momentum thickness which will be u by u into 1 minus 1 by u

dy. So, the momentum thickness is nothing it measures the momentum deficit in terms of length. Obviously, it can also give you some kind of an what you can do you can actually get the shear states and all these things. So, boundary layer approximations for delta which is very very small less than the small x which simplifies the Navier-Stokes equation leading to the several exact solutions. And it's quite common or known is that the boundary layer solutions or laminar boundary layer flow over a flat plate which gets you the Blasius solution.

So these are the solutions from the CS profile. Boundary layer approximation is applicable for high Reynolds number flow only. Additional restriction will definitely follow. Now, coming back to that, some of the important findings of laminar boundary layer. So, you have a uniform flow coming in going over the boundary layer there is a boundary layer growth so you can find out delta by x so which will give you the at a different location of x the boundary layer thickness or with the delta the displacement thickness delta star momentum thickness delta double star.

# Important results for laminar boundary layer flow over a flat plate



All these are again, these are laminar boundary layers so these correlations are well established so one can okay so essentially it's a self-similar velocity profile. Similarly, you can find out this delta double star and from there you can find out the drag force you

can estimate the shear state at one so these are simple system or simple equations of the finding for laminar boundary layer flow over a flat plate which you can just get it done through some integration and equating the forces with the other things so if i put those correlations of the finding together so my one shear stress is going to be rho u squared d dash star by dx I have skin friction coefficient which can be estimated like that, I have drag coefficient which I can drag force, then I have drag coefficient which I can estimate like that, then combining I can get something like that.



So, all these are some of the findings for laminar boundary layer profile over a flat plate which one can estimate. If I recall and bring things back here, my boundary layer thickness is the thickness over a solid surface on the flat plate where the velocity profile becomes 0.



99 of the infinity. Then I have the displacement thickness, which I can find out by doing integration at any point of time. I have this velocity profile, then I can integrate that. Momentum thickness I can find similar like that. And what I have, I have, there is no pressure gradient, so del P del X zero, and also there is normal pressure gradient is zero. So, that means these are the boundary layer growth or profile under zero pressure gradient.



So, I have u by e equals to f y by delta, which is a self-similar profile. I can estimate the drag force, and also I can find out the shear state at the wall, drag coefficient, skin friction coefficients and all these things. So, what i can find out is the approximate solution of this. Okay! So, i have this shear stress which can correlate it to u square d delta and my u star is then eta then momentum thickness i can write u star one minus u star one this then delta then i can use shear stress which is mu del u del y so if this is the solid surface so at shear stress it's a velocity gradient at y into mu, if i put that back i get this so then i can estimate the wall shear stress using this. So, these are two things if i equate and write down this then i can calculate recall so i get del y x equals to So, you remember del double star equals to C1 delta, then you can estimate that.

So, you can evaluate the shear stress in this fashion. So, these are some of the calculation

that one can do for this. So, what you can find out, that means when there is a boundary layer growth over solid surface, if you know the velocity profile, then you can find out everything. So, simplest velocity profile let's say u by u infinity is y by delta or if eta is a plus b eta then at y 0 which is at the solid surface obviously the velocity is 0 and y delta it will become the free stream velocity this is the free stream velocity. So, you find out a and b then you can find out this velocity profile so once you this use this velocity profile So, that means as long as there is a boundary layer, so up to delta, there is a y by delta and beyond that u equals to u.

$$c_{2}\mu \frac{U}{\delta} = \rho U^{2}c_{1}\frac{d\delta}{dx} \qquad \delta(x=0) = 0$$

$$\Rightarrow \frac{c_{2}\mu U}{\rho U^{2}c_{1}}dx = \delta d\delta \qquad \Rightarrow \delta^{2} = \frac{2c_{2}}{c_{1}}\frac{\mu x}{\rho U} + 2c_{3}^{2} = 0$$

$$\delta^{2} = \frac{2c_{2}}{c_{1}}\frac{\mu x}{\rho U} = \frac{2c_{2}}{c_{1}}\frac{\mu}{\rho Ux}x^{2} = \frac{2c_{2}}{c_{1}}\frac{x^{2}}{\operatorname{Re}_{x}} \Rightarrow \frac{\delta^{2}}{x^{2}} = \frac{2c_{2}}{c_{1}}\frac{1}{\operatorname{Re}_{x}}$$

$$\frac{\delta}{x} = \sqrt{\frac{2c_{2}}{c_{1}}}\operatorname{Re}_{x}^{\frac{1}{2}} \qquad \operatorname{Recall} \quad \delta^{**} = c_{1}\delta \qquad \frac{\delta^{**}}{x} = \sqrt{2c_{1}c_{2}}\operatorname{Re}_{x}^{\frac{1}{2}}$$
Evaluate
$$\tau_{w} = \rho U^{2}\frac{d\delta^{**}}{dx} \qquad F_{D} = \rho b U^{2}\delta^{**}$$

This is your capital U. Then you can estimate C1, you can estimate C2, then I can put del yx, I can put del star yx. So what I get this ratio is 3.46 exact solution 4.91 here. So, you can estimate all these things.



