Wind Energy

Prof. Ashoke De

Department of Aerospace Engineering, IIT Kanpur

Lecture 51: Mechanics: Flapping blade model (Forced motion)-contd...

Welcome back. So, we continue the discussion on derivation of this flapping model for free motion. So, we are looking at different forces. So, we started with centrifugal force and then find out the expression for centrifugal force and then now we are talking about gravitational force. So, the gravitational force actually acts downward on the center of mass of the blade and when the blade is up gravity tends to increase the flapping angle and when the blade is down it tends to decrease it. Now, obviously gravitational force is independent of your rotational speed.

And so now, we can actually write the magnitude of the gravitational force. Let's say the magnitude of the gravitational force Fg is g cos psi dm. So, clearly it varies with the cosine of the azimuth. So, since it varies with cosine of psi, so it can be said as cosine cyclic input.

That's a kind of a notation system that we have been following, now the restoring moment due to the gravitational force depends on the sign of, so, what we have Mg equals to r sin beta g cos psi dm. So, that is what we get both the gravitational force and the momentum. Similarly, the next one we can estimate is the inch spring force. So, this creates a moment at the inch so obviously this moment is proportional to the flapping angle this is proper to the angle okay, now, The spring usually what happens is that since the spring tends to back into the plane of the hinge, in turn is the plane of rotation. So, the magnitude of hinge spring moment is a beta into beta.

So, as the spring always tends to bring the, so, spring tends to bring the blade into the plane of rotation. So, this is force creation. Now, the one that we include is acceleration. This flapping acceleration inertial force caused by mass dm. So, this is estimated by the acceleration in the flapping direction that is beta double dot and which is multiplied by the distance from the flapping axis the flapping direction multiplied by the distance from the Now, there will be a moment which is created due to this force.

So, the moment created due to this force can be estimated as r square beta double dot dm. So, this is what is the acceleration moment that you have. Now, once we combine all the effects, we consider the effects of all the forces so, then what we have so, the some of the moments which is some of the moments due to forces could be zero because there is no external force. So, which is essentially one can write Mf, Mc, Mg, Ms equals to zero. Now, if we integrate over the entire blade, what we get 0 to R Mf plus Mc plus Mg plus Ms equals 0.

Acceleration : flapping acceleration interial force caused by mass dm
- this is estimated by the angular acceleration in the flapping direction $(\ddot{\beta})$ × distance from the flapping axis.

Moment created due to this force $(M_f) = r^2\ddot{\beta}\, dm$

Now, if we expand the term, expanding the terms what we have 0 to R r square beta double dot r cos beta sine beta omega square r g cos psi sine beta into dm K beta into beta equals 0. Now, you can recall that the blade mass moment of inertia Ib is 0 to R r square dm and rg is the the mass and distance to the center of mass, then we can correlate rg into mbeta. So, this is what we have. So, therefore we get Iv beta double dot, Ib cos beta sine beta, cos psi sine beta Mb rg kb beta zero. Now, again we using small angle approximation that means, cos beta would be 1 and sin beta would be beta and defining gravity as g mb rg by Ibeta and if we rearrange What do we get? Beta double dot.

The effects of all the forces :

$\sum M$ (sum of the moments due to forces) $= 0$ (there is no external force)

$\Rightarrow M_f + M_c + M_g + M_s = 0$

Integrate over the entire blade, $\int_0^R (M_f + M_c + M_g + M_s) = 0$

Expanding the terms: $\int_0^R [r^2\ddot{\beta} + r\cos\beta\, r\sin\beta\, \Omega^2 + rg\cos\psi\, \sin\beta]\,dm + K_\beta \beta = 0$

Recall, $I_b = \int_0^R r^2 dm$, r_g = mass & distance to the centre of mass

$\int_0^R r\, dm = r_g m_\beta$

Plus. Omega square. g cos psi. K beta by Ib. Zero. So, it's a pretty similar that we have already obtained earlier as well.

Now, you can recall what we assume that recall the assumption that is blade model as a uniform section. We can add the offset e and define the offset and epsilon. So, epsilon as mB erg R by Ib which is 3e by 2 into 1 minus e. So now, including the offset in the analysis, the final flapping equation of motion becomes beta double dot omega square one plus epsilon g cos psi Ib. This is what one get.

Okay. So, what essentially so, this is the equation for Free motion. That means, there is no external forces which are acting on it. So, what we started to derive this three equation. So, we have considered all these different forces and moment, starting with the centrifugal one, then gravitational forces, hinge springs forces and acceleration forces. And subsequently, the moment that has been created by those forces, which is considered.

