Wind Energy

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Lecture 55: Mechanics: Solution of Flapping Equation

We will now continue the discussion on this full flapping equation. And now, what we finally derived in the last session is that the complete, as you can see here, the complete flapping equation of motion, including all the forced effects. effect due to aerodynamic forces and moments due to aerodynamic forces your effect so, this is the host motion of flapping equation and now we are going to look at some of the solutions for this particular equation, okay! so, before we continue with the solution or different kind of solution there are some discussion that look like like to have here few aspect of it on this the final equation includes a damping term for some discussion of flapping equation so, that is what discussion of flapping equation. Final equation includes a damping term. So, that term multiplying beta prime depends on a log number. So, this is 0 when arrow dynamic are not included, okay so, this means that the only damping in this model is due to aerodynamic so, the log number which is the log number that is gamma which is rho c Cl alpha r to the power 4 by Ib this can be thought about aerodynamic forces to inertia forces, so, one can see So, that means swing the lock number is zero.

That means the aerodynamic effect is not considered. Now, if the lift card slope is zero or negative as in stall, gamma will also be zero or negative. Hence there will be no damping. OK.

$$\text{Lock number } (\gamma) = \rho c C_{l\alpha} R^4 / I_b := \frac{\text{Aerodynamic forces}}{\text{Inertia forces}}$$

Now, this could be a problem for a titered rotor with its large range of flapping motion. So, this can be a problem. or a heated rotor with its large range of traffic motion. Similarly, This can be a problem in rigid rotors with dynamic coupling. lead lag and edgewise motion.

So, these are situations where there could be some problem. In such cases, negatively damped flap motions may cause edgewise vibration. in such cases, negatively damped motion may cause edge-wise. So, please note the derived linear model does not include solve okay, because we have assumed the slope curve is linear so, it requires to be modified to capture the general trends such as the effect of negative lock number. So, with no crosswind or yawing the damping ratio, can be written as gamma by 16, 1 by omega beta by omega, where omega beta is lapping frequency.

And for tectered or articulated blades omega beta is omega. So, this xi is approximately 1 by 16. Sorry, it could be gamma by 16. So, this is what you can have. That means the, I mean, one of the issue here in the final equation is that the some damping term which is included and the term which is multiplying by beta prime is includes the lock number and lock number one can treat like an ratio of aerodynamics forces to inertia forces so lock number goes to zero and there is no aerodynamic forces and if lift curve slope is zero or negative which is also installed gamma is also going to be log number going to be zero or negative.

With no crosswind or yawing the damping ratio can be written as: $\xi = \frac{\gamma}{16} \cdot \frac{1}{\omega_\beta / \Omega}$

ω_β = flapping frequency

For teetered or articulated blades, $\omega_\beta = \Omega$, $\Rightarrow$ so, $\xi \simeq \frac{\gamma}{16}$

But, this can be a problem for titered rotors or can be a problem for rigid rotors when the dynamic coupling is there because negatively done flap motion will cause edgewise vibration. But, our derived linear model doesn't include stall. So, that is something one has to take account those by some modification. But if you do not have crosswind or even the damping ratio can be slightly modified. And this can also provide some kind of solution to the situation that may arise.

Similarly, we can see for rigid blades, for rigid blades, Omega beta is order of 2 to 3. 2 to 3 times times higher than omega and the flapping ratio is correspondingly smaller. With log numbers ranging from 5 to 10, the damping ratio is on the order of 0.

5 to 0.16. So, this amount of damping is enough to damp the flapping mode vibration. So, this amount of damping is enough to damp the trapping mode vibration, okay! so, another thing to be noted here is that the details of lead lag motions are not kind of discussed in details or considered. I mean, but one can note that full development of lead-lag equation of motion. So, no aerodynamic damping. So, what the issue is that lack of damping in lead lag can lead to Now, if we go back here and look at this equation, there is a constant term on the right-hand side, which is gamma by two.

So, this constant term that right-hand side constant term that is gamma A by 2. This describes blade coning which is a constant deflection of the blades which is a constant deflection of the blades away from the plane of rotation due to the steady force of the wind. This coning is in addition to any pre-coning. Okay, so this is important. So, that term, if there is any pre-conning, the pre-conning is sometimes incorporated in the rotor design for a number of reasons.

So, pre-conning is sometimes incorporated in rotor design. for a number of reasons. One, it keeps the tip away from the tower to it helps to reduce root flap bending moments on a downwind on a downwind rigid rotor. And three, it contributes to your stability. So, if you see that this thumb which is sitting there on the right hand side of the equation, and gamma by two.