So, there are some calculations which one can always refer to. So, these are exact solutions which are compared to the approximate solution. So, the Blasius solution approximation is 5. there is an Cf into rx and then this is drag so if you have a linear profile then then approximated like this if you have a parabolic profile it approximates very closely, if you have cubic profile this, so the base solution or the blessure solution kind of considered to be an exact solution of navier-stokes equation once you apply the boundary layer approximation So, what's going to happen is that when this boundary layer profile grows now the flow is growing so if you see there is a uniform flow coming in so that means now the flow is transiting from laminar to turbulent how that happens if you see this so there would be stage where you have laminar boundary layer.



Okay! and then you have a transition layer and finally turbulent layer so obviously this region this is the viscous dominated region and outside that obviously so empirical turbulent flow profile could be some kind of a power law profile and you can estimate boundary layer thickness, displacement thickness, momentum thickness, local skin friction, everything you can estimate so just like in laminar case where we have the blessure solution to estimate all these things in this case also one can estimate all of this very easily and this is the drag coefficient for flow over a flat plate with REl that means if this is my plate along this the this is the x direction so along this at distance L, I have this different CD data and using this CD data one can estimate the, so, that is common flow which is known as cylinder flow over cylinder or sphere But boundary layer theory can be expanded or can be extended to any geometries including cylinder sphere.

What it does, the boundary layer is that, let us see if I have a flow like this, so when the flow comes, the flow remains attached and then it starts separating, so this is the region. the boundary layer theory actually predicts the onset of flow separation and then transition leap to drag coefficient or forces but yes as soon as the flow separates your boundary layer assumption fails then that time you need exact solution due to the specifics so here you can see a simple example of flow separation there is a cylinder and,

you can rightly see that the how this boundary layer separates then you have a separation zone where you get separated flow and this case so obviously this is with increasing Reynolds number okay so with a Reynolds number is very low flow remains kind of an attach that means the flow comes and around the cylinder this is along a stream line it goes, so that there is no separation. But when you increase the Reynolds number to 50 something, the flow separates at certain point and then you get this recirculation of separation law. And when you increase further, then you get this kind of weak region for these things. So, boundary layer separates from the wall when it was generated.

So, concept of boundary layer still exists. Approximate equation fails. So, pressure gradient plays a significant role. Geometry is more important than Reynolds number. So, the pressure gradient is essentially we talked about from drag which increases necessary condition adverse pressure gradient.

That means when the flow separates like this in this region. So, this is a low pressure zone and this is high. Okay. But this separation occurs because of this adverse pressure gradient. So that means dp by dx is greater than 0 along this direction.

But once separated, this is it. So viscous region thickens. Basic assumption of boundary layer appropriation becomes big question. Now, if you look at an example like an flow through this kind of a convergent divergent section so here is a nozzle this is a diffuser so when the flow comes in you can see how the boundary layer grows and obviously it going through the contraction so velocity increases then it comes to a kind of a nozzle and then it starts again i mean here you will start seeing the separation because now you are increasing the area so the nozzle there is a area decreasing say decreasing pressure so the velocity will increase in so this is a favorable gradient in the throat that is pretty much constant pressure and area velocity is constant zero gradient but diffuser area is decreasing pressure is increasing so velocity decreases and it becomes the advanced system gradient and then boundary layer thickens similarly one can see the flow over a cylinder, flow past a cylinder. So, this is the cylinder surface. So, the flow accelerates and then decelerates.

Obviously, it goes through this favorable to adverse pressure gradient. Now, for viscous flow, adverse pressure gradient overcomes the momentum near the wall, generates inflation point and inorganicity profile. So, this is a region where you see adverse pressure gradient. This is favorable pressure gradient.

So, this side, there will be flow separation. So, you will get to see this kind of a separated zone. And this picture is very well-known picture, which is available in any textbook,

internet, wherever you look at. I mean, very low Reynolds number, flow remains attached. You increase certain number, you start seeing this kind of separated flow. At certain Reynolds number, you can get in this kind of vortex heading, which is periodic in nature.

Increase Reynolds number, turbulent wake. More Reynolds number, more turbulent wake. So, there is a drag versus Reynolds number plot, and you get to see the no separation, steady separation, vortex heading, boundary layer, turbulent boundary layer, and there is a drag. Similarly, one can correlate with the sports ball aerodynamics where there are dimples. This is a cricket ball. So, cricket ball, golf ball, these are obviously there are flow separation happens and that kind of makes the ball to swing for a bowler to do things.

