Wind Energy

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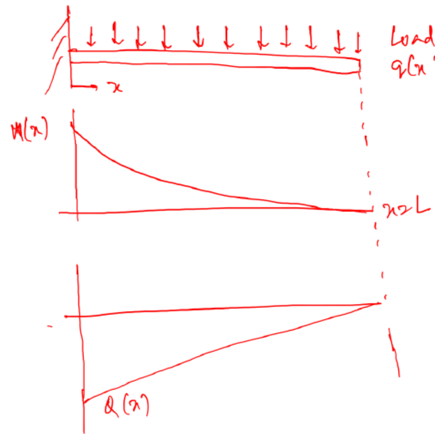
Lecture 41: Mechanics: Beam analysis

Yeah, we'll come back. So, we'll continue our discussion on this mechanics and the dynamics of the wind turbine. So, the last session, what we discussed about the Euler beam equation, okay, essentially the static beam, essentially static beam equation, or Euler Bernoulli equation. And from this equation, we have looked at one of the example, which is cantilever beam with end load. Now, we'll continue the same and look at other example of cantilever beam. with constant loading.

So, we had this. So, second example that we will look at it cantilever beam with constant loading. okay so, what we can we can draw the picture for cantilever beam with constant loading so, let us have that let's say this is the beam and you can have continuous loading okay which is let's say, uh load q and q of x so this is x q of x is constant along beam so, what we have ux equals to q constant and EI is also constant okay and so, we can have this diagram we can get this diagram once we find out the system okay so, let's keep that so what we have here u by e dot i is d4 by dx4 okay So from here what we have this Q by E dot I X4 by 24 C3 X cube C2 X square C1 X plus C naught. So, this is what we get as a solution.

now we can put the boundary condition so like we have at 0 0 which gets c 0 equals to 0 and also we have d of is 0 that gets c 1 equals to 0 so we will have m of x equals to d2w by dx2 e dot i and where m at l is 0 d2w by L is zero. So, what we get QX equals to DMX by DX and Q of L that means D3W by is zero. So, let's say this is Solution two, now what we can have, we can use this boundary conditions and what we can write, let's say from equation two, we can write x squared by two plus six three x plus two c two at x equals to l equals to zero.



Load - q
$q(x)$ = Const. along beam

$q(x) = q$ = Constant
E, I = Cst.

$$\frac{q}{E \cdot I} = \frac{d^4 w(x)}{dx^4} \quad - - - - \quad (1)$$

$$\Leftrightarrow \quad w(x) = \frac{q}{E \cdot I}\left(\frac{x^4}{24} + c_3 x^3 + c_2 x^2 + c_1 x + c_0\right) \cdots (2)$$

B.C.'s : $w(0) = 0 \Rightarrow c_0 = 0$

$\frac{dw(0)}{dx} = 0 \Rightarrow c_1 = 0$

$$M(x) = \frac{d^2 w}{dx^2} \cdot E \cdot I \quad \rightarrow \quad M(L) = 0 \Rightarrow \frac{d^2 w}{dx^2}(L) = 0 \cdots (3)$$

$$Q(x) = \frac{dM(x)}{dx} \quad \rightarrow \quad Q(L) = 0 \Rightarrow \frac{d^3 w}{dx^3}(L) = 0 \cdots (4)$$

We can write, essentially let's say two this is three this is four so this is from three from equation four we can write x plus six three minus one by six then from here what we get l square by 2 minus l square we get 1 by 4 l square so eventually we get its q by ei x 4 by 24 l by 6 x cube in squared x squared which is u x squared by ei x squared minus or lx then mx equals to ei L squared by two. And, we have u x equals to u into x minus L.

So these are the solution for that. And, if you now plot this, this is x equal to L. And this is MX. This goes like like that. And this is your.

From eq. (3): $\quad \frac{x^2}{2} + 6C_3 x + 2C_2 \Big|_{x=L} = 0$

From eq (4): $\quad x + 6C_3 \Big|_{x=L} = 0 \Rightarrow C_3 = -\frac{1}{6}$

$$\frac{L^2}{2} - L^2 + 2C_2 = 0 \Rightarrow C_2 = \frac{1}{4}L^2$$

$$w(x) = \frac{q}{E\cdot I}\left(\frac{x^4}{24} + \frac{L}{6}x^3 + \frac{1}{4}L^2x^2\right)$$

$$= \frac{qx^2}{EI\cdot 24}\left(x^2 - 4Lx + 6L^2\right)$$

$$M(x) = E\cdot I\cdot\frac{d^2w}{dx^2} = q\cdot\left(\frac{1}{2}x^2 - Lx + \frac{L^2}{2}\right)$$

$$Q(x) = q(x-L)$$

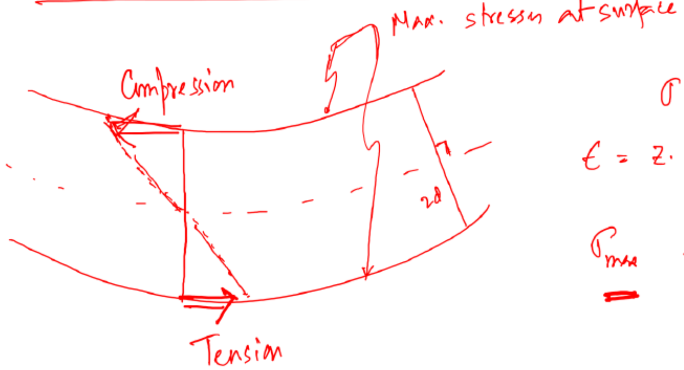
You have X. So the card would. Look like. That.

