Wind Energy

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Lecture 56: Mechanics: Solution of Flapping Equation-contd...

Welcome come back. So, we continue looking at the solution of this whole flapping equation and as we have said the solution could be expressed can be expressed as a Fourier series expression which would have constant term cosine term and sine term and we discussed about the effect of the Now, similarly, the sign term, I mean, we can have the positive sign constant means that the bleed which is rising, tends to be bent, downwind, when horizontal. When descending, the blade goes upwind. Overall, the plane described by the tip would tilt to the left in the direction of positive yaw, so, which has been shown in this figure so, that means, you can also have positive sign constant which would mean the blade when it rising tends to be I mean rising tends to bend. Downward. Descending the blade goes up.

So, you can see that that makes it plain like this. What is shown in this figure on the sign done. So. What we can.

So we can. Summarize. One. The constant term. The zero.

That indicates. That. The blade. Bends. Downward.

By a constant. amount as it rotates so as in only beta 1c say that indicates that the plane of rotation downwind when the blade is pointing down and upwind when pointing up. And beta 1s indicates that the plane of rotation tilts downwind when the blade is rising upwind when descending, okay! so, this is what you have in summary of that particular solution to the full flapping equation where beta 0 which is the coding term that indicates that blade, i mean, blade beds downward by constant amount as it rotates. Beta 1c indicates that the plane of rotation tilts downward when the blade is pointing down and upward when pointing up. And beta 1s indicates that the plane of rotation tilts downward when the blade is rising and upward when descending.

So this is in summary. We have different terms which are, So, now these constants are functions of the various parameters in the model. So, the coding term is related primarily to the axial flow, blade width and spring constant. So, if you see the coding term that is related to axial flow blade weight.

And the. Spring constant. The sign and cosine terms depend on. So the. Sign and. Cosine terms.

They depend. On. Your rate. Okay. We'll see here.

Crosswind. Which is. Your. And as well as the same time that.

Plus. Comes. That. Effect.

$$\beta(\psi) = \beta_0 + \beta_{1c}\cos\psi + \beta_{1s}\sin\psi$$

Okay. So. Now. We apply assumption that the flapping angle can be expressed as this beta naught. So, the beta naught would be plus beta would be beta naught plus beta 1c, So this, once we using this, then we can possibly using this, we can possibly solution in closed form. So, one can take derivative to this particular equation and substitute the results into the full bending flapping equation and then collecting the terms to match coefficients of the function of . So, what we finally get Finally what we have is a matrix K B minus gamma q bar d bar 2B K minus 1 gamma by 8 gamma U naught by 6, gamma by 8, K minus 1. where beta naught, beta 1c, beta 1s equals to you have gamma by 2A, minus 2q bar minus gamma by 2, V naught, A3, knus, U bar by 4, then we have minus gamma by 8 q bar, so, we get a matrix for this so now this is more like an again a generic form of the solution, so, what we are doing as we say that the beta can be expressed in terms of this beta non beta 1 c and beta 1 s And then take the derivative and put it in the full flapping equation that we have here.

So, collecting the term and the coefficients and match the coefficient, then we can. I get this system which is a generic in nature. Now, we can have solution of the fapping equation for different. Now we can obtain the simple solutions so.

Simple. solutions so now we can either we can use this is a matrix can use a Cramer's rule find out then the approach will in fact be taken in as it is I mean it's an simple linear system so one can find out multiple with this solution but, So, this is not a big task as such, but what would be more interesting to see the effect of different terms. Okay, once we consider and how the turbine respond to that, or rather the dynamic response of the turbine with respect to those terms would be pretty interesting to see. So, which kind of it's like an. We want to see.

The dynamic response. Of a turbine. To a.

