Wind Energy

Prof. Ashoke De

Department of Aerospace Engineering, IIT Kanpur

Lecture 49: Mechanics: Flapping blade model (Free motion)

Welcome back. So, we continue the discussion of this development of the in-spring model, where primarily we're focusing on the flapping flapwise motion. So, in the last session, we have defined this coordinate system where again, just to start with the X, Y, Z is attached to the earth. X prime, Y prime, Z prime is sitting in the turbine itself. And, then X double prime, Y double prime and Z double prime rotates with the blade. And, then we have looked at the top view, define all these things.

So, what we continue that we'll talk about then Flapping blade model. So, this is a dynamic model. Okay, so that we are going to talk about. So, this is a dynamic model.

Okay. So, that uses the int an offset blade to represent a real blade. So, the hinge offset and spring stiffness are chosen such that the rotating hinge and spring blade has the same neutral frequency and flapping inertia as the real blade. Okay! So, before we move to the details of this offset blade model we can look at the dynamics of the simplified hinge blade so, essentially we are focusing on the primarily focusing on the flapwise motion okay! so. That means what we are saying here is that this flapping blade model that we are going to develop, this is going to be a dynamic model which will use this hinge and offset blade to kind of represent as a real blade so that this equations of motion or the expressions could be used.

for analyzing the turbine. Now, the hinge offset and the spring stiffness will be chosen in such a way that the rotating hinge and the spring blade have the same natural frequency and flapping inertia as the real blade. So, that this model parameters constants are going to be chosen such that it represents a real blade so, that what you have kind of an replication or pseudo representation of the real blade so that, this simplified analysis can be used for design purposes. Since, we want to take this detailed offset blade and all this, before that we will look at the simplified one. So, in general, so the blade flapping characteristics So, if we look at the blade flapping characteristics which is a differential equation with constant coefficients would have a equation beta double dot plus function of restoring moments beta equal to function of forcing moments.

Here, the restoring moments are due to gravity, rotor rotation and in spring and the forcing moments are primarily due to yaw motion and aerodynamic. So, what we have is that, so, we have a standard this blade flapping characteristics which would be a simple differential equations with some constant coefficients which represents as here as a beta double dot beta into some function of restoring moments and the function of forcing movement so the restoring moments are caused due to gravity rotor rotation hinge spin and the forcing moments arises due to your motion and aerodynamic forces so, so obviously this linearized hinge spring model development that we are talking about will start with the development of the equation of motion for a few simplified models, assuming no functions. And, then finally we take them together to the actual model. So, it starts, so, We start with development of the equation of motions for a few simplified blade models. obviously, assuming no forcing functions okay so, these equations of free motion so these equations of free motion will have the form as beta double prime plus restoring moments into beta equals to zero.

These eqs. of free motion: $\ddot{\beta} + [f(\text{Restoring moments})]\beta = 0$

So, once we find out the solution of this equation that would provide the characteristic dynamic response of the rotor blade. So, solution will provide the dynamic response of rotor blades ok. I mean, now, we can derive the full hinge spring blade model with all the equation, I mean, free motion. So, the full equation of motion, that kind of includes the forcing moments that requires linearized model for the forcing moment due to wind, yaw motion, yaw error, wind shear and all this. So, once we derive all this term, then the complete equation of motion can be derived by ascending all these different terms.

And then finally, we would find the final form of the flap angle. Now, we would look at the dynamics of a simplified tapping blade model. So, this would be I mean the obviously, the the complete in spring model would be better understood by, by looking at the dynamics of the flapping blade. So, the first we try to understand the dynamics of a flapping blade with no offset so once we understand that then the full model would be easier to understand um so what we are considering so considering the effects of the screen and then of rotation of the blade with and without a spring. So, what we try to understand this dynamics so to understand the complete model we need to understand the flapping blade model and to flapping mode model first we try to understand the flapping blade model with no offset so that once we understand that or try to derive that equation then we will move to include the offset and all this so but while doing this we do consider the effect of the spin and the rotation of the blade with or without spin okay! so, the first situation to start with is a spin no rotation no offset so, that's the situation to start with.

So, this is first case to consider the natural frequency, flapwise frequency. So, first case to consider is the natural frequency which is flapping frequency or one can say flapwise frequency of a non-rotating blade hinge system blade hinge system so essentially try to find out the, so, there could be uh so, this is uh essentially it is analogous to spin mass damper system as we have discussed earlier and that is why we discuss the that spring mass damper system would be using. So, then what we can find out, we can find out the natural frequency for vibration about the flapping hinge through the equation. So, the equation of motion would be beta double dot minus K beta by Ib into beta. So, here IB is the blade mass moment of inertia about the cupping axis.

Spring, No Rotation, No offset

· first case to consider is the natural frequency (flapping / flapwise) of a non-rotating blade-hinge system.

(analogous to spring-mass damper system)

eq: $\ddot{\beta} = -(K_\beta / I_b)\beta$: I_b = blade mass moment of inertia about the flapping axis

K_β = flapping hinge spring cont.

