Wind Energy

Prof. Ashoke De

Department of Aerospace Engineering, IIT Kanpur

Lecture 54: Mechanics: Full Flapping blade model

I will come back. So, we'll continue the discussion on this aerodynamic forces and moments. So, this is what we, in the last session, we have written this flapping moment equation. Now, and these are all the non-dimensional term. I mean, they have been defined either by non-dimensionalizing based on the tangential component of the velocity or in terms of the angular velocity. So, we have all the non-dimensional inflow, cost flow, total cost flow, lock number, your rate term, wind velocity, everything.

So, now we look at the derivation of these components. So, this aerodynamic forces and moments. So, for the linear model, the component of the blade forces in tangential and normal directions are already calculated. now, what will account here will account for, number one the flapping angle and number two zero drag, these are two things that will account for linearized system.

Okay. Normal force, the normal force per unit length is Fn that is L cos phi cos beta. And, tangential force per unit length FT is L sin phi. These are the two different components. Now, the all the forces so the various forces must be integrated or summed over the blade to give the shear force okay or multiplied by the distance first and then sum to give the moments so or multi by the distances first and then some to give moments.

Aerodynamic forces & moments : account for ① flapping angle ② zero drag } Linearized system

Normal force per unit length $(\breve{F}_N) = \breve{L} \cos\varphi \cdot \cos\beta$

Tangential force per unit length $(\breve{F}_T) = \breve{L} \sin\varphi$

As before, simplification is made : $\sin\phi = U_P/U_T$, $\cos\phi = 1$, $\cos\beta = 1$

the flapwise shear force at the root of the blade is the integral of the normal force per unit length of the blade :

$$S_\beta = \int_0^R \tilde{F}_N \, dr = \int_0^R \tilde{L} \cos\phi \cos\beta \, dr = \int_0^1 \tilde{L} R \, d\eta$$

$$(\eta = r/R)$$

So, as before, we do uh simplification is made and what we have sine of phi is UP by UT also cos phi is one also cos beta is one, so, these are due to small angle approximation. Now, the flap wise shear force at the root of the blade is the integral of the normal force per unit length of the blade. So, what we have S beta, 0 to R, Fn dr, 0 to R, L cos phi, cos beta dr, 0 to 1, L R d eta. Here eta is r by R. So this is the flapwise shear force.

Similarly, we have flapwise bending moment. The flapwise bending moment at the root of the blade. So, is the integral of the normal force per unit length multiplied by the distance at which the force acts over the length of what you have M beta 0 to R FN r dr 0 to R cos phi cos beta r dr 0 to 1 L R squared eta d eta. Now this flapping moment, so this can be expanded. So, M beta is 0 to 1 R squared eta d eta.

0 to 1 of rhoc Cl alpha UP UT minus thetaP UT square R square eta d eta. Now we can do other substitution as required. Now, if we put back the complete equation of motion. So, this includes moments due to aerodynamic and urate or gyroscopic effect and then can have some algebraic manipulations so, it is beta double prime one plus epsilon G by omega square.

Cosine. K beta. By. Omega square. I beta into B.

$$M_\beta = \int_0^R F_N^{\sim} r\,dr = \int_0^R \tilde{L} \cos\phi \cos\beta \, r\,dr = \int_0^1 \tilde{L} R^2 \eta \, d\eta$$

This can be expanded, $M_\beta = \int_0^1 \tilde{L} R^2 \eta \, d\eta = \int_0^1 \left[\frac{1}{2} \rho c C_{l\alpha} \left(U_P U_T - \theta_P U_T^2 \right) \right] R^2 \eta \, d\eta$

Complete eq. of Motion : includes moments due to aerodynamics & yaw rate

$$\beta'' + \left[1 + \epsilon + \frac{g}{\Omega^2} \cos\psi + \frac{K_\beta}{\Omega^2 I_b} \right] \beta = \frac{M_\beta}{\Omega^2 I_b} - 2\bar{q} \cos\psi$$

