Wind Energy

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Lecture 57: Mechanics: General Solution of Flapping Equation

Well, welcome back. So, we'll continue the discussion on this simple solution, this full flapping equation. We have looked at two situations, one case which is only rotation here and which leads to coning. And, then we have included hinge spring and the offset. now, what we would like to include the gravity term so as soon as we include the gravity so, this actually complicates the solution okay! so, obviously like we have done in the other cases we assume no yaw, no crosswind, no wind shear, okay! then the solution matrix that becomes K, B, 0, 2B, K minus 1, gamma by 8, 0, minus gamma by 8, K minus 1, beta naught, beta 1 c, beta 1 s, gamma by 2 A, 0, 0 okay! I mean, obviously, we can use Cramer's rule. So, using typical Cramer's rule, we can find out the determinant.

(iii) Rotation + Hinge-spring + Offset + Gravity → complicates the solution.

(No yaw, No crosswind, No wind shear)

(*) becomes
$$\begin{bmatrix} K & B & 0 \\ 2B & K-1 & \gamma/8 \\ 0 & -\frac{\gamma}{8} & K-1 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_{1c} \\ \beta_{1s} \end{bmatrix} = \begin{bmatrix} \frac{\gamma}{2}A \\ 0 \\ 0 \end{bmatrix}$$

using Cramer's rule, $D\,(\text{determinant}) = K \begin{vmatrix} K-1 & \frac{\gamma}{8} \\ -\frac{\gamma}{8} & K-1 \end{vmatrix} - B \begin{vmatrix} 2B & \gamma/8 \\ 0 & K-1 \end{vmatrix}$

Okay. So, that would be K, K minus 1. gamma by 8, K minus one, minus B which is 2B, gamma by 8, so, what it gives the determinant is K, K minus 1 square plus gamma by 8 square, minus 2 B squared, k minus one. So obviously, i mean, once you find the determinant then like in camera school one can find the desired values by substituting the right hand side vector into the corresponding column of the matrix and then finding the new matrix and dividing by the original determinant so, what will happen now the first term that is beta naught will have now sine and cosine of algebra so the solution that one can get gamma A by 2 D, K minus 1 square plus gamma by 8 square, okay! so, this is what you get now similarly you have beta 1c which is minus B A by D, gamma K minus 1 and beta 1s will be minus B A by D gamma square by 8.

So, here, since the cosine and sine terms are negative the rotor disc is tilted downwind and to the left. So, now this magnitude of the sine and cosine terms can be related by cyclic sharing. So, the cyclic sharing would give beta 1s is gamma by 8k minus 1, beta 1c. Som this indicates the relative amount of movement integrates the relative amount of movement backwards compared to sideways. So, for a rotor with independent freely hinged blade, rotor with independent freely hinge blade means he is approaching towards one.

$$D = K\left[(K-1)^2 + \left(\gamma/8\right)^2\right] - 2B^2(K-1)$$

Sol. $\beta_0 = \dfrac{\gamma A}{2D}\left[(K-1)^2 + \left(\dfrac{\gamma}{8}\right)^2\right]$, $\beta_{1c} = -B\dfrac{A}{D}\gamma(K-1)$

$\beta_{1s} = -B\dfrac{A}{D}\left(\dfrac{\gamma^2}{8}\right)$

- Since the cosine & sine terms are negative, the rotor disc is tilted downwind and to the left.

'cyclic sharing' $\Rightarrow$ $\boxed{\beta_{1s} = \dfrac{\gamma}{8(K-1)}\beta_{1c}}$ $\longrightarrow$ indicates the relative amount of movement backwards compared to sideways.

- For a rotor with independent freely hinged blade ($K \to 1$), there will be mostly yawing.
  - For a stiffed machine, there will be mostly tilting.

So, there will be mostly join. Okay. Similarly for a skipped machine there would be mostly tilting this can also be considered in terms of phase lag also if you recall the gravity is a cosine input for flapping for a steep blade the response is mostly a cosine response so, there will be little phase lag for a theta rotor the response to a cosine input is only a function of the sign of the azimuth angle that means response has a 90 degree phase lag from the disturbance okay so these are the outcome that you would have on now we will consider wind shear plus hinge spring. So, this again here one can ignore gravity, yaw, cross flow terms etc. so, that matrix becomes K, 0, 0, 0, K minus 1, gamma by 8, 0, minus gamma by 8, K minus 1, beta 0.

So, the determinant of the coefficient matrix from here, what we get is K into K minus one square plus gamma square by 8. Again, once you apply Cramers rules, you can find out the solution. So, beta naught would be gamma by 2 A by K, beta 1c is minus one by D, gamma by 8, K nu s, K, U bar into k minus 1, and beta 1s is minus 1 by D, gamma square by 8, K nu s, U bar, K. So, what you have essentially the winds here is a cosine input.

Okay. If you recall winds here is a cosine input. In response a steep rotor will have both cosine and sine responses. So, in response A steep rotor will have both cosine and sine responses. A heated rotor with K equals to 1 will only have sine response. So obviously, if you see this, that means when you consider the wind shear, which is essentially a cosine input, it allows the turbine to respond to a different way.