Therefore, we get,

$$I_b \ddot{\beta} + I_b \cos\beta\, \Omega^2 \sin\beta + g \cos\psi \sin\beta\, m_B r_g + K_\beta \beta = 0$$

use small angle approximation: $\cos\beta \simeq 1$, $\sin\beta \simeq \beta$, $G = g m_B r_g / I_b$

Rearranging: $\ddot{\beta} + [\Omega^2 + G\cos\psi + K_\beta/I_b]\beta = 0$

Recall, the assumption → blade model has a uniform cross section, we can add the offset (e), and define the offset term (ϵ)

$$\epsilon = m_B e r_g R / I_b = 3e/[2(1-e)]$$

Including the offset in the analysis, the final flapping eq. of motion becomes:

$$\boxed{\ddot{\beta} + [\Omega^2(1+\epsilon) + G\cos\psi + K_\beta/I_b]\beta = 0}$$

And then, once we combine all these forces together, so in the absence of any external force, the sum of the moments are zero. This is what we have written here. And, then we integrate over all the blades that would be integration. I mean, entire blade that goes from zero to R. So we opt in this.

And then once we expand the term and put back all the individual expression for these different moments. So, and correlating the moment of inertia, we get back to this equation. And then, we use some small angle approximation for beta, which is the flapping angle and the gravitational force. Then, we rearrange that and we get here. And then we have one of the assumptions at the beginning for deriving this free motion is that the blade has a uniform cross section.

So, using that, we could add the offset term. or rather include the offset. And, then once we include everything together, this is the final equation that we obtain for flapping motion under free, I mean flapping equation of motion for free motion. That means there is no external force. So, this is how we derive that and the solution would provide the characteristics and the dynamics of these things now, what we'll do we'll look at the equation of motion under post condition okay! now, we move to uh equation of motion which is forced.

So, here when we talk about the forced condition, we will expand the analysis by incorporating the effect due to your motion and speed. So, this is what we are going to see. Your motion. So, this results in gyroscopic moments acting upon the blade. So, here we can find out this gyroscopic moment.

So, the gyroscopic moment in the flapping direction due to steady yaw motion of rate q that we write Myaw here 2 q omega cosine psi Ib, so, this is what we get due to the yaw. Now, we include effects of wind. Here, we will use the linearized aerodynamic model. So, that means the effect of wind on blade will be analyzed by incorporating linearized aerodynamic model. So obviously, this will be similar to the aerodynamic forces and moment that we have already discussed.

<u>Yaw Motion</u> : results in gyroscopic moments acting upon the blade

Gyroscopic moment in the flapping direction due to steady yaw motion of rate q

$$M_{yaw} = -2q\Omega\cos\psi\, I_b$$

But the all the detailed one that we have derived that would be little complicated to be used here in this analysis. So, that detail aerodynamic expressions that we have derived would be complicated to incorporate in this simplified system. So, we will use some linearized model, whereas we would include the vertical wind shear, crosswind, yaw error, these things we will include. So, what we include that crosswind effect, vertical wind shear, yaw error, all these that we will include. obviously, when we talk about this linearized system we might ignore the drag during our analysis okay! so, in this linearized model ignores drag force Okay, so, that is another thing.

Obviously, this is a reasonable approximation because drag force has less impact compared to the lift forces during normal operation. I mean, obviously, if you talk about the complete dynamic model, then that should or must include both the lift and drag force and everything. Okay. So, now this required this linear is aerodynamic models. This requires linearized description of lift force on blade then linearized characterization, linearized characterization of the axial and tangential components of wind at the rotor which is essentially UP UT obviously, the lift force is function of this UP and UT, so, which means this is also a function of mean wind speed blade Flapping speed.

Your rate. Posh wind. Winds here. OK. So. That means what we have to kind of do that find out this linearized force. So, what essentially it says that this linearized aerodynamic model that we are talking about, that means, I mean, if you want, I mean, somebody wants to include the complete aerodynamic model, that would be complicated.

So, here we would be using linearized one that completely neglects or ignores the drag force. And that's a fairly reasonable assumption. because the drag effect of the drag force is significantly less compared to the lift force and the lift force is function of this axial and tangential component of the wind and the rotor so, which in turn is a function of mean wind speed blade flapping angle your rate across wind wind shear and all this so, once we include all these forces then we can find out the linearized system for the lift force. And once we find the lift force, then we can include that along with the yaw motion for getting the flapping equation of motion under forced conditions. So, because the worst condition arises while taking care of the air motion and the wind speed, which in turns having effect due to wind shear, crosswind effect, also the perturbations associated with the gust and all these things.

So we'll... continue this linearization process of the lift force in the next session thank you