Okay. So, this is how you have the this beam with constant loading. You can have. Now we can. see the maximum stress at boundary.

Okay. So, we had this This is compression. This is tension. So, this is max stresses at surface. This is 2D. Now, what you have sigma is E into epsilon.

Epsilon is small z, d2w by dx2, mx, i of x, z equal to d, sigma max, ed, mx, yeah so that's uh gives you an idea about the maximum stress at the surfaces where it's getting active. Now, we can see some, I mean, we can expect some numerical value. Let's say, if sigma max equals to 250 MPa, and D equals to one meter. So, one can find out M max, what would be M max? So, here, since D equals to one meter, you have I, which is, pi by four d four or i by d pi by four d cube m max would be sigma max i by d that is 250 into pi by four into one cube So effectively you get two into 10 to the power eight Newton meter. You can see a very high moment. So, typically a higher moment will lead to plastic deformation. So, that is what is going to happen.

Maximum stress at Boundary



$$\sigma = E \cdot \epsilon$$

$$\epsilon = z \cdot \frac{d^2 w}{dx^2} = z \cdot \frac{M(x)}{E \cdot I(x)} \qquad (z = d)$$

$$\underline{\underline{\sigma_{max}}} = E \cdot d \cdot \frac{M(x)}{E \cdot I(x)} = \frac{d}{I(x)} \cdot M(x)$$

eg. If $\sigma_{max} = 250\,\text{MPa}$, and $d = 1\,\text{m}$, $\longrightarrow$ $M_{max} = ?$

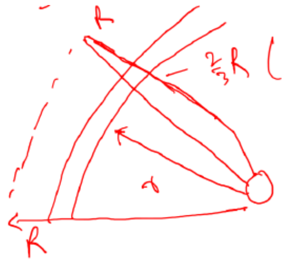
$$I = \frac{\pi}{4} d^4, \quad \frac{I}{d} = \frac{\pi}{4} d^3$$

$$\underline{\underline{M_{max}}} = \sigma_{max} \cdot \frac{I}{d} = 250 \times \frac{\pi}{4} \cdot 1^3 \simeq 2 \cdot 10^8\ \text{Nm}$$

$\longrightarrow$ A higher moment will lead to plastic deformations.

Now we can look at what we can do look at the loads at blade so we can look at the loads at blade root okay so in clockwise direction so So typically for a blade in an ideal design, the distributed load, which is Q of R is given by one by b the thrust of the corresponding annulus the first the first of the corresponding annulus okay So, if we recall, let's say if this is the root, and there is a grid like that, typically, and this can be R and this is two-third of R. So essentially at this point assuming all first force is and B is number of blades let's say 3 and this would be R complete that's the radius of the rotor blade okay so what we have already have df equals to 4a 1 minus a half rho q infinity square 2 pi r dr which is essentially this was corresponding to coefficient of thrust into a half rho u infinity square 2 pi r dr that was equals to b into q of r and q of r is So, this already we have seen. Now, what one can do, now the bending moment bending moment at the blade root that is r equals to 0 can be found as m of 0, 0 to r, q r, r dr.

So what we can write 1 by B CTA up rho Q infinity square 2 pi 0 to R R square D R. This is R cubed by 3. So, what we have 1 by B cta of rho u infinity square into two-third pi r cubed one by b two-third r cta of rho u infinity square. So, this is essentially our total force on actuator disc. This is total force on actuator disc.



$\frac{2}{3}R$ (assuming all thrust force is concentrated here)

B = no. of blades = 3

$$dF = \underbrace{4a(1-a)}_{G(a)} \cdot \frac{1}{2}\rho U_\infty^2 \cdot 2\pi r \cdot dr$$

$$= \underbrace{G(a) \cdot \frac{1}{2}\rho U_\infty^2 \cdot 2\pi r\, dr}_{\Rightarrow B \cdot q(r)}$$

$$q(r) = G(a) \cdot \frac{1}{2}\rho U_\infty^2 \cdot 2\pi \cdot \frac{1}{B}$$

Bending moment at the blade root ($r=0$), can be found as

$$M(0) = \int_0^R q(r)\, r\, dr = \frac{1}{B} \cdot G(a)\, \frac{1}{2}\rho U_\infty^2 \cdot 2\pi \cdot \underbrace{\int_0^R r^2 dr}_{:= \frac{R^3}{3}}$$

$$= \frac{1}{B} \cdot G(a)\, \frac{1}{2}\rho U_\infty^2 \cdot \frac{2}{3}\pi R^3$$

$$= \frac{1}{B}\, \frac{2}{3} \cdot R \cdot \underbrace{G(a) \cdot \frac{1}{2}\rho U_\infty^2 \cdot (\pi R^2)}_{:= F_T(a)} \text{ (Total force on actuator disc)}$$

