Wind Energy

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Lecture 37: VAWT performance calculation

Welcome back so, we'll continue our discussion on vertical axis wind turbine, so, what we have been talking about this different kinds of vertical axis wind turbine and their advantages and also how I mean the operational of the vertical axis wind turbine primarily we talked about two different kinds one is the lift base other one is the drag base and in the lift base we talked about this different kind of blade design of the vertical access wind turbine where you can, I mean all these different design that you can see here which we have already talked about, these are just to the purpose of these different kind of blades to have improved performance of the turbine. So, sometimes we may use kind of a twisted blade sometimes use helical kind of configuration sometimes different configuration. Then you can have kind of a flapper, you can have pitch controlled or variable pitch foils, control turbines. And, there are ways or different ways one could actually design those kind of turbine. But essentially, the idea here is that all these different strategies would lead to the different kind of different kind of design to so here is another another example of the frame design the frame essentially supports the blade and deals with the statics and dynamic load so, the frame design is very very important this is a each type rotor you can see the base frame which is quite straightforward then you have a one main shaft and connecting airfoils And, then the frame, so you have two blade various rotor, you have three blade areas rotor, you have Y type rotor, you have double Y type rotor. So, the thing is that what you could actually do is that once you design this frame and they can actually have this different structure to connect with the mainframe shaft and all these things so obviously these designs are having some advantages some disadvantages and all their issues but overall vertical axis wind turbine will have its own advantages over horizontal axis wind turbine so this is insensitive to wind direction which is very very important and turbulence level because your vertical axis wind turbines are smaller in size and then you have the wind which is coming from this direction or that direction so that can be easily handled for that you don't require any your pitch mechanism and the extra mechanism, so, and the most important thing since they are smaller in size they are installed closer to the surface and you don't have a big structure to be there.

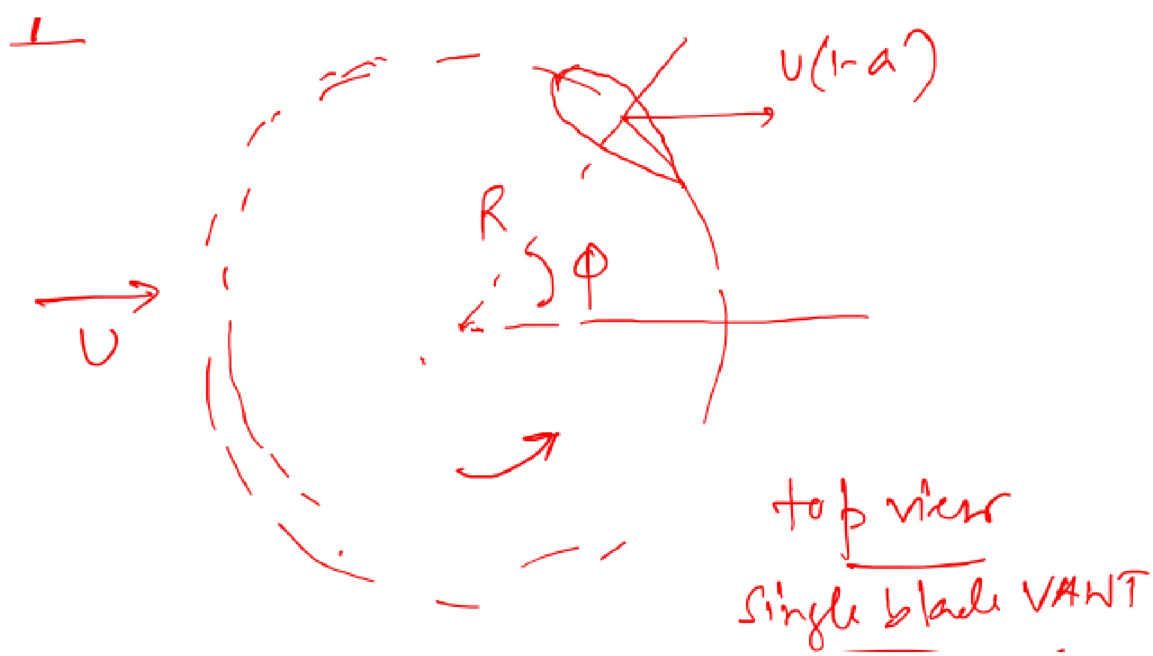
Obviously, that means this will be cheaper to build, install and maintain. You have a very simple tower so the blade sweeps the conical area. you have replacement and maintenance are much simpler compared to the horizontal access wind turbine because in horizontal access wind turbine your gearbox and everything sitting inside the nacelle which is few meter above the ground level where someone has to go up there and do the maintenance, so, the maintenance become absolutely simpler or easier Vertical axis wind turbines can be installed on horizontal axis wind turbine wind pump and also buds and all these things so, there are certain advantages of the vertical axis wind turbine but it is not like that everything is advantages over horizontal axis wind turbine but there could be some disadvantages as well for example one of the major drawback of the efficiency of the turbine because the amount of power that you could extract out using horizontal axis wind turbine you may not be able to extract out but yes at the same time you can make use of this vertical axis wind turbine for the small power generation purposes then a well-located large horizontal axis wind turbine is continuously driven by the wind source but that may not be possible for vertical axis wind turbine, okay so, what you can have these are some of the and then if you look at the power transmission part so, again you see this connect to the generator this is the blades with the main tram then you have a rotating sap to gearbox to generate an electrical load and you can see some simple power transmission system which is there so these are the different kinds of vertical axis wind turbine that you can i mean vertical axis wind turbine that one can design and have advantage over that so now what we'll talk now since, we have a good idea about this how they look and all these similarly like what we have done so we will do some kind of a momentum simple analysis for power extraction and all these things so so so what would continue to do that like we will do some aerodynamics aerodynamic analysis of this vertical axis wind turbine. So, that's what we are going to do.

