**Wind Energy**

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# Lecture 50: Mechanics: Flapping blade model (Forced motion)

Welcome back. So, we continue discussing this flapping-lead model dynamics. So, just to recap with that, we have looked at the simplified system with no offset and considering different combination like with rotation without rotation with spin without spin and both having spin and rotation with no offset now we'll extend that same analysis for with the offset case so now what we'll do again we'll go case by case so first we'll take rotation ring and offset. So, in general the real blade does not behave as if it had a hinge spring right axis of axis of rotation. So, what that means, in general, what it is that omega square would never be equals to omega in our square plus omega square, which we have found earlier, since the real blade doesn't behave as it had a hinge being right at the axis of rotation. So, now to correctly model blade motion and the blade dynamics, which are represented by both omega r and omega nr, we need to properly describe the expression for that.

So, how we can, we can achieve the same by including the non-dimensional Inch. Upside. Okay. So.

That's how. We are going to. Find that. Out. Okay.

That is. E. So this. E. can be thought of as the fractional distance from the axis of rotation to the location of the blade hinge.

So, now what we can do that we can actually estimate all the constant of the hinge-spring model. Once, we know the natural frequencies of rotating and non-rotating real blade. So, that means knowing omegaR and omegaNR of real blades allows to estimate the constants in hinge-spring model. So, once we know that. we can actually estimate all this.

So, all this constant in these things and the offset can be defined as two into minus one divided by three plus. So, this is as per the literature given by Eggleston and Stoddart in their book. So, where Z is given as omega R square minus omega N square by omega square. So, we have this offset, which is the fractional distance from the axis of rotation. And, this is what we have shown in the picture here.

if you remember this that is what we have taken that e introduced here which is the offset so, okay! so, now in addition to the offset um i mean or this can be accounted by adjusting the moment of inertia. So, that means this offset can be adjusted by accounting in moment of inertia. So, the mass moment of inertia now will have Ib equals to mB R square by 3, 1 minus e whole cube. So, this is mass moment of inertia of the hinge blade, okay! so, this is where you take into account the then my flaping spring constant that is K of beta can be estimated as OmegaNR square into ib. So here, one has to be careful that the rotating and non-rotating natural frequency may be calculated by other method or from measurement and data, whatever is available.

So, what it shows that the flapping characteristics flapping characteristics of the blade can model by uniform blade with an offset hinge. Uniform blade with an offset hinge. and spring that has one degrees of freedom and that responds in the same manner to forces as the first vibration mode of the real blade. So, essentially this response in the same manner to forces as the first vibration mode of the real plate. Similar constants can be obtained for lead-lag motion of a real blade.

Knowing $\omega_R$ & $\omega_{NR}$ of real blades $\rightarrow$ allows to estimate the constants in hinge-spring model.

$$e = 2(Z-1)/[3+2(Z-1)] \quad , \quad Z = (\omega_R^2 - \omega_{NR}^2)/\Omega^2$$

This offset can be adjusted by accounting in moment of Inertia.

$$I_b = m_B \left(\frac{R^2}{3}\right)(1-e)^3$$ [ mass moment of inertia of the hinged blade ]

Flapping spring constant $(K_\beta) = \omega_{NR}^2 I_b$

Torsion requires a Deepness constant not require any offset hinge model. Okay. So, with all this constant, the blade model allows for three degrees of freedom. So, all these, all of these allows three degrees of freedom, which are flapping lead-lag and torsion. So, what we now, once we introduce that offset characteristics in the mass moment of inertia, then the spring constant can be obtained.

And, then one can find out this flapping characteristics, but obviously, which is modeled with an uniform blade with an uniform offset and hinge. So, that would have one degree of freedom and that responds in the same fashion to the forces as the first vibration mode of the real blade. So, that means how the real blade would respond to the vibration mode, this would behave in the same fashion so, that you obtain the similar kind of characteristics. then also, if you include the lead-lag motion of the real blade then we get some constants then torsional stiffness can be included so, with all these all the three degrees of freedom would be included which includes flapping lead-lag motion torsion and then we can write the complete equation of the full flapping blade model with free motion So, that kind of, so we write equation of motion, full cupping blade model, which is free motion.

Okay. So here, including gravity and an offset. So, the free motion looks like beta double dot omega square one plus epsilon g cos psi plus K beta by Ib. Complete system where g is the gravity term which is kind of equals to mb rg by Ib. rg is the radial distance to the center of mass. Epsilon is the offset term.

which is given as p e divided by two into one minus e is the azimuth angle. Okay. So, this is what the complete equate looks like that includes the free motion of the full flapping blade model. That means there is no extra force in term. So, that would be our next set of complications to incorporate.

This is where you have all the term. Now, we try to kind of derive this how we obtain that okay, now we'll go to the derivation of this derivation of free motion so let us define a system here first okay, We have this. So, this is GDM. This is Psi. This side R cos beta omega square DN.

This term is G. So, this is kind of a flapping blade viewed from down wind direction. Here, g is the gravitational force. m would be mass. r is radial distance from axis of rotation. beta is starting angle omega is rotational speed azimuth angle.

Okay! so, here so, this is, so what we get is that the blade is turned out of the plane of rotation towards the veer by the flapping angle beta. So, and the azimuthal angle is psi. So, if you remember that. So, this flapping angle beta is the blade is turned out of the plane towards the veer. And zero azimuthal angle corresponds to the blade tip pointing downwards.

