Wind Energy

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Lecture 43: Mechanics: Dynamic Beam analysis

Welcome back. So, we continue the discussion of this eye oscillation and eigenmode where initially we looked at the spring mass damper system and then we are talking about this two degrees of freedom or more than one degrees of freedom system where your displacement could be a vector and you essentially deal with this kind of matrix system and the solution vector could be found using Raleigh method. So, now we have already looked at based on the guess values this Raleigh method can be quite accurate for estimation of this frequency of oscillation. Now, what we would like to look at is the error of Rayleigh's method. So, we can estimate the error of Rayleigh's method. So, what we can say that assume, let's say omega naught which belongs to a real number and omega naught belongs to Rn are the true eigen pairs.

So, that means what that is they satisfy. So, once we assume that this omega naught and omega naught are the true eigen pair, then they satisfy this equation k. naught omega naught square m w naught okay now what let's say w the actual eigen vector is w naught plus delta w and where delta w is the error in our guess. So, what we get then essentially what we get omega square which is half of w bar transpose a w bar half of w bar transpose in w bar so this is f of w bar then what we can have above w naught transpose a w naught m w naught.

So, this is f of w naught square plus this is w naught transpose into del w square. So, what we have here here del f of w naught is half of W naught transpose M W naught K W naught half of W naught transpose K W naught M W naught half of square, which is k w naught minus omega naught square m w naught, half of w naught transpose 0. what we, so, the error that you have that is omega square omega square omega naught square plus w square which is essentially order of I mean is essentially second order. Okay. So, we now, we can see for no different example we can see let's say we have x1 this is m2 x2 this is k2 this is k1 so this is m 2 x 2 double dot k 2 x 2 minus x 1 0 m 1 x 1 double dot k 1 x 1 minus k 2 x 2 minus x 1 0.

Error of Rayleighs Method

Assume: $\omega_0 \in \mathbb{R}$ & $W_0 \in \mathbb{R}^n$ are the true eigen-pair, that is, they satisfy

$$KW_0 = \omega_0^2 M W_0 \quad , \quad \tilde{W} = W_0 + \Delta W \quad (\Delta W = \text{error in our guess})$$

We get,

$$\omega^2 = \underbrace{\frac{\frac{1}{2}\tilde{W}^T K \tilde{W}}{\frac{1}{2}\tilde{W}^T M \tilde{W}}}_{:= f(\tilde{W})} = \underbrace{\frac{\frac{1}{2}W_0^T K W_0}{\frac{1}{2}W_0^T M W_0}}_{:= f(W_0) = \omega_0^2} + \nabla f(W_0)^T \Delta W + O\left(\|\Delta W\|^2\right)$$

$$\nabla f(W_0) = \frac{\left(\frac{1}{2}W_0^T M W_0\right) K W_0 - \left(\frac{1}{2}W_0^T K W_0\right) M W_0}{\left(\frac{1}{2}W_0^T M W_0\right)^2} = \frac{KW_0 - \omega_0^2 M W_0}{\left(\frac{1}{2}W_0^T M W_0\right)} = 0$$

error: $\omega^2 = \omega_0^2 + O\left(\|\Delta W\|^2\right) \longrightarrow$ 2nd order

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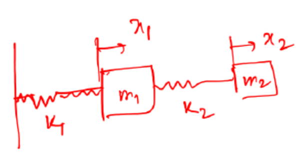
so my either the vector w vector is x 1 and So, this belongs to R2. Then, if we write down the matrix equation, this would be M1, 0, 0, M2, W dot, K1 plus K2, K2 minus K2, W equals to 0. So, this is nothing but M which is belongs to 2 by 2 this is belongs to K matrix also 2 by 2 so here two different system what you can see is that so similarly we can look at another example where let's say M1 X1 which is A1 m2 x2 which is two millimeter let's say one millimeter this is k2 okay this is another um system okay so, what you can see is and wt is w bar e to the power j omega t you can assume m2 greater than m1 k1 is k2 so this is one two which are the eisenvector or eigenvector then we have wt j omega t 2 into j omega t so, what we can have kinetic energy would be half w bar transpose m w bar m 1 plus 4 m 2 So, now this is the kinetic energy expression that you get, then what you have is like potential Similarly, we can find out the potential energy. That would be half of W bar transpose A W bar. So, here what we can do now, we can find out the potential is kW bar.

ex.