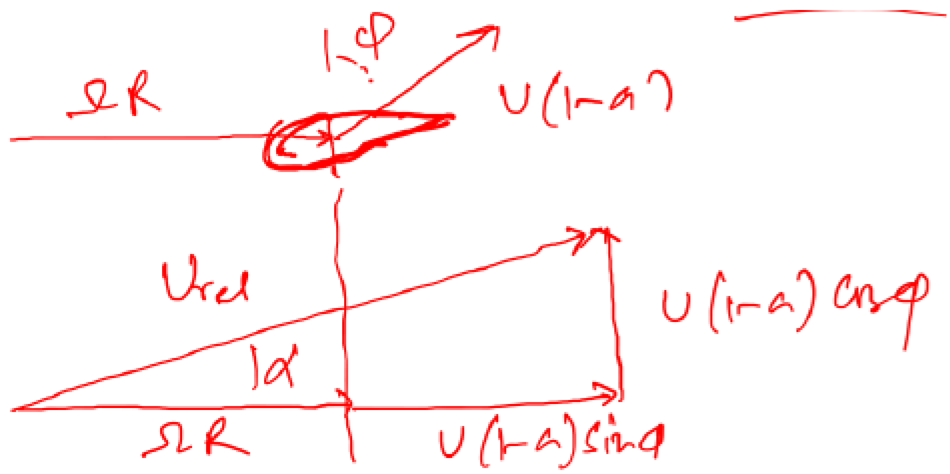
So, that means what we are going to look at it is the aerodynamics of let's say straight blade vertical axis wind turbine. Okay. So, we'll look at that and carry out the analysis the similar way. So, what we'll try to look at it, we'll first look at simple single stream tube analysis. Okay.

Single stream tube analysis. and, then we can also look at the multiple stream tube method obviously you can have the darius rotor can be modeled with the modification of the state blades, or so, what we do in the single blade vertical axis wind turbine so, if you let's say the it is let's say let us draw a circle here or maybe here then we will have one arrow foil okay and we'll cut through this axis this is u this is u into one minus a and this is let's say radius r this is p, so, this is an top view so some a single blade single blade vertical axis wind turbine which is seen from the top view and this would so it's a rotating in this direction which is counter clockwise and wind is seen impinging on the rotor from the left to right. So, that is what it has been shown. So, typically in vertical axis wind turbine the airfoil which is being used is symmetric. So, obviously the blade is oriented so that the quad line is always perpendicular to the radius of the rotation.

So, blade is oriented so that the quad line is perpendicular to the radius of the circular. The radius obviously defines the angular position, which will normally meet the chord and makes an angle phi with the wind direction. Now, what we can draw a velocity triangle for this, let's say, If we draw this airfoil and through which we will pass through this line. Obviously, this is coming here, which is omega of r. This goes that way, which is u into 1 minus a.