So, these are all Example of so same thing happens in tennis and soccer ball as well. So, there are different surfaces where boundary layer separates and their separations actually going to play a role. Obviously, when you talk about this boundary layer separation, whether it is desirable or not, but most of the time it is not desirable. Because once the boundary layer separates, it's leading to higher drag. Some birds make use of boundary layer separation for long distance gliding.

I mean just some albatross can fly very long distance with near zero energy expenditure. Okay. That kind of, give you some sort of an idea about this external flow the boundary layer and all these things. So, what now we're going to touch upon quickly the turbulence and their properties and characteristics so few questions that we would like to answer what is turbulence how do the turbulence flow behave how can it be quantitatively described and what are the physical processes. So this is a picture of a flow around this uh here where the fiction diagram so this is a fiction coefficient across Reynolds number you can see how it varies so it's again different Reynolds number Reynolds number very low Reynolds number at high and then there is a sudden drop in the friction factor which is known as the drag force or even drag crisis and obviously the drag force one can estimate like this.

Similarly, you can see this is a Reynolds number 15,000 this is 13,000 But interestingly, that means this reynold number to this reynold number, your inertia has gone up. So, you have a higher level of turbulence. So, turbulence is actually reducing the overall drag. Because behind this bluff body, if you have a less separated zone, this is a highly separated zone.

So, this will have higher drag. So, that means one of the advantages of the turbulence is that it can reduce drag. Some other example of turbulent flows, this you can see in the volcano. This is the Ash flume at a very height. So, these flows are happening in nature and they are highly, highly turbulent.

There is another, this is a smoke-formed cigarette. This is downwards of the aircraft when it is flying. So, you can see these are, I mean, these things one can, see in nature. What is important of the turbulent flow is that scale similarity. So, that means small scale structure for different Reynolds number.

So, this is Reynolds number 4300. This is 10 to the power 7. If I zoom in the views, you can see this meter is pretty similar. So, that's the important characteristics of the scale similarity. That means the Reynolds number changes, the intensity changes. This is where transition to turbulence happens.

So, this is a top view of an experimental picture. There is a nice uniform flow, goes through the boundary layer growth. So, it goes through transition, then transition becomes completely turbulent. This is a Reynolds number.

I mean, it's a J. You can see it's a 1,520,000. So, increase in Reynolds number, range of scale with increase in Reynolds number. That means you see the structure here and you see the structure here. So, once you increase more and more, this kind of, I mean, the NSA effect or the Reynolds number effect, you get to see. So, this is to characterize at a particular point what turbulent flow behavior looks like.

So, it is more like an instantaneous randomness in the flow field. okay! So, that is how so if one has to define that turbulence flow field, so, obviously it's a fluid motion where velocity pressure and other flow quantities are fluctuating or fluctuate irregularly with time and space so the turbulent fluid motion is an irregular condition of fluid in which various quantities show randomness so essentially it's a randomness is the system which so turbulence is due to formation of point or line vertices on some component of the velocity becomes infinite, so, this is generally who quoted that so, if we put them together in slightly different way so the turbulent flow these are essentially unsteady and very irregular chaotic motion which is transported quantities like mass momentum so one can identify swelling patterns enhance mixing So, here the fluid properties exhibit random variations. Their statistical averaging results get to some measurements. Characteristics allow for turbulence modeling. Then wide range of turbulence scales or the ADs. Actually, what happens in turbulence is there is an energy cascading that takes place.

So, that means primarily the large scale structure are very much geometry dependent and then the breaks down to the smaller structure and then finally it comes to a small structure where it gets away through the viscous dissipation. So, this is where the energy cascading. So, this is called energy cascading so, that means large scale structure to small scale medium scale to small scale, small scale that's how that happens the properties of this turbulent flow if i have to talk about they are unsteady and three-dimensional in nature essentially there are irregular and chaotic means categorized by vortices structures or areas or things like that there are different length scale there are large scale structure which are known as macro structure and they are most of them are dependent on the geometry there are small scale structure which are known in the microstructure these are essentially energy dissipation universal nature turbulence are often characterized at the high Reynolds number so the inertia effect is very dominant They are dissipative in nature, that means there is a rapid loss of energy. Just now we talked about the energy cascading process.

It allows the effective mixing, enhances heat and mass transfer. So, how one can look at the turbulent flow, whether the flow is turbulent or not. So, you can look at the nondimensional number like Reynolds number and try to identify based on whether the internal flow or natural convection or external flow you have to characterize based on the Reynolds number. If it is beyond certain limit, then the flow is considered to be turbulent. Obviously, then it shows the property of the physical property of the turbulent Reynolds. So, these are very, very idea about the turbulent flows and things like that.

Because this would be required when we talk about the wind turbines and all these things. but, any other details of this flow field and things like that you can actually refer to the textbooks and read, okay! so that much i mean that's pretty much would like to cover or face your basic idea about fluid mechanics turbulent flows and all these things now we'll move to them slowly the discussion on the turbines technologies and all this.