Where, $\beta'' = \ddot{\beta}/\Omega^2$ = azimuthal second derivative of β

M_β = aerodynamic forcing moment

M beta. Omega square I b 2 q bar Cos psi. beta double prime is beta double dot by omega square is the azimuthal second derivative of beta and M beta is aerodynamic forcing moment. Now, if you see this equation now is expressed in terms of azimuthal derivative which we will discuss further because this will have some once we write in terms of beta double prime it would be easier to solve so, now from here so this actually taking care of the your moment or moment due to your rate or moment due to aerodynamic forces so this is what in terms of beta second derivative of beta we get the complete equation of motion okay, now we will further detail out this equation So, now if you recall the original clapping equation of emotion when the aerodynamic and gyroscopic moments are included that is written as beta double prime beta double dot omega square one plus epsilon g cos psi A beta by IB into beta N beta by IB to Q omega cos theta. Now we divide by omega square and what we get beta double dot by omega square 1 plus epsilon G by omega square of cos psi K beta by omega square I beta m beta by omega square Ib 2 q bar cos psi. So, if you see this particular equation here it is in time domain.

The rotational speed is assumed to be constant. It is more interesting to express the equation as a function of angular or azimuthal position. so, now we use chain rule beta dot is d beta by dt you can have d beta by d psi d psi by dt which is omega d beta by d psi omega beta prime. Similarly, one can find out beta double dot equals to omega square beta double prime. So, obviously you can see the dot stands for time derivative and that prime or single prime or double prime stands for derivative with respect to azimuth now once we rearrange this flapping equation of motion first now we'll rearrange so what we'll do rearrange flapping equation of motion for solution.

Original flapping eq. of free motion, when the aerodynamic & gyroscopic moments are included,

$$\ddot{\beta} + \left[\Omega^2(1+\epsilon) + G\cos\psi + \frac{K_\beta}{I_b}\right]\beta = \frac{M_\beta}{I_b} - 2q\,\Omega\cos\psi$$

Divide by Ω^2 $\Rightarrow$

$$\frac{\ddot{\beta}}{\Omega^2} + \left[1+\epsilon + \frac{G}{\Omega^2}\cos\psi + \frac{K_\beta}{\Omega^2 I_b}\right]\beta = \frac{M_\beta}{\Omega^2 I_b} - 2\bar{q}\cos\psi$$

It is in time domain!!

Since, the rotational speed is assumed to be constant, it is more interesting to express the eq. as a function of angular (azimuthal) position -

Use chain rule: $\dot{\beta} = \frac{d\beta}{dt} = \left(\frac{d\beta}{d\psi}\right)\left(\frac{d\psi}{dt}\right) = \Omega\left(\frac{d\beta}{d\psi}\right) = \Omega\beta'$

Similarly, $\ddot{\beta} = \Omega^2\beta''$

So, now once we incorporate or substitute everything there what we have is that beta double prime plus gamma by 8, 1 minus 4 by 3, v bar cos i beta prime plus one plus epsilon G by omega square cos psi gamma v naught by six sin psi K beta omega square Ib beta minus two q bar cos psi. gamma by 2. Minus theta p by 4. Gamma by 8.

q bar. sin psi minus. cos psi. Gamma by 2. b two plus two theta by three K nu s U bar by four okay! so, this is v bar okay now this can be written into slightly simpler form like we can have beta double prime and plus gamma by eight one minus four by three cos psi b naught bar plus q bar d bar beta prime k plus 2 B cos psi. Now by six V not sine psi beta gamma A by two gamma q bar by eight sine psi two q bar gamma by two, A3 plus q bar D bar.

