Wind Energy

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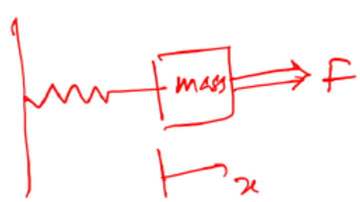
Lecture 42: Mechanics: Oscillations and Eigenmodes

We'll come back so we'll continue the discussion uh on this structural analysis so we have looked at the static beam equation and then also looked at how the for constant loading and distributed loading for cantilever beam how that acts and then try to estimate the bending moments at the root. So essentially, from or using the basic structural mechanics equation, we have kind of tried to correlate with the puts us on the blade roots and that now what we will continue is the oscillations and eigenmode okay so essentially the vibration part so oscillations and eigenmodes So, this essentially the vibration of the body that we would like to continue to. Obviously, here while talking about that first thing that we'll introduce, we'll introduce the spring mass damper system so the spring mass damper system will first introduce which is essentially a simple kind of a this is mass and this is where the force is applied this is how when the spring is exposed to a external force so then and the forces relate the body actually oscillates this typical fundamental uh vibrational analysis so they follow simple um equation where m x double dot plus beta x dot plus kappa x equals to f of t. So that's a so essentially with time the force which is applied and here the few things that is x is displacement displacement m would be mass f of t is external force a of x is being forced beta is damping parameter which could be viscous damping or linear damping so you have all these parameters for the analysis but one can now so once we find out the solution for this particular system and then from there we can again try to correlate with our rotor system and there so how we can go about it so let's say for f of t which is given at f naught e to the power j omega t where if not greater than zero and usually we take the real part of the design, real part of the solution and design okay so now the solution can be written.



$$m\ddot{x} + \beta\dot{x} + \kappa x = F(t)$$

For, $F(t) = F_0 \cdot e^{j\omega t}$, where $F_0 > 0$

so here the solution which can be written as x t x naught j omega t okay where x naught belongs to complex then x dot is j omega e to the power j omega t and you have x double dot minus omega square j omega t now if we put that back in that basic equation of spring mass damper system of this equation so what we have is that m x naught e to the power j omega t plus beta j omega e to the power j omega t kappa x naught j omega t f naught j omega t. So, what we can write x naught a minus m omega square j beta omega equals to f naught. So, this portion is the real part of the solution.

This is imaginary part of the solution. So, here x naught is a complex number. x naught is a complex number with magnitude of x naught would be f naught root over of k minus omega square beta square omega square so if we try to take the maximum x naught that is maximum x naught magnitude is approximately taken at neutral resonant which is Eigen frequency omega nr and we can say that k minus m you know, square equals to zero. Omega nr is root over kappa by m.

So that is what you get for this. Now, the question is that how much this if not can be amplified. Okay. So, we can say that how much can if not be amplified. So, our being forced which is if not in this game to X so what we have magnitude of if not spring is K magnitude of X naught which is F naught by one minus omega by omega nr square plus beta square by k square omega square.

Soln. $x(t) = x_0 e^{j\omega t}$, $x_0 \in C$

$\dot{x} = (j\omega)\cdot x_0 e^{j\omega t}$

$\ddot{x} = -\omega^2 x_0 e^{j\omega t}$

$$-m\omega^2 x_0 e^{j\omega t} + \beta j\omega x_0 e^{j\omega t} + K x_0 e^{j\omega t} = F_0 e^{j\omega t}$$

$$x_0 \cdot (\underbrace{K - m\omega^2}_{\text{real}} + \underbrace{j\beta\omega}_{\text{imaginary}}) = F_0$$

$x_0 \to$ complex number

$$|x_0| = \frac{F_0}{\sqrt{(K-m\omega^2)^2 + \beta^2\omega^2}}$$

Max. $|x_0|$ is approximately taken at neutral resonant "Eigen frequency" ω_{NR}

$$K - m\omega_{NR}^2 = 0 \Leftrightarrow \omega_{NR} = \sqrt{\frac{K}{m}}$$

How much can F_0 be amplified?

Spring force $F_0^{spring} = K\cdot x \to |F_0^{spring}| = K|x_0| = \dfrac{F_0}{\sqrt{\left(1-\left(\frac{\omega}{\omega_{NR}}\right)^2\right)^2 + \frac{\beta^2}{K^2}\omega^2}}$

So, what we get, let's say at omega is omega nr we get f naught spring a by beta omega so which clearly so this clearly tells one thing smaller the damping the higher is the amplification okay so if you can like plot this let's say omega let's say this is omega nr this is f naught being by f naught This is K by beta omega nr. Okay. And this is 1. This is known as Bode diagram of F0 spin of omega divided by f naught so typically the amplification factor can be 5 to 10. so amplification factor can be 5 to 10.

so the resonance cell typically be avoided. So, resonance typically be avoided. Now at very low frequencies in force equals applied force that is our static analysis is sufficient. So, which you can correlate that how this resonance and amplifications are connected and that can be avoided and where your static analysis is sufficient to estimate that this. So, where we are heading with this particular thing is that, so this is kind of giving you an idea about the simple spring mass damper system that you have.

at $\omega = \omega_{NR}$, we get, $\dfrac{|F_0^{spin}|}{F_0} = \dfrac{K}{\beta \omega_{NR}}$

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smaller the damping, the higher is the amplification.