Finally,

$$(*)\quad \begin{bmatrix} K & B & -\gamma\bar{q}\bar{d} \\ 2B & K-1 & \gamma/8 \\ \frac{\gamma\bar{U}_0}{6} & -\frac{\gamma}{8} & K-1 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_{1c} \\ \beta_{1s} \end{bmatrix} = \begin{bmatrix} \gamma/2\, A \\ -2\bar{q} - \frac{\gamma}{2}\left[(\bar{V}_0 + \bar{q}\bar{d}) A_3 + \left(\frac{K_{vs}\bar{U}}{4}\right)\right] \\ -\frac{\gamma}{8}\bar{q} \end{bmatrix}$$

Fairly. Simple. Imports. Even. Without. considering the effects of turbulence and non-linear aerodynamics. So, that would be something interesting to see. So, what we can do, we can go a little bit of case by case basis.

And let's say first we say that rotation only situation. So, we are considering rotation here, but what we have gravity is zero. Crosswind effect is zero. Yaw rate is zero.

Offset is zero. Inch spring is zero. So, it's only rotation only considered. Okay. And rest of the terms here are considered to be 0. So, then the equation, this matrix, so these equations I would say it becomes only 1, 0, 0, 0, 0, gamma by 8 0 minus gamma by 8 0 beta naught gamma by 2a 0 0.

(i) Rotation only : (Rotation only considered)

gravity = 0
crosswind = 0
yaw rate = 0
offset = 0
hinge-spring = 0

$(*)$ becomes:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & \gamma/8 \\ 0 & -\gamma/8 & 0 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_{1c} \\ \beta_{1s} \end{bmatrix} = \begin{bmatrix} \gamma/2\, A \\ 0 \\ 0 \end{bmatrix}$$

$\beta_0 = \frac{\gamma}{2} A$! Only coning in this case.

balance between aerodynamic thrust and centrifugal force $\rightarrow$ flapping angle.

So, the solution to this is pretty straightforward so what we get beta naught equals to gamma by 2 a there is only coning in this case so, only owning in this case. I mean, what happens is that there is a balance between aerodynamic thrust and the centrifugal force which determines the clapping angle. So, balance between aerodynamic thrust and centrifugal force determines the flapping angle. So, there is no dependence on azimuth. So, this is probably the simplest of the, I mean, so that's what I said it would be interesting to look at the impact of different terms in the final solution.

So, if you only consider rotation, this gives you the coning effect only and the other centrifugal force and irrotational force balance between each other. So, the, which determines the flapping angles and there is no dependency of the adjuvant. So, that is one of the simpler situation that one can have. The second and what we can consider is we will keep rotation then we add in spring plus offset okay so that is what we will add to that now adding the spring and offset terms give the same form of the solution. So, adding the ring and offset terms give same form for the solution.

So, flapping angle is, however, the flapping angle is reduced. So, the star becomes K, 0, 0, 0, k minus 1, gamma by 8, 0, minus gamma by 8. So, gamma by 2a, 0, 0. So, the solution here is beta naught is gamma A by 2K. So, the cooling angles now results from a balance between the aerodynamics moment on the one hand and the centrifugal force the hinge spring moments opposing them.

So, as one can expect that stiffer the spring the smaller the coning angle. So, this tells that stiffer the spring, smaller the coning angle. So, essentially that's what you get from this particular situation where you have this balancing act between aerodynamics moment and the centrifugal force and the other side is the instant moments which would oppose to those two moments. Now, we can include some more time here. Now, we can include rotation plus inch spring plus offset plus gravity.

(*) becomes

$$\begin{bmatrix} K & 0 & 0 \\ 0 & K-1 & \gamma/8 \\ 0 & -\gamma/8 & K-1 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_{1c} \\ \beta_{1s} \end{bmatrix} = \begin{bmatrix} \frac{\gamma}{2} A \\ 0 \\ 0 \end{bmatrix} \Rightarrow \beta_0 = \frac{\gamma A}{2K}$$

— stiffer the spring, smaller the coning angle.

Obviously, when we add gravity, it will, so if you see that what we are doing, essentially, once we get this matrix, we're trying to see the impact of different terms and the components on the final solution. So, what is happening is that rotation gives you coning only then once you add rotation to hinge spring and offset that also gives you coning but there is an effect of the hinge spring moment and which balances out the aerodynamic moment and the centrifugal moment and then that you get to see stiffer the more stiffer spring you have then smaller the coning now we want to include But once you include gravity, it complicates the solution. That we will discuss in the next session. Thank you.