Wind Energy

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Lecture 40: Mechanics and Dynamics

Welcome back. So, we'll continue our discussion on this dynamics or mechanics of this wind turbine. So, what we have kind of looked at it, if you recall, we have talked about different kind of loads. Okay, so primary resource or the primary source of those loads are aerodynamics load which causes your lift and drag. There could be gravity force, inertia force, electromechanical, operational forces which could arise due to your pitch mechanism. then, we talked about types of loads whether it's steady, cyclic, resonant kind of things and plastic so all this and we took an example to talk about that so, we'll go a little bit details now and try to analyze this loads and all these things on the wind turbine.

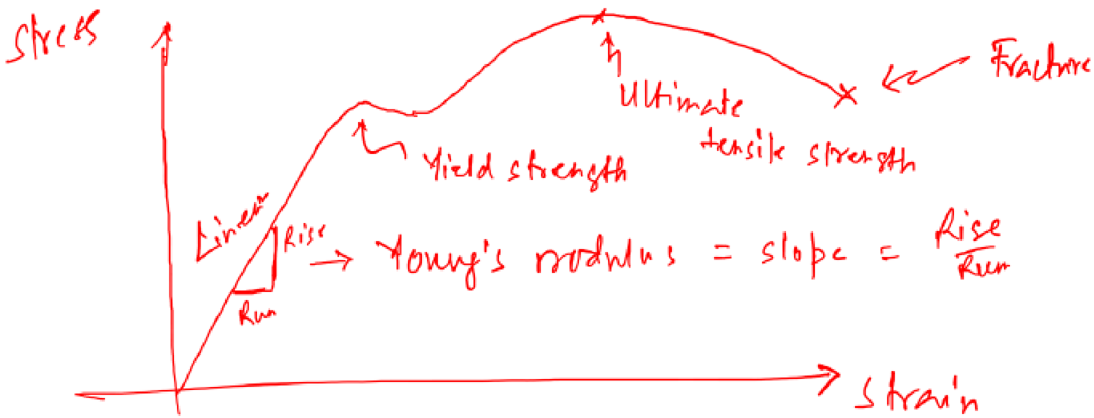
Okay. So, what we do, we start with simple space and strain definition. Space and strain. Okay.

So, we start with that. Now, let's say material. Okay. And, we kind of bifurcate that. This is the length L.

Let's say this is delta L and there is a force which is acted on it. Okay. And, this you consider as a cross-sectional area. Okay. So, this is this material is under this material is under tension.

Now, what this which is Sigma force by area okay obviously the unit should be Pascal strain we designate the epsilon which is delta L by L. That means how much length has been increased and for a material, typical stress-strain curve, so if you do, so this goes Okay. So, this portion is linear. And, the slope of this curve, this is gives my, essentially this is the slope. This slope is, gets me the Young's modulus.

which is essentially a slope and which is a linear curve so, it could be let's say this is rise and this is run then this should be rise over run this point is yield strength somewhere here this is ultimate tensile strength and this point is finally it goes to fracture. Okay. So, that's typical stress strain curve. So, for example, let's say, okay, still E is 200 gigapascal. Y is 250 MPa and whereas ultimate strength is 500 MPa.

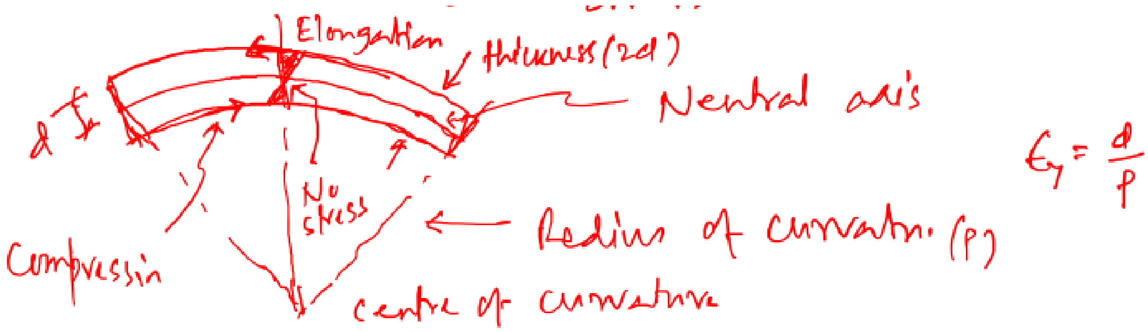


Now, one can find out at which strain does the steel start to deform. which strain does a steel start to deform plastically or permanently. So, what we have sigma y is K into epsilon y Okay. And Sigma y is essentially y. So my epsilon y is y by e 250 into 200 into 10 to the power 6, 3.

