Wind Energy

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Lecture 53: Mechanics: Linearized aerodynamic model - contd...

We'll come back. So, we'll continue the discussion on this crosswind effect. So, we have already shown some figure that crosswind work from left to right. So, the effect of crosswind now will incorporate in these changes in terms of some perturbations and things like that. So, what it's going from left to right now since the blade is turning counter clockwise looking upwind, the crosswind decreases the tangential wind speed when the blade is Below the. R-axis.

And. Reversely. Increases. It.

When. Above. Okay. So, the. That.

Change in. tangential component of wind velocity which is due to crosswind is deltaUT crosswind effect minus Vnaught cos psi. Now, when the blade is out of the plane of rotation by the a flapping angle beta there will be a component that decreases the perpendicular when speed and blade. DeltaUP due to crosswind effect is minus V naught sine beta by using small angle approximation. Your error is predominantly a cosine cyclic disturbance affecting the tangential velocity so, this is cosine this is sine okay! that's how this effects now we consider the yaw motion so, this also affects the blade velocities.

I mean, apart from the gyroscopic moments and et cetera. Let's consider the blade. set UP and a yaw rate of q. So the blade will experience a velocity due to the yaw. So, the blade will experience velocity due to the yaw.

$$\Delta U_{T,crs} = -\dot{V}_0 \cos\psi$$

$$\Delta \dot{U}_{P,crs} = -V_0 \sin\beta \sin\psi \simeq -V_0 \beta \sin\psi$$

Now rotation of yaw due to yaw rotation which is where dO is the distance from the axis of tower to the center of the rotor. dO is the distance from the axis of the tower to the center of the rotor. If the blade is inclined away which is downwind from the plane of rotation by the trapping angle beta, there will be increase the effect so not there this will increase the effect by a factor which is rq sine beta now the effect is high or greatest when the blade is up or down and the non-existent when blade is horizontal this effect is highest when blade is up or down and non-existence coil horizontal. Now we use that small angle approximation using small angle approximation so what we can write that d yaw beta plus r would be r, r beta dyaw could be d yaw. So, what we have right delta UT yaw q d yaw cosine psi, delta UP yaw minus qr sin psi.

These are similar to the crosswind terms with the tangential term varying with the cos psi and the axial or perpendicular one varying with the sine side. Essentially, these are cosine and cyclic sine variation. Now, we need to take into account vertical wind shear. Vertical wind shear. So, that kind of vertical wind shear means essentially the boundary layer variation that would be there in the wind profile or free stream wind profile.

There could be effect of roughness, terrain and other effects. So, there is a gradient that exists as we go along the perpendicular direction from the surface, earth surface. So, those gradients are usually that creates that kind of a shear stress these are called the shear stress due to wind velocity and this is termed as wind shear and that can be so in a nutshell this talks about the wind velocity variation in the perpendicular direction and the shear stays due to those velocity gradients and things like that. So, we can model that. Let's say U1 by U2 as h1 by h2 to the power alpha, where alpha is power law.

$d_{yaw}\ \beta + r$

$r\beta + d_{yaw}$

$$\Delta U_{T,yaw} = -q\, d_{yaw} \cos\psi$$

$$\Delta U_{P,yaw} = -q r \sin\psi$$

$$\frac{U_1}{U_2} = \left(h_1/h_2\right)^{\alpha}$$

α = power law exponent

1, 2 → correspond to different height

Vertical wind shear is linear across the rotor

Free stream wind at height 'h' : $U_h = U\left[1 - (r/R) K_{vs} \cos\psi\right]$

Exponent. And. One and two. Correspond to. Different.

Right. Okay. So, also in simplified model. that is I mean, in simplified aerodynamic model it's assumed that the vertical wind shear is linear across the so vertical wind shear is linear across the rotor across the rotor. This is what we consider in the simplified. Now, so what will happen? We can write free stream wind at height h can be written that Uh equals to U into 1 minus r by R k nu s cos psi where h is H minus R when blade tip is down and h is H plus R when blade tip is up.

It is. All right. In us is. Linear. Wind shear.

constant. OK. And. New is. This dance for. Basically vertical shear. So, now the center of the at the center of rotor and on the horizontal. So, at the center of the rotor Uh is U ,at the top with zero flapping, Uh is u into one plus k nu s.

And at the bottom, Uh is u into one minus k nu s. The incremental effect of vertical shear is predominantly on the perpendicular component of wind. Okay. So, the tangential wind shear can be assumed to be zero. So, that means the predominant effect of vertical wind shear is on perpendicular component.

Hence delta UT nu s on tangential component it is assumed to be zero. Well, Up nu s. minus U r by R K nu s cos psi. So, similarly. One can include the high end and we see it as well.

So. we can consider the small angle approximation. So, at different, then we can have a variation. So, this is talk about wind speed perturbations. There could be axial flow, crosswind, yaw rate, vertical This is delta UP.

This is delta UT. So, axial flow, this is U. 1 minus a r beta dot. Crosswind minus v naught.

Beta sine psi. Minus q. R sine psi. Minus U r by R K nu s. cos psi. This is omega r.

Minus V naught. cos psi. Minus q dyaw cos psi and this is 0. So, obviously this using this small angle approximation. So, this is a table that you can have for wind speed perturbation, I mean the summarized thing. So, what it shows that now Vertical wind cell is a cosine perturbation. Crosswind is primarily a cosine perturbation to the tangential component.

