Wind Energy

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Lecture 38: VAWT performance calculation (contd..)

Welcome back. So, we'll continue our discussion on this vertical axis wind turbine. So, what we are trying to look at it. So, we are now trying to look at this performance parameter calculation. So, we are using the concept of, again, the velocity triangle. but when you compare with horizontal axis wind turbine the complexity of the whole thing is much more lesser because you have less complications you have few number of blades your sizes so, we'll try to establish those equations and then analyze the whole system, okay so, now last time what we stopped here is that this um power coefficient calculation and as you can see this particular equation that equation 19 you cannot solve analytically you need some numerical methods to solve this Obviously, one can solve this particular equation under some idealized condition.

You may not be possible for all generic condition, but some idealized conditions you can do that. So, that's what it is going to happen. So, now we'll take this equation and some specific situation we can analyze let's say start with particularly let's say particularly we start with high tip speed ratio so, which is lambda quite high so in this case what would happen the angle of attack will be relatively small. That means alpha is small.

so, then again when you have small angle of attack that means here you can use that the small angle approximation first what happens is that for small angle of attack let's say below stall alpha which is small what you can have the lift coefficient is linearly so lift coefficient which is cl varies linearly with alpha so also since Symmetric. Airfoil. Is considered here. So, that allows us to write the lift coefficient. Basically Cl equals to.

Cl alpha. Into. Alpha. So, which is. There.

And here. This is the. Slope. of the lift curve. So, the angle of attack from equation 5, the angle of attack is in And in addition, we can write cos phi plus alpha as cos phi because alpha is small.

So, then what we can write from equation 12 one can approximate as one by one minus a which is one plus one by eight c by r one by two pi zero to two pi okay y plus sine phi, full square, cos square phi, into Cl alpha, alpha, cos phi. So, that is equation 21. So, what we have termed in the bracket here, and this can be expanded. So, this can be expanded.

Small angle approximation &

α (below stall) small $\rightarrow$ Lift Coeff (C_L) varies linearly with α

Since, symmetric airfoil is considered $\rightarrow$ $C_L = C_{l,\alpha}\,\alpha$ .... (20)

$\hookrightarrow$ slope of the lift curve

From eq. (5), the angle of attack is in the appropriate form, and, in addition, $\cos(\phi+\alpha) \simeq \cos\phi$.

From eq. (12), one can approximate as:

$$\frac{1}{1-a} = 1 + \frac{1}{8}\,\frac{BC}{R}\,\frac{1}{2\pi}\int_0^{2\pi} \left\{ (y+\sin\phi)^2 + \cos^2\phi \right\} C_{l,\alpha}\,\alpha \cdot \cos\phi \cdot d\phi \quad \text{.... (21')}$$

 Okay. And, once we expand this plus we use equation five and 21. So, what we use a five and 21 can be rewritten as 1 by 1 minus a 1 plus 1 by 16 BC by r 1 by pi 0 to 2 pi y square 2 y sin phi and square phi L alpha 1 by y power square Okay. So, here we can use that trigonometric identity of this which is 1. Then this equation we can write that 1 minus 1 by a equals to 1 plus 1 by 16 bc by r 1 by pi 0 to 2 pi. It's a bit of an algebra, nothing more than that.

 So, one has to kind of do it a little bit carefully. Cos square phi d. So, this integration if we perform, then what happens? So, once we perform this integration, this gets you the, essentially the sine terms become zero cosine terms let's say square terms become pi or integrate to pi whatever so, if you rewrite that this would be 1 plus 1 by 16 BCR L alpha Y plus 1 by Y which approximated as 1 by 1 by 16 or one can say A is 1 by 16 BC by R into lambda. Similarly, the expression for power coefficient can be simplified. Similarly, expression of Cp can be simplified using small angle.

 approximation and by assuming the drag coefficient to be constant. When you say that drag coefficients to be constant, constant drag coefficient gives us CD of alpha, CD of zero. So, this is a constant drag term that you have. So, what you can write equation 19, which is the power coefficient expression. So, we can say equation 19 can then be integrated to field Cp 4 pi C l alpha one minus a to the power four divided by lambda four square plus one half d zero lambda one minus a square four square plus one.