So, this is 1.25 into 10 to the power 3. Something around 0.125 percent. So, that you get like that.

So, similarly one can say Let's say, you can say, when does a beam start to deform? If you look at the beam structure or if you draw it, let's say, This is our center of curvature. This is radius of curvature. This is neutral axis. This side would be on compression. This is tension or elongation.

So, this distance is D. and this is thickness 2D. So, neutral axis to upper surface is D and neutral axis to lower surface is also D. So, this point is no space okay so here simply epsilon y is d by rho which is your i mean that radius of curvature, if you say okay, row here the typical convention that you have in solid mechanics. So, you can see the beam situation that one surface would be under tension, the other surface would be on compression and It depends on this radius of curvature.

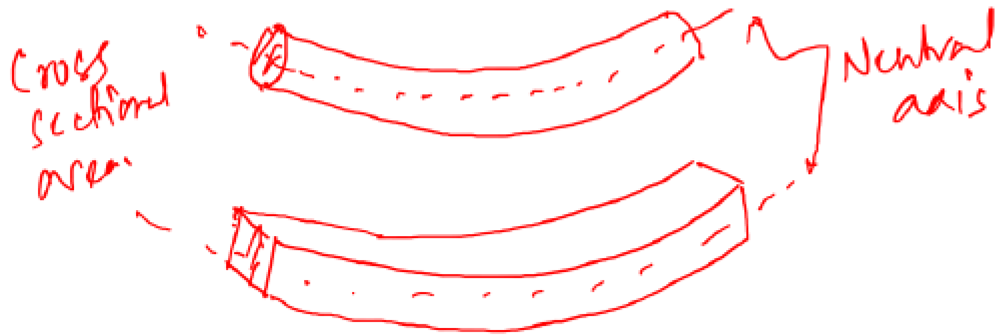


Now, we find out static beam bending, which is essentially my Euler Bernoulli theory. So, what we know from Hooke's law, sigma is E into epsilon. Basically stress, where sigma is stress, E is Young's modulus, epsilon is strain or deformation, whatever you call it. So, you can have structure, different kind of structure under different kind of, obviously this is the central axis. I mean, basically, these are neutral axes.

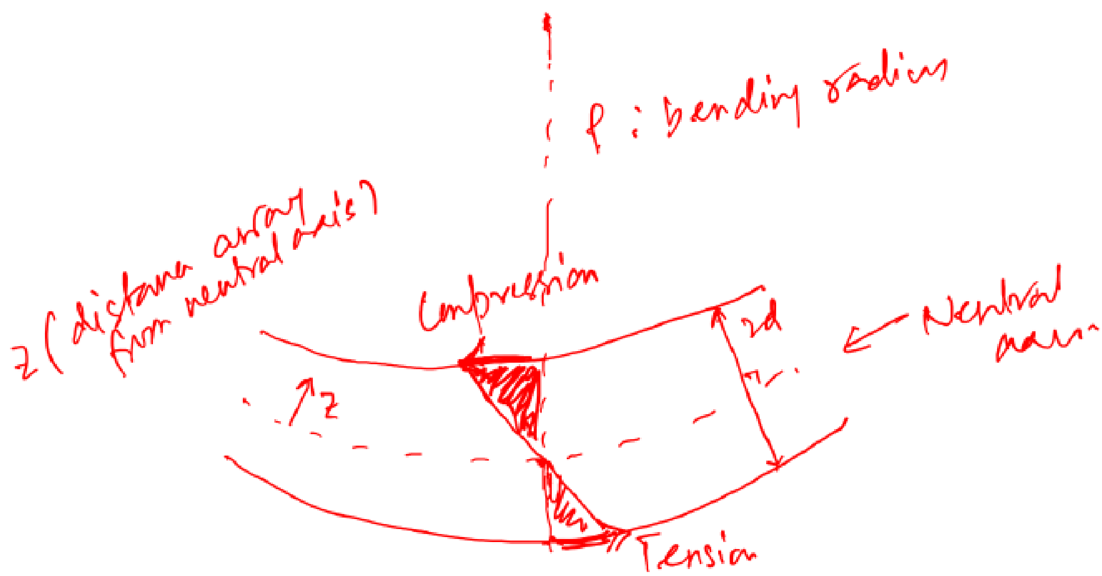
Okay. And these are cross-sectional radii. You have Here you can see the bending obviously, assuming the displacement is orthogonal. So, now what we will do? Obviously, one can draw little bit better picture. Here, the idea is to be important. This is, let's say, bending radius or radius of curvature.

