Wind Energy

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Lecture 52: Mechanics: Linearized aerodynamic model

Well, welcome back. So, we'll continue the discussion on this equation of the flapping blade motion under force conditions. So, here we are considering the yaw effect and the wind effect where we are going to look at the linearized lift force. And linearized lift force we are going to consider because it's a simplified system. Essentially, we would be ignoring the drag force assuming that doesn't have too much of impact on the system. Now, we can find out this lift force per unit length that is L bar can be written as half of rhoc Clalpha UP UT minus thetap UT square.

So, here is the chord length. L alpha is the slope of the lift curve. So, up is the wind velocity perpendicular to the rotor plane, UT is the wind velocity tangential to the blade element, thetap is each angle. Okay.

So now, the equations for wind velocity component could take into account axial flow, flap rate. So, the equations for wind velocity components that take into account axial flow, flap rate, yaw rate, yaw error and wind shear. So, obviously the details of this individual components can be derived, later on, so what we have is UP is U into 1 minus a minus r beta dot v zero B plus q r sin psi minus U r by R Kvs cos psi and Ut equals to omega r minus v zero plus q d yaw cos psi. U is the system velocity. A is axial induction factor.

V0 is crosswind velocity, which is due to yaw error. due to yaw error. We have K nu s which is wind shear coefficient. Then, you have which is the distance from rotor plane to your axis. So, this is what we are going to get as the wind velocity components and while getting those wind velocity components which will take into account this axial flow flap rate your rate your error wind shear everything and you would get this uh where U is the free stream velocity, A is the axial induction factor, V0 is the crosswind velocity, K is the wind shear coefficient, D is the distance from the rotor plane to the axis, all this.

So, considering individually all this expression, we'll finally get this. So, now we'll derive this one by one. So, to do this, we need to have a velocity diagram which is important so let us have that let's say we kind of draw that. That is there. This is Omegar is Ut.

Lift force per unit length, $\mathcal{L}$, $\approx \frac{1}{2}\rho c C_{l\alpha}\left(U_P U_T - \theta_p U_T^2\right)$

c = chord length, $C_{l\alpha}$ = slope of the lift curve, U_P = wind velocity perpendicular to the rotor plane

$$U_P = U(1-a) - r\dot{\beta} - (V_0\beta + qr)\sin\psi - U(r/R)K_{vs}\cos\psi$$
$$U_T = \Omega r - (V_0 + q\,d_{yaw})\cos\psi$$

U = free stream velocity

a = axial induction factor.

V_0 = crosswind velocity (due to yaw error)

K_{vs} = wind shear coefficient

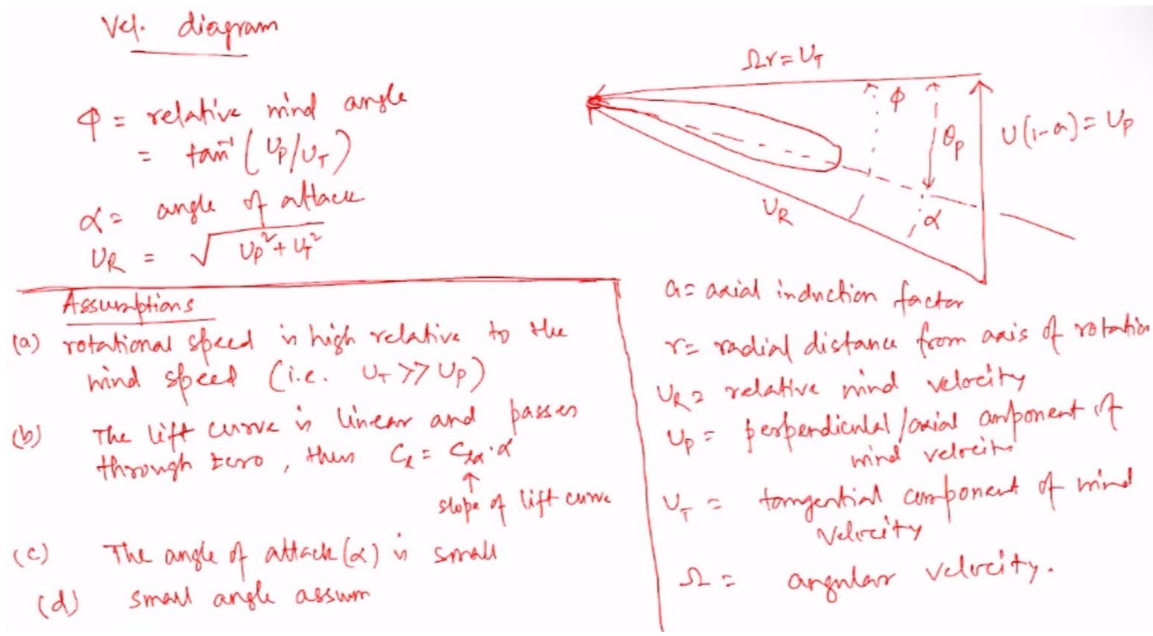
d_{yaw} = distance from rotor plane to yaw axis

This is U 1 minus a is UP. This is alpha, theta p. So, and this one is uR. So, here what you have, In this, A is axial induction factor. R is radial distance from axis of rotation.

uR is relative wind velocity. UP is perpendicular or axial component of wind velocity. It is tangential component of wind velocity and omega is angular velocity. So, here we will consider the steady wind. I mean start with a steady wind then subsequently we will consider the motion and all these things.