$$\frac{1}{1-a} = 1 + \frac{1}{16}\frac{BC}{R}\frac{1}{\pi}\int_0^{2\pi}\{y^2 + 2y\sin\varphi + \underbrace{\sin^2\varphi + \cos^2\varphi}_{=1}\} C_{l,\alpha}\cdot\frac{1}{y}\cos^2\varphi\, d\varphi \quad \cdots (22)$$

$$\frac{1}{1-a} = 1 + \frac{1}{16}\frac{BC}{R}\frac{1}{\pi}\underbrace{\int_0^{2\pi}\{y + 2\sin\varphi + \frac{1}{y}\} C_{l,\alpha}\cos^2\varphi\cdot d\varphi}_{\Downarrow} \quad \cdots (23)$$

Performing this $\Rightarrow$ Sine terms become 'zero'
(Cosine)^2 " " 'π'

$$\frac{1}{1-a} = 1 + \frac{1}{16}\frac{BC}{R}C_{l,\alpha}\left(y + \frac{1}{y}\right) \simeq 1 + \frac{1}{16}\frac{BC}{R}C_{l,\alpha}\, y \quad \cdots (24)$$

or, $$a \simeq \frac{1}{16}\frac{BC}{R}C_{l,\alpha}\lambda \quad \cdots (25)$$

Now, this particular equation, one can further simplify this equation. I mean, let's simplification for simplification what you can say that since y is greater than one so which is true that y square plus one becomes y square and one minus a square y square become lambda square So, equation 27 can be simplified to CP for a 1 minus a square minus half of lambda k. So, this is quite simplified situation. there is no drag which is more like an idealistic condition. So, then you can see the CP optimum would be at A equals to 1 by 3.

So, then CP max is again becomes 16 by 17 which is again 5, 9, 2, 6. That's the base limit that we have already talked about in the context of horizontal axis wind turbine. So, in this particular case also same restriction would apply. I mean what you could see here that you obtain the base limit, similar base limit that we have obtained for the horizontal axis wind turbine. so, when you get this maximum all coefficient that restriction of minimum limit restriction so, restriction on a the minimum a is still there which is validative of less than 0.

5 so that means if the induction factor becomes higher which could be the high flow condition due to turbulent or something, this optimum theory may not hold good. So, that kind of gives you a simplified analysis. Obviously, this particular thing that we have seen here, there is one thing that we started is that or high tip speed ratio. That means lambda is quite high. And also, we have some small angle approximation so that the CL curve is linearly proportional to alpha.

Cst Drag Coef: $\Rightarrow$ $C_d(\alpha) \simeq \underline{C_{d,0}}$ ... (26)

Eq. (19). can then be integrated to yield:

$$C_p = \frac{1}{4\pi}\frac{BC}{R} C_{l,\alpha}\frac{(1-a)^4}{\lambda}(y^2+1) - \frac{1}{2}\frac{BC}{R} C_{d,0}\,\lambda(1-a)^2(y^2+1) \quad \ldots (27)$$

For simplification: Since $y \gg 1$, which is true that $y^2+1 \simeq y^2$

& $(1-a)^2 y^2 = \lambda^2$

eq. (27). can be simplified to:

$$C_p \simeq 4a(1-a)^2 - \frac{1}{2}\frac{BC}{R} C_{d,0}\lambda^3 \quad \ldots (28)$$

And then all the sine alpha, cos alpha terms can be approximated based on that small angle approximation. that is what the actual expression was this which is the generic expression and that one can apply to any kind of turbine I mean vertical axis wind turbine I mean straight blade obviously this is for to mention that straight blade so then we could got this simplified situation for this. Okay. Now, this was single steam tube analysis. So, if you recall that we started this discussion as a single steam tube analysis.

So, now we can move to multiple steam tube momentum theory. So, previous one was, so let us, here let's draw the circle first. Then we have the aerofoil. Okay. Now, what we do we have this point we have this point this is your u into 1 minus a so this is p this is r so, So this angle is delta phi.

Then this is my r cos phi into delta phi. Okay. This is omega r component and this is my u. So this is my multiple stream tube geometry look at from the top. So, this is also multiple stream tube theory is also used for sometimes used for analysis of vertical axis wind turbine.