$\Leftarrow$ Bode diagram of $\dfrac{|F_0^{spin}(\omega)|}{F_0}$

and their oscillations and oscillations frequencies you can estimate. And once you can estimate that, then we'll use this information or the similar way to correlate when the turbine blades exposed to this kind of vibration. So, that is the whole idea that how we correlate this natural frequencies or eigenmodes and all these things. So, this gives us the oscillation frequency or eigenfrequency and natural eigenfrequency. now we can find out the eigen modes and that okay so now we look at again modes okay now for spin mass damper system with more than one degree of freedom.

So, we can have the displacement which can be so the displacement would be represented as vector, let us say Wt, which belongs to Rn. And the equation of motion becomes M w dot d w dot k w equals to f of p. So the previous analysis of spring mass damper system, we had one degrees of freedom. So, essentially, if you try to schematically draw that then now here in this equation what we have M is a mass matrix which belongs to our.

In my in. D belongs to. Damping mattress. K belong to. It's just metrics.

$$\boxed{M\ddot{w} + D\dot{w} + Kw = F(t)}$$

M = mass matrix, $\in \mathbb{R}^{n\times n}$

D = damping "

K = stiffness ", $\in \mathbb{R}^{n\times n}$

If damping is neglected $(D = 0)$, natural resonances must satisfy $\bar{w} \in \mathbb{R}^n$

$$w(t) = \bar{w}\, e^{j\omega t} \rightarrow M\ddot{w} + Kw = 0$$

That is, $-\omega^2 M\bar{w} + K\bar{w} = 0 \Leftrightarrow (M^{-1}K - \omega^2 I)\bar{w} = 0$

With. In my. No. what if damping is neglected that is D0 then natural resonances must satisfy that Rn. So, what we have in the solution vector Wt, W e to the power omega t. And we have M w double dot plus k w equals to 0. and what we can write m equals to zero m inverse k minus omega square i a zero.

So, this is an eigenvalue. So, this is an eigenvalue equation. So, eigenvalue equation for matrix M inverse kappa N by N. and, we know there are n eigenvalues with n eigenvectors which is essentially eigenmodes. Now, here, interestingly, since these are now we are dealing with the metric system.

So, as both M and K are positive definite Eigen values of N inverse K are real and positive. So, what we are interested is often or often we are often only interested In the. I can.

With. Lowest. As in frequency. Okay. So, this is what we are essentially. Now the. Thing is that once you get the eigen modes.

how you can estimate the solution of this. So, the point here is that in multiple degrees of freedom case, we are dealing with a metric system. And from the metric system, we try to find out the solution. So now, how we find out the solution? So, there is a method called Raleigh method.

So, we use that. Let's say Raleigh's method. Okay. Obviously, we have a, good guess of an eigen mode vector is w bar Rn. And, now to find the corresponding omega square. So, we have to find to find the corresponding omega square we can use the equation which is k w bar omega square Now let's say let's say this could be over data mint if W bar is fixed.

Okay. Then we can multiply by half of W bar transport and that gives that half of w bar transpose k w bar is omega square half of w bar transpose m w bar. So, this system here, this is over determinant if that vector w bar is fixed. then we can multiply by this hat of W by transpose which gives us this. Now, this component is essentially nothing but elastic or potential energy at maximum displacement. And this is kinetic energy at max speed, which is zero displacement.

Rayleighs Method

Assume, we have a good guess of an eigenmode vector, $\bar{w} \in \mathbb{R}^n$.

To find the corresponding $\omega^2 \in \mathbb{R}$, we can use the eqn's:

$$K\bar{w} = \omega^2 M\bar{w}$$

→ overdetermined if $\bar{w}$ is fixed, then we can multiply by $\frac{1}{2}\bar{w}^T$, that gives:

$$\underbrace{\frac{1}{2}\bar{w}^T K\bar{w}}_{:=\ \text{elastic/potential energy at max. displacement}} = \omega^2 \cdot \underbrace{\frac{1}{2}\bar{w}^T M\bar{w}}_{:=\ \text{kinetic energy at max. speed (zero displacement)}}$$

$$\omega = \sqrt{\frac{\frac{1}{2}\bar{w}^T K\bar{w}}{\frac{1}{2}\bar{w}^T M\bar{w}}} := f(\bar{w})$$

So, from here, what we can get at Omega is half of W bar transpose A W bar divided by half of W bar transpose M W bar. I mean, one has to be careful that you cannot cancel this numerator and denominator here. You cannot cancel this numerator and denominator because you are dealing with matrix here. You are not dealing with scalar anymore. So, these are kind of a vector.

So, what it gives you that if the guess of W bar is good, um the method is surprisingly so what we can say that let's say if the guess of w bar is good the can be surprising accurate estimation of omega so that's what it does so essentially again just to point to note here when you try to find out this omega you cannot cancel each other out because you are dealing with the Matrix so all depends on this that what is the because we started with the guess of the eigenvector w bar the vector eigenvector w bar for the guess values so if the guess is good this particular method of rally method of analysis gives you a very good rate okay so we'll We can also look at the error of this method and other things and then try to extend for different example. We will continue that discussion in another session.