(iv) Wind shear + Hinge-Spring (ignore gravity, yaw, crossflow terms)

(*) becomes

$$\begin{bmatrix} K & 0 & 0 \\ 0 & K-1 & \gamma/8 \\ 0 & -\gamma/8 & K-1 \end{bmatrix} \begin{Bmatrix} \beta_0 \\ \beta_{1c} \\ \beta_{1s} \end{Bmatrix} = \begin{bmatrix} \gamma/2\, A \\ -\gamma/8\, K_{vs} \bar{U} \\ 0 \end{bmatrix} \Rightarrow D = K\left[(K-1)^2 + \left(\frac{\gamma^2}{8}\right)\right]$$

$$\beta_0 = \frac{\gamma}{2}\frac{A}{K}, \quad \beta_{1c} = -\frac{1}{D}\left[\frac{\gamma}{8} K_{vs} \bar{U} K(K-1)\right], \quad \beta_{1s} = -\frac{1}{D}\left[\frac{\gamma^2}{8} K_{vs} \bar{U} K\right]$$

So, stiff rotor will have both cosine sine response. Detailed rotor will only sine response but wherever the turbine response would be both sine and cosine cyclic responses. So, the turbine essentially the turbine will have both cyclic responses there. So, that would respond to both the cyclic patterns and all these things okay! now, if we try to get a general solution or write down the general solution of flapping equation of motion so, simply, i mean general solution one can find out just by using cramera's rule to the matrix and then the but that would not be very illuminating so one approach that helps to add some clarity is to explain the terms in the flapping angle as some of other constants So, which contributes, I mean, which represent the contribution of the various forcing effect. So, one can say beta 1C, beta 1C crosswind, beta 1C yaw rate, beta 1C nu s, beta 1S would be, beta 1S ahg, beta 1S crosswind, beta 1S yaw rate nu s, here ahg is subscript for axial flow and hinge spring gravity that is blade width shear for crosswind wire for yaw rate nu is for vertical wind shear okay So, this is what one can represent the flapping and the sum of other constant.

I mean, but one has to understand that each of these constants are also function of other parameters. So, we can have I mean, instead of rather expanding the matrix solution by itself, we can present the various subscripted terms which could be obtained from such a solution. The dominant owning term includes axial flow, hinge spring and gravity. which includes axial flow in spring and gravity that can be said that beta naught 1 by determinant gamma A by 2 plus gamma y A square. So now, if we put these things in a table, so like we have kind of an, So, this is ahg, cr, nu s, yr, you have cosine which is beta one c star, you have sine beta one star, So, ahg is 1 by d gamma, BA K minus 1, V naught by D gamma by 8 square gamma by 2 into 4 by 3.

A minus gamma. A3 by 2. K into K minus 1. Then this is.

Minus Ksh. U bar. By D. Gamma by 8. K into K minus 1.

General Solution of flapping eq. of Motion

by using Cramer's rule to (*).

$$\beta_{1c} = \beta_{1c,ahg} + \beta_{1c,cr} + \beta_{1c,yr} + \beta_{1c,vs}$$

$$\beta_{1s} = \beta_{1s,ahg} + \beta_{1s,cr} + \beta_{1s,yr} + \beta_{1s,vs}$$

'ahg' → axial flow, hinge-spring, gravity

'cr' → crosswind

'yr' → yaw rate

'vs' → vertical wind shear

Dominant Coning term. (includes axial flow, hinge-spring, gravity)

$$\beta_0 = \frac{1}{D}\frac{\gamma A}{2}\left[(K-1)^2 + \left(\frac{\gamma}{8}\right)^2\right]$$

contributions to flapping responses

*	Cosine, β_{1c*}	Sine, β_{1s*}
ahg	$-\frac{1}{D}\gamma BA(K-1)$	$-\frac{1}{D}\frac{\gamma}{8}BA$
cr	$\frac{\bar{V}_0}{D}\left[\left(\frac{\gamma}{8}\right)^2\left(\frac{\gamma}{2}\right)\frac{4}{3}A - \frac{\gamma A_3}{2}K(K-1)\right]$	$-\frac{4\bar{V}_0}{D}\left(\frac{\gamma}{8}\right)^2\left[\frac{4}{3}A(K-1) + A_3K\right]$
vs	$-\frac{K_{sh}\bar{U}}{D}\frac{\gamma}{8}K(K-1)$	$-\frac{K_{sh}\bar{U}}{D}\left(\frac{\gamma}{8}\right)^2 K$
yr	$\frac{K\bar{q}}{D}\left[\left(\frac{\gamma}{8}\right)^2 - 2(K-1) - \frac{\gamma A_3}{2}(K-1)\bar{d}_{yaw}\right]$	$-\frac{K\bar{q}}{D}\left(\frac{\gamma}{8}\right)\left[\frac{\gamma}{2}A_3\bar{d}_{yaw} + K + 1\right]$

This would be. K, q bar by D, gamma by 8 square, minus 2 K minus 1, gamma A3 by 2 into k minus 1 d bar yaw. So, these are cosine terms. Similarly, sine terms is one by D, gamma by eight, BA, 4 V naught by D, gamma by eight square, 4 by 3 A into K minus one plus A 3 K, minus K sh U bar by D square into K, K q bar by d, gamma by 8, gamma by 2, A 3 d yaw plus K plus 1, so, these are contribution to Flapping responses. OK. So, these are all cosine and sine terms, which are kind of summarized.