$\omega_{NR} = \sqrt{K_\beta / I_b}$: natural flapping freq. of non-rotating blade

and k beta is the trapping H string constant. So, the omega nr would be root over of k beta which is the natural flapping frequency of non-rotating blade. I mean, that's what we discussed the basic equations of the mechanics and the dynamics because now we can have the analogy between that with this while deriving the equation for the turbines. So, what we have, have this flatwise motion and find out the natural frequency of non-rotating blade in this case we are only considering this spring there will be no rotation no offset so, it's a absolutely simplified system and that's what you can simply have a correlations with the spring mass damper system and from there you can find out this natural frequency obviously, there are assumptions which we have one of the assumptions is that the blade has a uniform cross section. So, the mass moment of inertia of a blade of mass mB obviously with no offset we find out Ib 0 to R r square dm which is 0 to R r square mB by R into dr which gives us mB R square by 3.

$$I_b = \int_0^R r^2 dm = \int_0^R r^2 (m_B / R) dr \Rightarrow m_B R^2/3$$

Okay! so, that's what we have now we move to a case where we consider now the we consider rotation now we consider rotation but no screen and no offset so we are taking one effect at a time so when When the blade is rotating with a hinge at the axis of rotation and has no spring the flap wise natural frequency is the same as the speed of rotation. So, this can be seen like here the only restoring force is the centrifugal inertia, which is let's say Fc and its magnitude is proportional to the blade speed and cosine of the flapping angle. So, the restoring component can be determined by the sign of flapping angle. So, what we can write is that IB beta double dot 0 to R sine beta dfc which is 0 to R, R cos beta omega square r sine beta row blade dr okay so again assuming small angles this is a small angle approximation So, we have cos beta would be 1, sin beta would be your beta. So, what we will write beta double dot equals to minus of omega square by Ib, 0 to R, R square Bm which is minus omega square beta.

So the solution to this equation would be omega equals to capital omega. So that's what we said. Since there is only rotation associated with that and there is no spring, no offset, it's just the blade rotating around with a hinge and the axis of rotation. So the flapwise natural frequency would be exactly the same on the speed of rotation. so this is what we can see because the restoring force is only the centrifugal inertia so once we use that we can find that out okay so now we want to make it now we consider two effect now we consider rotation we consider but still no offset.

Only restoring force (centrifugal inertia) : F_c

We can write : $I_b\ddot{\beta} = \int_0^R [-r\sin\beta]\,dF_c = \int_0^R [r\cos\beta\,\Omega^2][-r\sin\beta]\,\rho_{blade}\,dr$

assuming small angles : $\cos\beta \simeq 1$, $\sin\beta \simeq \beta$

$$\ddot{\beta} = -(\Omega^2/I_b)\,\beta\int_0^R r^2\,dm = -\Omega^2\beta$$

- soln : $\boxed{\omega = \Omega}$

So, so far we have looked at effect of rotation alone. We have looked at the effect of spring alone. Now we will look at effect of both rotation and spring and but without any offset. So here, so the blade has no offset so it is only inch and also it includes a spring So the natural frequency can be determined by the sum of the spring solution and the rotation solution. So what we have is that approximate equation is can be written as beta double prime plus A beta by IB plus omega square to beta equals to zero.

So the solution that we have omega r square equals to k beta by i beta which is omega non-rotation plus omega square. So, here omega r is the rotating natural frequency. and omega nr is the non-rotating natural frequency so what one can see that the rotating natural frequency is greater than omega nr because there is a component so that is why We say that the rotation defends the blade. You can see that the natural frequency is higher, and that's why it can defend the blade.

So now what we... consider is that we'll consider the dynamics of lapping blade with offset. Now we'll include the offset. So now what we can do is that extend the analysis without offset. So what we have done is that first start with a simplified model where there is no offset and then we had initially just only considered the spin and find out the and no rotation then non-rotational natural frequency we have found out. then we incorporated the rotation but that time we have considered no spring is attached so it is only a rotation so that the natural frequency is the same with the rotational speed and then finally we include both the spring and the rotation but still the whole system is without any offset and that shows that the rotating natural frequency is always higher than the non-rotating natural frequency And that kind of states or suggests that why due to rotation the blades get more and more stiffness and that is why their natural frequency becomes higher.

Rotation, Spring, No Offset

- blade has no offset - it is only hinged and also includes a spring.

Approx. eqn: $\ddot{\beta} + \left(K_\beta / I_b + \Omega^2 \right) \beta = 0$

soln: $\omega_R^2 = K_\beta / I_b + \Omega^2 = \omega_{NR}^2 + \Omega^2$

ω_R = rotating natural frequency

ω_{NR} = non-rotating natural freq.

Thus, $\omega_R > \omega_{NR}$ $\rightarrow$ Rotation 'stiffens' the blade

Now we would like to take that this fact that we have derived now we would extend that for deriving the flapping grid with offset. So we'll continue that discussion in the next session. Thank you.