That kind of gives you an idea about blade coning. Okay. Though that is essentially a constant deflection of the blade from the plane of rotation due to the steady force of the wind. That is how that kind of takes care of. But this coning term that is the equation in addition to any pre-coding.

So, this is what that is in addition to any pre-coding. So, why pre-coning is done? It is sometimes incorporated in the design procedure of the rotor because it helps in numerous ways. It gives the tips from the rotor. It helps to reduce bending moments on a downward rigid rotor root flap bending moments it contributes to some extent your stability so, that is why some kind of a pre-coning is done in the rotor design also, so, these are some of the discussion that one should know that some contribution of the terms of that full flapping equation okay now we can look at the solutions of flapping equation of motion now we can look at that now solutions to mapping equation of motion.

Okay. So, which we can show mapping equation has constant terms, sines, cosines of azimuthal angles. So, the full solution would be written as a Fourier series. that is to say sum of sines and cosines of azimuth is positively higher frequencies so the full solution would be written as fourier series okay, now, this we can see by noting that the azimuth psi essentially, what we can put a psi is omega into t. So, that azimuth psi is actually equal to the rotational frequency multiplied by time. Now, the frequencies in the Fourier series would begin with sinusoids of the azimuth and increase by integer multiples.

increase by integer multiples. Now, so a good approximation would be sum of cosine and sine terms of the azimuth angles. So, that could be a good approximation. So, using these assumptions, the solution of the full flapping equation of motion can be expressed in terms of three constants. So, the solution of flapping equation of motion would be expressed in terms of three constants, beta naught, beta 1c, beta 1s.

and beta would be beta naught plus beta 1 c cos psi beta 1 s sin psi where beta naught is coning or collective response understand beta one C's cosine cyclic response constant and beta 1s is sine cyclic response constant. Okay. So, obviously, it would be important to note the directional effects of the different terms in the solution equation. So, I mean, one can see these effects like I mean, If I have a, then So, this is beta naught, this would be beta 1c, beta 1s. So, here beta naught is the collective response constant, beta 1c cyclic, cosine cyclic response, and beta 1s is the cyclic.

Solution of flapping eq. of motion could be expressed in terms of three constants: $\beta_0, \beta_{1c}, \beta_{1s}$, $\beta \simeq \beta_0 + \beta_{1c}\cos\psi + \beta_{1s}\sin\psi$

β_0 = coning or "collective" response constant.

β_{1c} = cosine cyclic response cnt.

β_{1s} = sine cyclic response cnt.

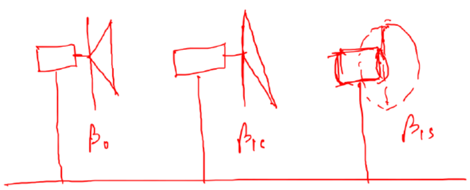

illustration of terms

So, this is how the illustration of terms. So, I mean, as we said that important to consider the directional effects of the different terms in the solution. Okay. So, now, what happens is that the coning term is positive.

Okay. So, that means it indicates that the blade bends away from the freestream wind. So means the blade bends away from the freestream wind. From the freestream wind. So now, I mean, as for this figure here that we have drawn, a positive cosine term, a positive cosine term or cosine term or constant indicates that when blade is pointing down, it is pushed further downward. And when pointing upwards, the blade tends to bend upward.

when pointing upwards, the blade tends to bend a point. In either horizontal position, the cosine of the azimuth equals to in either horizontal position cosine of the azimuth is zero. So, does a plane determined by the path of the blade tip would tilt about a horizontal axis upwind at the top, downwind at the bottom. So thus a plane determined by a path of the blade tip would tilt about a horizontal axis upwind at the top and downwind at the bottom.

So, this is what happens. I mean, this is the illustration of these different terms of the solution in the Fourier series that can be expressed either constant term beta naught, which could be a elective response term, beta 1c, which is a cyclic cosine term, beta 1s, cyclic sine term, and positive cosine term or constant indicates that the blade is straight pointing down. when pointing upwards, the blade tends to bend upward. But then when it comes to horizontal position, the cosine terms contributes to zero. So, that's what the plane determined by the path of this blade T would tilt about a horizontal axis upwind at the top and downward at the bottom, which is shown here. Basically, this would kind of a tilting nature that would come.

Similarly, We can also have the solution term for the sine term also. That we will discuss in the next session.