And similarly, what you can have the shear force, shear force at blade root is trivially bound by U of zero Ft by B. So, one can think about that moment is M of zero is Ft by B which is total force on blade into two-third R that is two-third of radius that is what we get the moment. I mean provided here the assumption that is there that assuming all forces acting on And, we get the right bending moment. I mean, one can illustrate with a simple example, let's say. And, we have a blade like that at R0.

So, here you have q0, here you have moment 0. And everything acting at, let's say, 2 third by r. Then f t by 3 at 2 3 by r on each blade. OK. So, what you have at P equals to P by 1 minus A into U infinity.

shear force at blade root is trivially found by: $Q(0) = F_T/B$.

$M(0) = \frac{F_T}{B}$ (total force on blade) $\times \frac{2}{3}R$ ($2/3$ of radius)

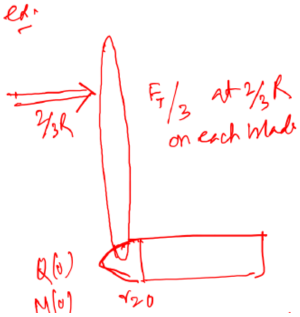
(assuming all forces acting on $\frac{2}{3}R$)

So, let's say 1 mega Newton or 6 megawatt at U infinity equals to 9 meter per second and radius is 75 meter. then, what will happen is that m of zero two-third by r into which is kind of 16 mega newton meter okay so this is what the measure of the bending moment that you have. Okay. Now, similarly, what can say, let's say what is the maximum bending stress at blade rotor. Obviously, you can, I mean, this is regarding the annulus cross section.

So, what we have, if you have a, yes, R1, R2, and this is D equals to R2. This is your neutral axis. So here, R2 my R1 equals to B, which is less than R2. So, my moment of inertia pi by 4 r2 to the power 4 minus pi by 4 r1 to the power 4 which will be pi r2 cube b. So i by r2 square into b.

Then I have sigma max r2 by i M0 which is M0 by pi by R2. So, essentially M0 by R2 square into B. So, let's say r2 equals to one meter. Sigma max equals to 250 megapascal. Then one can say how thick should the blade root be.

ex:

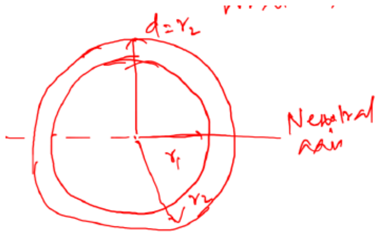


$F_T \sim \frac{P}{(1-a)U_\infty}$

Say, 1 MN for 6 MW at $U_\infty = 9$ m/s & $R = 75$ m

$M(0) = \frac{2}{3}R \cdot \frac{F_T}{3} \approx 16$ MN·m

What is the max. bending stress at blade rotor?



$r_2 - r_1 \doteq b \ll r_2$

$I = \frac{\pi}{4} r_2^4 - \frac{\pi}{4} r_1^4 \sim \pi r_2^3 b$

$I/r_2 = \pi r_2^2 \cdot b$, $\sigma_{max} = \frac{r_2 M(0)}{I} = \frac{M(0)}{I/r_2} = \frac{M(0)}{\pi r_2^2 b}$

So, one can kind of try to find out the thickness of the blade route. I mean, the one that we got that from here, straight away you can find out the thickness, which you can say m naught pi r two square into one by sigma max. So, if you put five mega Newton meter by 250 mega pascal one by 50 meter around two centimeter so you can easily estimate that the thickness of the root of the blade how much it would be i mean obviously we have kind of seen that how along the radius these things changes okay so root to half things would vary and that kind of variation one has to bring into the design purposes so these are simple talking about the basic structural analysis. These are again basic structural analysis, but one has to understand the complete picture about this basic structural analysis. And now what we would do, we would also look at the other part of the structural design.

Say, $r_2 = 1\ m$, $\sigma_{max} = 250\ MPa$ $\longrightarrow$ how thick should the blade root shall be?

$$b = \frac{M(0)}{\pi r_2^2} \cdot \frac{1}{\sigma_{max}} = \frac{5\ MN\text{-}m}{m^2} \cdot \frac{1}{250\ MPa} = \frac{1}{50}\ m = 2\ cm.$$

and then connect back things with the aerodynamics forces like that. Okay. So, we'll continue the discussion in the next session.