So, here in this particular approach, so what is assumed is that A which is the induction factor may vary in the direction perpendicular to the wind. However, but it is constant in the direction of wind. that means if this is my wind direction if this is my wind direction then in this direction a is constant whereas in this direction a may vary, i mean, defining this system so, that is what it so that's uh so that is one of the assumptions associated with this multiple stream cube theory. So, that means what we are seeing here in this particular picture or the drawing so, from here from this figure what we can say that each stream tube of constant A is parallel to the wind. So, as we have shown here so from this figure we can draw that interference each stream tube of constant A is parallel to the wind.

So, the force on a stream tube of width r cos phi delta phi can be related to the change in momentum of the passing through it. That is what I mean when you compare with the single stream tube analysis to this multiple stream tube analysis. That is one of the difference that you have. So, now what we can write, we can write the force per unit height which is similar to equation 7. what we can say that delta FD R cos phi del phi rho 2 A 1 minus A v square.

So, this is the force per unit height which is again analogous to. So, now what you can do that let's say during any given rotation, a single blade will pass through the stream tube twice which was not the case in the earlier scenario when we did single steam tube analysis so obviously you can find the forces on the blade by using blade element the theory which is primarily so one can find forces on blade can be found using blade element theory which is commonly used so that's not very uncommon here so what it turns out it turns out that the force on the blade at both the point and downwind crossing position is the same. So, now we can find the force which is analogous to equation 9. delta FD, B into 2 by 2 pi, phi 2, phi plus delta phi, f of rho, u real square, Pcl, cos phi plus alpha, d phi. So these two equations may be equated and one can find the expression for E.

Now the integral equation 31 can be approximated as like p to p plus delta phi half rho u del square T L cos p plus alpha d phi half rho you will square L plus P plus alpha del P, which is, so this is what we kind of write. Now, what you do using equation 30, and 31 and taking advantage of equation 32, one can show that A A minus 1 by A equals to 1 by 4 pi B C by R U real by U square. So, this power coefficient and power coefficient which is Cp can be solved as before equation 27 but in this case it is to be called iteratively so that's what you get the power coefficients and all this thing that you kind of so you can So, it has to be solved iteratively. The reason, because here A is a function of alpha.

Force / height (similar to eq. 7) : $\Delta \tilde{F}_D = R \cos\varphi \, (\Delta\varphi) \, \rho \, 2a(1-a) U^2$ ... (30)

During any given rotation, a single blade will pass through the stream tube twice.

Forces on blade $\Leftarrow$ using blade element theory

It turns out, the forces on the blade at both the upwind & downwind crossing position is the same.

Force (analogous to eq-9) $\Rightarrow$ $\Delta \tilde{F}_D = B \frac{2}{2\pi} \int_{\varphi}^{\varphi+\Delta\varphi} \frac{1}{2} \rho U_{rel}^2 \, c \, C_l \cos(\varphi+\alpha) \, d\varphi$ ... (31)

Integral in eq (31), can be approximated as :

$$\int_{\varphi}^{\varphi+\Delta\varphi} \frac{1}{2} \rho U_{rel}^2 \, c \, C_l \cos(\varphi+\alpha) \, d\varphi = \frac{1}{2} \rho U_{rel}^2 \, c \, C_l \cos(\varphi+\alpha) \, \Delta\varphi \quad \ldots (32)$$

That is the difference. So, one can look at the book by Vyas or So, they have given some detailed calculation procedure for this, but this much will be good enough to have an understanding how, but one can also put this in the numerical programming and get this equation solved. So, this is how double stream tube analysis can be also done for vertical axis interval. So, we will stop here and continue this somewhat discussion on this in the next session.

Using eq- (30) & (31), and taking advantage of eq-(32), one can show that

$$a(1-a) = \frac{1}{4\pi} \left(\frac{Bc}{R}\right) \left(\frac{U_{rel}}{U}\right)^2 C_l \frac{\cos(\varphi+\alpha)}{\cos\varphi} \quad \ldots (33)$$

Power coeff (C_p) can be solved as before eq-(27), but in this case it is to be solved iteratively, because here 'a' is a function of 'α'.

Thank you.