Rearrange flapping eq. of Motion for solution

$$\beta'' + \left[\frac{\gamma}{8}\left(1 - \frac{4}{3}\bar{V}\cos\psi\right)\right]\beta' + \left[1 + \epsilon + \frac{G}{\Omega^2}\cos\psi + \left(\frac{\gamma \bar{V}_0}{6}\right)\sin\psi + \frac{K_\beta}{\Omega^2 I_b}\right]\beta$$

$$= -2\bar{q}\cos\psi + \frac{\gamma}{2}\left(\frac{\Lambda}{3} - \frac{\theta_p}{4}\right) - \frac{\gamma}{8}\bar{q}\sin\psi - \cos\psi\left\{\frac{\gamma}{2}\left[\bar{V}\left(\frac{\Lambda}{2} + \frac{2\theta_p}{3}\right) + \frac{K_{\gamma s}\bar{U}}{4}\right]\right\}$$

can be written into slightly simpler form!

$$\beta'' + \frac{\gamma}{8}\left[1 - \frac{4}{3}\cos\psi\left(\bar{V}_0 + \bar{q}\bar{d}\right)\right]\beta' + \left[K + 2B\cos\psi + \frac{\gamma}{6}\bar{V}_0\sin\psi\right]\beta$$

$$= \frac{\gamma A}{2} - \frac{\gamma\bar{q}}{8}\sin\psi - \left\{2\bar{q} + \frac{\gamma}{2}\left[A_3\left(\bar{V}_0 + \bar{q}\bar{d}\right) + \left(\frac{K_{\gamma s}\bar{U}}{4}\right)\right]\right\}\cos\psi$$

Where: K = Flapping inertial natural frequency (includes rotation, offset, hinge-spring)

$= 1 + \epsilon + K_\beta / I_b\Omega^2$

A = First axisymmetric flow term $= (\Lambda/3) - (\theta_p/4)$

A_3 = Second axisymmetric flow term $= (\Lambda/2) - (2\theta_p/3)$

B = Gravity term $= G/2\Omega^2$

$\bar{d}$ = Normalized yaw moment arm $= d_{yaw}/R$

K nu s U bar by four. Cos psi. Where you have where K is flapping inertial natural frequency so, which includes rotation offset hinge spring So, this is given as one plus epsilon plus A beta bar IB omega square. Now A is first axi-symmetric no term which is theta p by four a three second axis symmetric four term which is gamma minus two theta p by 3. b is gravity term which is g by 2 omega square and d bar is normalized your momentum normalized your moment which is d your by r okay so what we have now this situation actually includes all the forces i mean essentially, includes all the restoring forces and all of the forcing terms and now this can be used to find out the rotor behavior under variety of wind and dynamic condition so this is a complete this equation that we have here this is a complete nothing motion uh includes all the i mean forced motion but obviously once we make the all the aerodynamic forces under your rate effect then it becomes a free motion so the free motion is a special uh special case of this particular generic equation with we have now for the force condition.

So, all these different terms like defined here, they are kind of, so, they are all defined here and this is an equation which includes all of the restoring forces and moments and this now can determine the rotor behavior under variety of wind conditions and dynamic conditions as i have said the free motion could be a special case of this equation if we make the right hand side zero which is due to the aerodynamic force I mean, if it all the force and moments due to aerodynamic force linearized aerodynamic force and force and moment due to your rate so now once we have this generic equation we can now look at different condition or different situation to find out the solution now again the impact here is that so what we just quickly go through so this is where we derive the flapping equation for the free motion And we can see, right hand side, there is no effect. And then we start deriving the forced equation, which is incorporating the effect due to yaw and wind speed. And to do that, we have taken the linearized aerodynamic forces, so that we have derived. And for that, we get these velocity components, which will include all the effects of wind shear and things like that. This is the diagram that we started off.

And then finally, we take care of crosswind effect, wind shear effect, your motions, everything, vertical wind shear. All these we have included. And then we got this. And then the moment equation, that flapping moment, we have. And then that also component-wise derivation we carried.

And finally, we reach to this particular equation. So, this includes all kinds of forces moments everything included so that it allows us again this linearized systems or simplified equation model is very very handy or useful for getting initial design data okay, we'll now look at the different solutions and all these things of this in the next session thank you