$$m_2\ddot{x}_2 + k_2(x_2 - x_1) = 0$$

$$m_1\ddot{x}_1 + k_1 x_1 - k_2(x_2 - x_1) = 0$$



$$W = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2$$

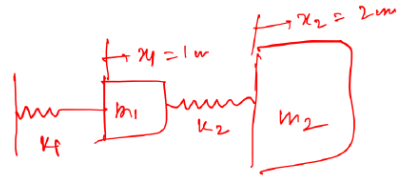
$$\underbrace{\begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}}_{:= M \in \mathbb{R}^{2\times 2}} \ddot{W} + \underbrace{\begin{bmatrix} (k_1+k_2) & -k_2 \\ -k_2 & k_2 \end{bmatrix}}_{:= K \in \mathbb{R}^{2\times 2}} W = 0$$

$$w(t) = \bar{w} \cdot e^{j\omega t}$$

Assume! $m_2 \gg m_1$, $k_1 \simeq k_2$

$$\bar{w} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (\text{eigenvector})$$

$$w(t) = \begin{bmatrix} e^{j\omega t} \\ 2 \cdot e^{j\omega t} \end{bmatrix}$$



$$E_{kin} = \frac{1}{2} \bar{w}^T M \bar{w} = \frac{1}{2} (m_1 + 4m_2) \omega^2 A_0^2$$

$$E_{pot.} = \frac{1}{2} \bar{w}^T K \bar{w}$$

So, this one can write half of A naught square 1 2 transpose k1 plus k2 minus k2 k2 k2 1 2. Half of A naught, 1, 2 transpose, A1 plus K2, minus, 2 K2, 2 K2, half of A naught, A1 minus K2 plus, 2 K2, which is above A naught, k1 plus k2 so omega square is k1 plus k2 m1 plus 4 m2 k2 by 4 k1 by 2 by m2 so this is the estimation for the kinetic energy So, that's how you get. So, we have looked at two different examples of multiple degrees of freedom and finding the eigenvectors and estimating the kinetic energy and potential energy, things like that. so, these are all what we are looking at it is that first i mean the basic structural analysis in terms of static beam equation then we see that how that gets applied to the rotor blade and then we look at the basic vibration analysis for a single spring mass damper system and then two degrees of freedom spring mass damper system. From there, we would extend this analysis to other, we call the other rotor blade thing.

$$E_{potential} = \frac{1}{2} \bar{w}^T K \bar{w} = \frac{1}{2} A_0^2 \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}^T \begin{bmatrix} (k_1+k_2) & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$= \frac{1}{2} A_0 \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}^T \begin{bmatrix} (k_1+k_2) - 2k_2 \\ -k_2 + 2k_2 \end{bmatrix}$$

$$= \frac{1}{2} A_0 (k_1 - k_2) + 2k_2$$

$$= \frac{1}{2} A_0 (k_1 + k_2)$$

$$\omega^2 = \frac{k_1 + k_2}{m_1 + 4m_2} \simeq \frac{k_1 + k_2}{4} \cdot \frac{1}{m_2} \simeq \frac{k_1}{2} \cdot \frac{1}{m_2}$$

But now what we would look at the dynamic beam equation. Okay. So, now we'll look at the dynamic beam equation. Okay. So, we start with the dynamic beam equation.

Okay. So, we have earlier seen the Euler Bernoulli's equation. Now, this is a combination of Euler Bernoulli and Lagrange kind of produce the dynamic beam equation so obviously since it's a dynamic beam equation this depends on time and hence the equation can be written as del 2 by del x 2 e of x i of x, q of xt minus mu of x del 2. So, that is what is kind of represented as dynamic beam equation. So here, what you have, you have mu of x, which is mass density per length, per length, U of Xt, which is distributed load. Omega of Xt, which is time varying solution.

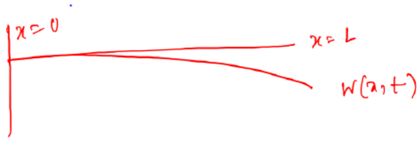
No damping. Okay. So, what we have, it will look at it. This goes like that x equal to L. And this is w of x t.

This is x equals to zero. Okay. so, this particular dynamic beam equation that we have this is a linear partial differential equation so after special discretization we can have a of w m of w dot so essentially where E of kinetic energy 0 to L m of x del by del t square into dx. And elastic energy up of 0 to L e x del to w by del x to square dx.