So, here in this particular figure the P is relative wind angle. So, that is essentially tan inverse up by ut alpha is angle of attack. Okay. UR is root of UP squared plus UT squared, okay! so, these are all the associated information that one would need to now to derive the thing will have some these are all nomenclature or information that is required. So, the assumption a rotational speed is high relative to the wind speed that is UT much higher than up be the lift curve is linear and passes through zero.

So, Cl equals to Cl alpha into alpha where Cl alpha is the slope of lift curve. The angle of attack alpha is small. Okay. Small angle assumption may be used as appropriate. No wake rotation and then the blade has a constant arc, no twist.



Although, the model can be extended to include a non-constant chord and black twist. So, this is what we need the velocity diagram to start with where we have this axial component up or the perpendicular component of the wind, then you have the tangential component UT, you have the axial induction factor a, radial distance from the axis of rotation r, relative velocity uR, angular velocity omega, their angle of attack, relative wind angle, pitch angle. And, then the whole derivation starts, I mean, based on certain assumption and which is quite obvious because this is a simplified model. So, this would definitely include some assumptions. So, as listed out here, ut would be much higher than up that means the tangential component of the wind is much higher than the axial component lift curve is linear so that CL could be integrated at the cl alpha into alpha then the alpha is small we have small angle approximations so that we could use that there is no weak rotation blade has constant twist i mean no twist and constant chord but yes this model can be extended further to incorporate the blade twist or non-uniform chord those things so, with this assumption we are going to derive the linearized model so, what we have now axial flow i mean, once we take the i mean, consider the considering the assumption about lift card which is says that the lift curve is linear, then we can write L bar up rho CL uR square up rho CL alpha U R square into alpha.

Now, we use small angle approximation that is tan inverse of angle would be that angle. So, what we have is that alpha is theta minus theta p which is tan inverse UP by UT minus theta P. We can write UP by U T minus theta p. Therefore, what we get L half rhoc CL alpha UR square into UP by UT minus theta p. Now, if you recall, UR square is UP square plus UT square, which is again UT square, because we use that approximation that UT is higher than, so this approximation allows us to use this.

Then we can write, this would be half of rhoc, CL alpha, UT squared, UP by UT minus theta p half of rhoc Cl alpha UP into UT minus theta p UT squarec, so, this is what we get okay! now, what we have for flapping blade tangential velocity as usual the radial position multiplied by the angular speed which is actually the tangential velocity which is UT is actually reduced by the cosine of flapping angle. However, due to small angle approximation, this is ignored. But, usually what happens is that the tangential velocity gets reduced by, so now since that is ignored due to small angle approximation, that is ignored thus we can have UT is omega into r the and what we have the perpendicular the axial component UP is U into 1 minus a minus r beta dot because this is the reduction due to tapping velocity that is r beta dot so, use the free stream velocity here so, so there are other factors such as yaw error, crosswind, vertical wind, wind shear, et cetera, that will also affect the wind actually experienced by the blade. So, this can be accounted as perturbation on the main flow. Obviously, these perturbations are assumed to be small.

Axial Flow

Considering the assumption about lift curve

$$\hat{L} = \frac{1}{2}\rho c C_L U_R^2 \approx \frac{1}{2}\rho c C_{L\alpha} U_R^2 \cdot \alpha$$

Use, small angle approximation: $\tan^{-1}(\text{angle}) \approx \text{angle}$

$$\alpha = \phi - \theta_p = \tan^{-1}(U_P/U_T) - \theta_p \approx U_P/U_T - \theta_P$$

Therefore, $\hat{L} = \frac{1}{2}\rho c C_{L\alpha} U_R^2 [U_P/U_T - \theta_P)$, Recall: $U_R^2 = U_P^2 + U_T^2 \approx U_T^2$ (since $U_T >> U_P$)

We can write: $\hat{L} = \frac{1}{2}\rho c C_{L\alpha} U_T^2 (U_P/U_T - \theta_P)$

$$= \frac{1}{2}\rho c C_{L\alpha} (U_P U_T - \theta_P U_T^2)$$

For flapping blade: tangential velocity (U_T) — actually reduced by the cosine of flapping angle.

Due to small angle approximation, that is ignored.

Thus, $U_T = \Omega r$ & $U_P = U(1-a) - r\dot{\beta}$

compared to the axial point speed. So, we'll refer here as deltas in our Discussion. So. These are other factors.

Which are. Also. Effecting. This. Main flow. Okay. And there would be impact.

And that can be considered as. So, what we can do, so, now we say that crosswind and yaw error. So, crosswind which is V0 that is perpendicular to the axis of rotor. This is perpendicular to the axis of rotor and parallel to ground. So, this crosswind, this arises due to error.

There could be, I mean this error could be due to some misalignment of the rotor axis and the wind or a so this could be sudden engine wind direction or misalignment of the rotor axis and the wind so, this error would come from all this. So, what it does this crosswind tends to increase the tends to increase the tangential velocity at the advancing blade and decreased at the retreating blade. So, these different factors, which are this crosswind, these things, obviously arises primarily due to the error. And, the error could come for sudden change in the wind direction or misalignment of the rotor axis and the wind. So, what happens that this crosswind tends to increase tangential velocity at the advancing blade and decrease it at the retreating blade.

So, we'll I mean we have already defined the crosswind from left to right in one of the earlier figure or diagram. So, considering that we would take into account this effect. So, we'll derive that in the next session.