Perpendicular component is a sine perturbation. Now, once you add all these components, And the contribution. In. For axial flow and all these things.

Where, $h = H - R$ when blade tip is down
& $h = H + R$ when blade tip is up

H = hub height
K_{vs} = Linear wind shear constant.
'vs' → stands for vertical shear.

at the centre of the rotor : $U_h = U$
At the top (with zero flapping) : $U_h = U(1 + K_{vs})$
At the bottom (") : $U_h = U(1 - K_{vs})$

Predominant effect of vertical wind shear is on perpendicular component
Here, $\Delta U_{T,vs} = 0$, $\Delta U_{P,vs} = -U(r/R) K_{vs} \cos\psi$

Wind speed perturbations
(using small angle approximations)

Case	ΔU_P	ΔU_T
Axial Flow	$U(1-a) - r\dot{\beta}$	Ωr
Crosswind	$-V_0 \beta \sin\psi$	$-V_0 \cos\psi$
Yaw rate	$-q r \sin\psi$	$-q\, d_{yaw} \cos\psi$
Vertical Wind shear	$-U(r/R) K_{vs} \cos\psi$	0

So, what we get. Is. UT Omega r minus. V naught. q dyaw cos psi. UP equals to U Into 1 minus a minus r beta dot.

V naught beta. qr sine psi minus U r by R K nu s cos psi, so this is what you get the velocity component and this is what we kind of started to i mean, so that, yeah for simplified thing this is what we said the velocity component would be like this and now we completed that derivations for both axial flow and tangential component where we consider different effects of crosswind urate vertical wind shear effect and finally we get this velocity components like here. So, once we have this axial or perpendicular component and tangential component, then we can use them in the, use them to find out aerodynamic forces and moments. So, already we have found out the simplified lift force per unit length. And now we can use this simplified. So, this is what can be used for design purposes.

So, let's look at the aerodynamic forces and moments. So, this is essentially the lapping moment due to aerodynamic.

Forces. Which is. M of beta. half. Gamma. Ib.

Omega square. By three. Minus B by four. Theta p by four. cos psi.

V bar. So, theta p by 3. K nu s u bar by 4. Sin psi. V naught. Beta by 3. q bar by 4. So here, this is non-dimensional info, which is given as u into 1 minus a by r omega.

V naught 1 is non-dimensional cross flow. which is V0 by omega R. V bar is non-dimensional total crossflow, which is V0 q d yaw divided by omega R.

The azimuthal Derivative. Of. flap angle. Which is. Beta dot by omega. Gamma.

Lock number. which is rho CL alpha C R power 4 IB. U bar is non-dimensional yaw rate term, which is U by omega and u bar is non-dimensional wind velocity which is u by omega r which is 1 by lambda and lambda is the tip speed so what we right now after deriving those velocity component axial or perpendicular component and the tangential component we can have the flapping moment due to aerodynamic forces so this expression looks bit messy but this is what you get because these are something you are trying to analyze analytically so that you can use this for initial or preliminary design purposes so here all this term involved non-dimensional inflow so everything has been non-dimensional based on the tangential component or tangential I mean either are omega or in terms of omega. So, all these different parameters have been clearly defined here. So, all of them are non-dimensional number, whether non-dimensional inflow, non-dimensional crossflow, total crossflow, flap angle, Lock number, yaw rate, non-dimensional wind velocity. So, everything kind of consider here but this is what again we will kind of derive those equations I mean these individual components and things like that and I mean we consider term by term and then put them back in the complete moment and then get back to this particular expression. And, then later on, we'll try to see different solution, simplified solution strategy to figure out or find out the solution.

$$U_T = \Omega r - (V_0 + q\, d_{yaw})\cos\psi$$

$$U_p = U(1-a) - r\beta' - (V_0\beta + qr)\sin\psi - U(r/R)K_{x1}\cos\psi$$

Aerodynamic Forces & Moments : the flapping moment due to aerodynamic forces

$$M_\beta = \frac{1}{2}\gamma I_b \Omega^2 \left\{ \frac{\Lambda}{3} - \frac{\beta'}{4} - \frac{\theta_p}{4} - \cos\psi\left[\bar{V}\left(\frac{\Lambda}{2} - \frac{\beta'}{3} - \frac{2\theta_p}{3}\right) + \frac{K_{x1}\bar{U}}{4}\right] - \sin\psi\left[\frac{\bar{V}_0\beta}{3} + \frac{\bar{q}}{4}\right]\right\}$$

Λ = Non-dimensional inflow = $U(1-a)/\Omega R$

$\bar{V}_0$ = Non-dimensional crossflow = $V_0/\Omega R$

$\bar{V}$ = Non-dimensional total crossflow = $(V_0 + q\, d_{yaw})/\Omega R$

β' = Azimuthal derivative of flap angle = $\dot{\beta}/\Omega$

γ = Lock number = $\rho C_{l\alpha} c R^4 / I_b$

$\bar{q}$ = Non-dimensional yaw rate term = q/Ω

$\bar{U}$ = Non-dimensional wind velocity = $U/\Omega R = \frac{1}{\lambda}$

So ,we'll now start looking at how we can reach or derive this particular expression for flapping moment due to aerodynamic forces. So, we will continue that in the next session.