So, what you can see that once we talk about the general solution, then I mean, one can do that using that matrix by finding the determinant can find the complete solution, but it would be good to express this flapping angle in terms of different contribution. And these are the cosine and sin. So, this is something quite interesting too. But one important point to note here is that about the various subscript constant is that they can help to illustrate the amount of response that is due to a particular input. For example, in a particular situation, if the wind shear wind shear sine cyclic response were close in magnitude to the total sine cyclic response constant, that it would be immediately apparent from this system the other factors were of little significance so this table kind of suggests you to identify the dominance of the different inputs and their responses to that okay so this is how you can have a solution to this different, terms and components of this now along with this once we find out the complete solution so, this is the complete solution of the flapping motion so, once we find out the solution to this complete system, then it is good to also look at the loads like blade and hub loads for this.

So, that one can determine by estimating the forces on the blades and the root and the hub. So, good to know the blade and hub loads. So, moments and forces on the hub and tower can be determined. So, the moments and forces on tower and hub can be determined okay! so, for a rigid rotor turbine with cantilevered and not tethered blades both flapping and lead lag moments are transmitted to her so for a rigid rotor turbine with cantilevered and not teetered blades. Both flapping and lead lag moments are transmitted to hub.

Flapping is usually the predominant aerodynamic load and then so in teetered rotor on the other hand no flapping moments are transmitted to the hub. unless the teeter stops our heat. Okay. So, what we can do that if you recall the blade flapping angle which was approximated as beta 1 plus beta 1 c cos psi plus beta 1 s sin psi. So the corresponding, corresponding blade route bending moment for each blade can be estimated as m beta is k beta into beta.

So this is what one can estimate. So, I mean, so if you look at the hub reactions, hub reactions, and then you have retard rotor, and then you have cantilevered. Then you have flapping moment flapping shear lead lag moment lead lag shear blade tension blade torsion so this case it would be none this case would be total thrust on hinge this case it would be power torque then this case would be force producing torque this case centrifugal force, weight, this case would be, pitching moment. It would be full flap moment, full flap moment, thrust of each blade, this is power torque force producing torque centrifugal force weight one blade pitching one blade pitching moment So in similar to the development of the flap equation, one could develop a lead lag and torsion equation and full set of response to get hub loads and all these things. So, this give you an idea about those things.

Yaw stability

For yaw stability, if gravity, steady wind, wind shear are only effects considered,

$$\beta_{1s,er} = \beta_{1s,ahg} + \beta_{1s,vs}$$

$$\text{steady state yaw error } (\theta) \cong \frac{\Omega R}{V}\left(\frac{3B}{2(2K-1)}\right)$$

So, then top of that, you can have, I mean, power loads. Okay. So, power loads can come from multiple fronts. One could be from aerodynamic load aerodynamic power load then you can have power vibration you can have dynamic power loads, okay, so, All this you can have. Essentially, the aerodynamics power loads include the rotor thrust during normal operation, the movement of the rotor tug, extreme wheel load, then tower natural frequencies can be estimated.

Then the most important concept in the tower design is to avoid natural frequencies of the rotor frequencies like 1p, 2p, 3p. Then dynamic loads on the tower during. So all this then, I mean, in dynamic tower loads, you can have the different flap-wise, these things. now you can have your stability so your stability is an issue for few your turbines this complicated problem and simplified dynamic model is of limited utility in its analysis at this moment but it can provide some insight to some of the basic physics the First thing is to note that both vertical wind shear and gravitational force on both blades and I mean on the bent blades tend to turn the rotor out of the wound in the same direction. This means that the rotor tend to experience a crosswind from the direction rotor back in the other direction.

For EOS stability then for EOS stability If gravity, steady wind, wind shear are only effects considered, then the beta 1s, beta 1s ag plus beta 1s nu s. so using this and ignoring the wind shear and assuming smaller approximation one can find steady state your error which is that is omega r y v e b by 2 into 2 k minus 1. so the steady state EO error, then in the absence of vertical wind shear is greater at faster rotor speed. For softer rotors, which is smaller key, vertical wind shear here, if you see this expression, the vertical wind shear would increase the steady state EO error even more. Obviously, one needs to carry out a thorough analysis of these terms.

I mean, you can see this when you talk about these mechanics and the dynamics and all these things. So you have, these are giving the simplified systems gives you the, find out the analytical solution for estimating the initial design parameter so this pretty much gives you the complete picture of the dynamics of the wind turbine and there are few points which you will discuss in the next session to continue okay thank you