This is neutral axis. This is 2D thickness. This is Z axis which is distance I from neutral axis. So, this is my side view. this side would be tension and this is my completion, okay now, if you look at the of the section this is neutral axis this is my z variation this could be front view so, this is how the um so obviously, z here minus d to plus d if the neutral axis is zero then you have this symmetric these things.

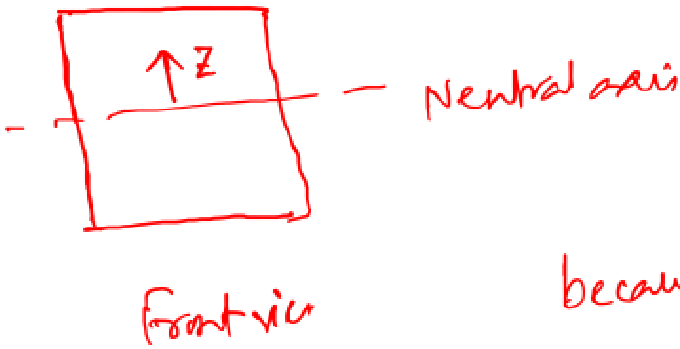
So, this is going from minus d to plus 0. What we can write? n is epsilon, which is squared by rho. because 1 by rho is d 2 w x by d x 2. So, what we get epsilon z into d2wx by d2.



Okay. Now, we can find bending moment. So, m of x is minus d2dz sigma zd which one can write z ez d2 wx by dx2 da e minus d2 dz square da. This term is nothing but I, which is the second moment of area. So, what we get EI d2wx by dx2. So, one can write from here M equals to e i 1 by rho okay that's what the uh i mean that's what you have as bending moment okay now what we write the static BIM equation for Euler Bernoulli.



$$z \in [-d, d]$$



$$\text{strain} = \epsilon = \frac{z}{\rho}$$

$$\text{because, } \frac{1}{\rho} = \frac{d^2 w(x)}{dx^2}$$

$$\epsilon = z \cdot \frac{d^2 w(x)}{dx^2}$$

So, d2 by dx2 Px Ix d x 2 equals to q x. Here you have shear force which is d m x by d x which is capital Q x. You have distributed load that is d 2 mx by dx2 which is small qs. So, this is what you have for the static Euler Bernoulli equation or beam equation. So, these are the, essentially, this is the basic equation.

Now, we take a simple example. With this, we take a simple example, which is quite often cantilever beam with end load, probably simplest pattern of loading so there are forces here and if this is x this is x equal to l and this is w l or here w0 equals to 0 w prime 0 equals to 0 okay and that's the length of l so it's an beam which has been fixed at one end cantilever beam which is fixed at one end and you have an end load acting at the other end of the beam. So, what we have mx equals to ei d2w by dx2 which is f into l minus x and dm by dx which is minus f so d u x by dx is zero because e i is constant and there is no distributed load of ux equals to 0. Also no gravity of the beam. So, if we that's what your this is your mx okay this is qx which is that's the variation how it looks like okay so what we write now d2w by dx2 equals to f by ei is minus x from the first equation which is simple you get that so here this is an second order ordinary differential equation so it can be simply solved and if you solve what you get f by ei L x square by 2 minus x cube by 6 c naught plus c1 x.

Bending moment:

$$M(x) = \int_{-d}^{d} z \cdot \sigma(z)\, dA$$

$$= \int_{-d}^{d} z \cdot E \cdot z \cdot \left(\frac{d^2 w(x)}{dx^2}\right) dA$$

$$= E\left(\frac{d^2 w(x)}{dx^2}\right) \underbrace{\int_{-d}^{d} z^2 dA}_{:= I \text{ (second moment of area)}}$$

$$= EI \frac{d^2 w(x)}{dx^2} \Rightarrow \boxed{M = E \cdot I \cdot \frac{1}{\rho}}$$

Static beam equation / Euler Bernoulli:

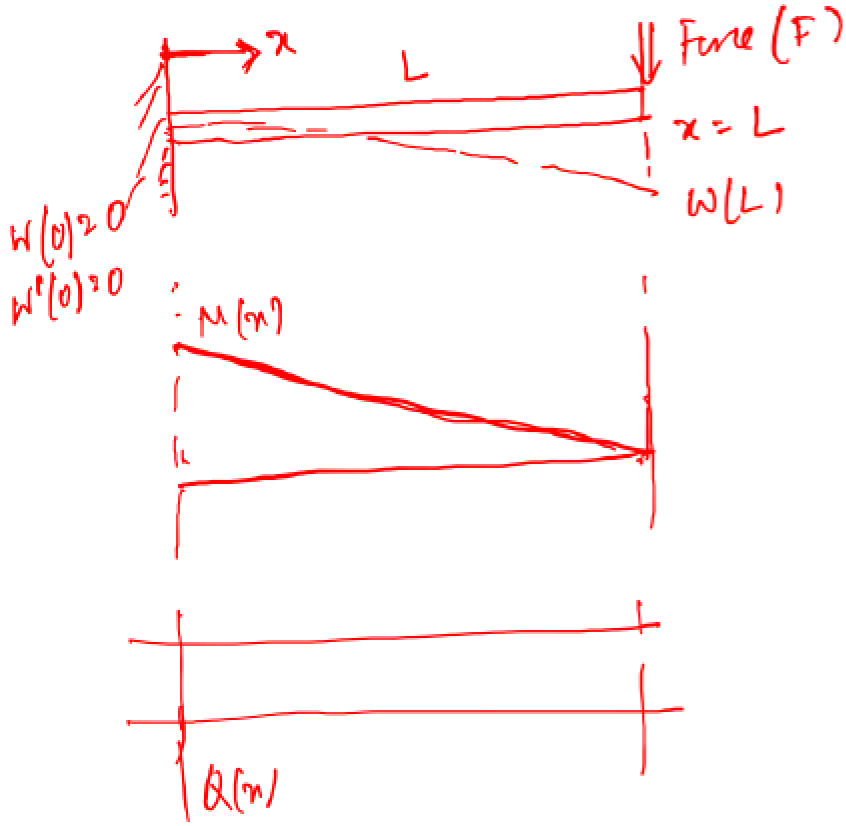
$$\frac{d^2}{dx^2}\left[E(x) I(x) \frac{d^2 w(x)}{dx^2}\right] = q(x)$$

Shear force: $\frac{dM(x)}{dx} = Q(x)$

Distributed load: $\frac{d^2 M(x)}{dx^2} = \frac{dQ(x)}{dx} = q(x)$

Now we have initial conditions where w 0 is 0, w prime 0 is 0. If I apply that, it gives me c1 equals to 0, c naught equals to 0. So, what eventually we get? W by X, if X square by 6 EI, minus x. So, that's a kind of a variation of the displacement along x. That means from here if you start along the axis at different axial location, one can find out the displacement distance or things like that.

So, at end point, which is x equal to L, W by L would be fL square 6EI into 2L. this one can write F into L cube by T E I. One can think about this is displacement, this is force, this is kind of a being constant okay this is what you get when you have simple cantilever beam with one with the load at one end and so the beam is connected at one end fixed and the other end you have loading and then you get the displacement equation using static Euler binary and once you integrate this is a simple integration here what you see and apply your initial conditions at one end and then find out the equation for displacement at different axial locations along the length or axis of the beam. And final displacement you can find out how much at the other end the beam is going to be displaced due to the load.



$$M(x) = EI \frac{d^2 w}{dx^2} = F(L - x)$$

$$Q(x) = \frac{dM}{dx} = -F$$

$$\frac{dQ(x)}{dx} = 0$$

(E, I is const. and there is no distributed load, $q(x) = 0$
- No gravity of the beam)

$$\frac{d^2 w}{dx^2} = \frac{F}{EI}(L - x)$$

$$W(x) = \frac{F}{EI}\left(L\frac{x^2}{2} - \frac{x^3}{6} + C_0 + C_1 x\right)$$

$$\text{I.C.}: \begin{matrix} W(0)=0 \\ W'(0)=0 \end{matrix} \Bigg\} \Rightarrow C_1 = 0,\ C_0 = 0 \qquad \boxed{W(x) = \frac{Fx^2}{6EI}(3L-x)}$$

$$\text{at end point } (x=L),\ \underbrace{W(L)}_{\text{displacement}} = \frac{FL^2}{6EI}\cdot 2L = \underbrace{F}_{\text{Force}} \cdot \underbrace{\frac{L^3}{3EI}}_{\text{spring cnt.}}$$

So, we will look at some other example of cantilever beam also and continue this discussion in other session. Thank you.