Euler-Bernoulli & Lagrange $\rightarrow$ $\underbrace{\text{Dynamic beam eq.}}_{\text{depends on time, and hence,}}$

$$\boxed{\frac{\partial^2}{\partial x^2}\left(E(x)\cdot I(x)\frac{\partial^2 w}{\partial x^2}\right) = q(x,t) - \mu(x)\frac{\partial^2 w}{\partial t^2}}$$

$\mu(x)$ = mass density/length
$q(x,t)$ = distributed load
$w(x,t)$ = time varying soln.
(no damping)



Linear PDE, after special discretization, we can have:

$$\boxed{Kw = -M\ddot{w}}$$

Where,

$$E_{kin} = \frac{1}{2}\int_0^L M(x)\cdot\left(\frac{\partial w}{\partial t}\right)^2\cdot dx$$

$$\underline{E_{ela}} = \frac{1}{2}\int_0^L E(x)\, I(x)\cdot\left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx$$

Okay. So, that means from the dynamic equation, one can estimate the kinetic and elastic energy as well. Now, we can find out tower is in mode because both tower and nacelles have mass. So, obviously, one can look at like this, where So this is kind of a lowest again mode. And, one can see Is the.

Second lowest. Eigen mode. So. So, here. Since both tower and nacelles. Have mass. So, one has to. Compute. So, the eigen mode.

So, one. Needs to. Compute. eigenmodes for a very unequal mass distribution. So, this is how the two lowest eigenmodes may look like. Okay. So, now what essentially we're trying to say here is that, again, now we are extending our fundamental analysis of vibration to our rotor blade.

But along with that, there is a structural constraint here because the tower and the nacelle, they are also having some mass. So, which is kind of represented here, which is just like a hammer with a rod and with a hammer, but they are having some kind of an oscillation. So, their eigenmode needs to be also considered. So, I mean, later on, when we'll talk about the linearized model and all this analysis for that, the idea of what we are talking here is that initially look at this basic example of calculations of this frequency and all these things, but later on then considering the aerodynamic forces, we look at the simplified model to analyze all this. So, this is for the tower Eigen mode that we have talked about.

Now, we can estimate this the kinetic energy and potential elastic energy using this dynamic beam equation so, here the issue is that the whole structure along with the root is exposed to dynamic loading so that is why they need to be taken into consideration So, now the kinetic energy is half of mu x x t by del x 2 sorry del t square dx and uh elastic potential energy e pot or e elast half of zero to y e of x i of x uh x t by del x 2 square dx okay so for this let's say assume omega x t is omega bar x into e to the power j omega t with omega bar x is a naught x square by l square so this is for a wrap approximation of the lowest eigenmode and assuming constant mass mu x e x and i x throughout the tower so,

lowest eigenmode

2nd-lowest eigenmode

Kinetic energy: $E_{kin} = \frac{1}{2}\int_0^L \mu(x) \left(\frac{\partial w(x,t)}{\partial t}\right)^2 \cdot dx$

Elastic pot. " : $E_{pot} = \frac{1}{2}\int_0^L E(x)\, I(x) \left(\frac{\partial^2 w(x,t)}{\partial x^2}\right)^2 dx$

Assume: $w(x,t) = \bar{w}(x)\cdot e^{j\omega t}$, with $\bar{w}(x) = A_0 \frac{x^2}{L^2}$ — for a rough approximation of the lowest eigenmode, and assuming const. mass $\mu(x)$, $E(x)$, & $I(x)$ throughout the tower.

Using Rayleigh's Method.

what we will get now we can using rallies method Raleigh's method we can get E kinetic energy is omega square half of 0 to L M tower A naught by L square dx m nacelle a naught square so what we have kind of done here if you see so one component this coming from power and this is the component coming from so both the component of this the whole unit they are now considered and since they are exposed to this dynamic condition so we try to estimate using this dynamic equation and estimate the kinetic energies and elastic potential energy so important point here to note is that not only the rotor blades and all these things because of the wind also your these things get exposed the structure and the other thing so we will finish this discussion or continue the discussion in the next session.

$$E_{kin} = \omega^2 \cdot \left(\underbrace{\frac{1}{2} \int_0^L \frac{m_{tower}}{L} \left(\frac{A}{L^2}\right)^2 (x^2)^2 \, dx}_{\text{Tower}} + \underbrace{\frac{1}{2} m_{nacelle} A_0^2}_{\text{nacelle}} \right)$$

Thank you