This is what if you remember earlier diagram when we were talking about the aerodynamic analysis. An azimuthal angle or it increases along the direction of rotation. Okay. And the flapping angle is positive in the downwind direction.

Eq. of Motion: Full flapping blade model (Free motion)

including gravity & an offset, the free motion: $\ddot{\beta} + \left[\Omega^2(1+\varepsilon) + G\cos\psi + K_\beta/I_b\right]\beta = 0$

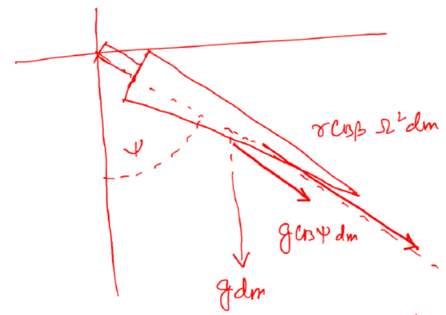
$G$ = gravity term ($= m_B r_g / I_b$ ; $r_g$ = radial distance to the centre of mass)

$\varepsilon$ = offset term ($= 3e/[2(1-e)]$)

$\psi$ = azimuth angle

Derivation of Free motion

$g$ = gravitational force
$m$ = mass
$r$ = radial distance from axis of rotation
$\beta$ = flapping angle
$\Omega$ = rotational speed
$\psi$ = azimuth angle




Flapping blade viewed from downwind direction

Okay. Now, what we do in our further discussion that it would inputs may vary as the sign or the cushion of the azimuth angle. will commonly use that as a cyclic. So, essentially cyclics the term that we use that is cyclics of cosine or sine cyclics. Okay. So, essentially these are the cyclic or cyclical inputs of the turbine response.

In this particular figure here, you see two forces act along the axis of the blade, on the element of the blade with mass dm. So, this is what two forces. The gravity component due to the weight of the blade depends on azimuth. Now the equation of motion of a flapping blade without an offset which is subject to restoring forces due to rotation restoring forces due to rotation which is essentially centrifugal force gravity, the inspiring and inertial forces due to acceleration. That is what we will develop here.

Okay. So, essentially now first we will start with the modified equation. So, the modified equation of motion for a blade with an offset is then first discussed and then the full equation would be developed which will include the offset. Now, we will draw another picture here where we will so, one is r square beta omega square dm, r beta double dot dm, the other force is g cos psi dm this is beta, this is r, omega, here you have k beta, okay! this is lapping blade viewed from above okay, now so this is essentially looking down the y double prime axis So, here we also include this flapping in constant and flapping acceleration. So, this includes a beta and flapping acceleration.

OK. So, both of them are included. So, now we can summarize all these forces. So, first you have centrifugal force. So, it starts to bring the flapping blade. Flapping blade.

back into the plane of rotation. So, this magnitude depends on the square of the speed of rotation and it is also independent of blade as in centrifugal force always act on the center of mass of the blade and which is perpendicular to the rotor's axis of rotation. So, now from this particular figure here, what we have drawn, we can write that Fc is r cos beta omega square dm. Okay! And, the moment about the flapping hinge axis, so, the moment about the flapping hinge axis which is Mc and due to Fc we get Mc equals to r sine beta r cos beta omega square dm okay! so, we get both centrifugal force and the moment. So, what it does, so if you see this particular picture here, which is looking down to that Y double prime axis, which includes both the spring constant K beta and the flapping acceleration. And then we summarize all the different forces.

Certifugal force : serves to bring the flapping blade back into the plane of rotation.

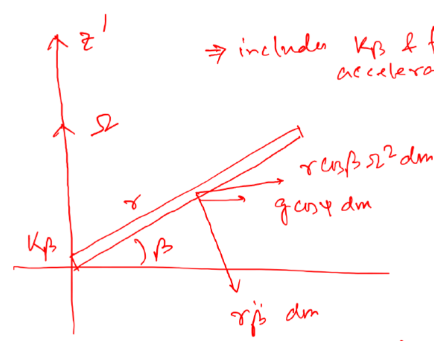
- acts on the centre of mass of the blade (perpendicular to the rotor's axis of rotation)

$F_c = r \cos\beta \, \Omega^2 dm$

Moment about the flapping hinge axis ($M_\beta$) due to $F_c$

$M_c = r \sin\beta \, [r \cos\beta \, \Omega^2 dm]$

⇒ includes $K_\beta$ & flapping acceleration.




Flapping blade viewed from above (looking down the y'' axis)

So, to start with, we have included this centrifugal force which is going to bring back the flapping blade into the plane of rotation so we have both flapping force and the centrifugal force and the moment so, the next one we will consider gravitational force so, acts On the.

Center of the. Center of. Mass.

Of the. Blade. So. When the. Blade is. Up. gravity tends to increase the flapping angle and when down it tends to decrease it. So, this is independent of rotational speed. There is no dependency on the rotational speed. So, this is what we get. essentially now, we have this we want to derive this free motion so, we have this blade that has been kind of given that and then we are considering looking down through the y double prime axis and considering each of these forces start with centrifugal force, then we are talking about gravitational force.

So, we'll stop here and continue this discussion in the next session.