This angle is phi. then what we have we have this component we have this component we have this component so that's complete the triangle this is alpha this is omega r u 1 minus a sine phi u 1 minus a cos phi okay so These are the velocity components how it is acting on the blade. Obviously, the component due to rotation is tangential to the circle of rotation. So, what we can say the component due to rotation is tangential to the circle of rotation and thus parallel to the pod line of the careful okay so so also one component of the wind acts tangentially and another component of the wind is normal to the circle. So, the perpendicular to the airfoil as you can see.





So, induction factor I mean A which accounts for the deceleration of the wind as it passes through the rotor. So, now from the velocity triangle what we write which is very U relative square. which is essentially this is u relative omega r plus one minus a u sine phi square plus one u into one minus a cos phi square that is let's say one it's a simple right angle triangle formula so this complete vector to this vector so that's a so, this we can rewrite u rel by u is root over lambda plus 1 minus a sin phi square plus 1 minus a cos phi square Okay. Where your lambda is omega r by u, which is again tip speed ratio. So, that's what you have.

Second term, here in the equation two, which is becomes small so the second term in equation 2 becomes small so what we can write u rel by u lambda 1 to minus a sine phi okay that's what it happens. So, let's say we will say it's 3. Since, the pod is perpendicular to the radius of the circle, we can find out since, the pod is perpendicular. Since, the quad is perpendicular to the radius of the circle, we can fight alpha which is tan inverse 1 minus a cos phi lambda plus 1 minus a sin phi which is 4. So, at higher tip speed ratios, this second term in the denominator, at higher tip speed ratio is relative, is small relative to the, so at higher lambda, this is small relative to lambda.

So, what happens is that. So, then tan of alpha is can be alpha. So, what we write alpha equals to 1 minus a cos p divided by lambda. So, it's a small angle approximation that is what we are using here. Now, what we can do, we can by analogy with momentum theory for horizontal axis wind turbine, horizontal axis rotor, let's say, horizontal axis rotors the forces on the blade can be related to the engine momentum in the, so, one can assume that as before that is far away velocity is reduced to so that means we can say that the far away velocity would be u into 1 minus 2a.

This, is like what we have done in the horizontal and at the rotor and rotor it will be u into 1 minus a. So, what we can write that the change in velocity is delta U, which is U into U1 minus 2A, 2AU. And, the force per unit height, force per unit height, which is FD, In the wind direction, one can write equals to m dot del u. So here, this is mass flow rate per unit height, mass flow rate per unit height. So, what we can write m dot v is rho 2ru 1 minus a.

$$\frac{U_{rel}}{U} = \sqrt{\{\lambda + (1-a)\sin\varphi\}^2 + \{(1-a)\cos\varphi\}^2} \quad \ldots \quad (2)$$

$$\lambda = \frac{\Omega R}{U} \quad \ldots \text{tip-speed-ratio}$$

2nd term in eq (2) becomes small, $\frac{U_{rel}}{U} \approx \lambda + (1-a)\sin\varphi \quad \ldots \quad (3)$

Since, the chord is perpendicular to the radius of the circle,

$$\alpha = \tan^{-1}\left[\frac{(1-a)\cos\varphi}{\underbrace{\lambda + (1-a)\sin\varphi}}\right] \quad \ldots \quad (4)$$

$\Downarrow$ at higher 'λ', is small relative to λ

Then, $\tan\alpha \approx \alpha$, $\alpha \approx \frac{(1-a)\cos\varphi}{\lambda} \quad \ldots \quad (5)$

By analogy with mom. theory for ~~HAWT~~ horizontal axis rotors, the force on the blade can be related

So r is the radius of the rotor, so r is the radius of the rotor, density of air. Then, the force is FD equals to 4R rho A 1 minus A U square. So, this force must be equal to the average force on all the blades during complete revolution. So, now this is what we have found using the blade element theory. by integrating around the circle and taking the number of blades into account.

So, what we can get by integrating around the circle and using blade element theory what we get b by 2 pi, 0 to 2 pi, 1 by half rho u rel square pod C L cos alpha plus pi d pi. So, B is number of blades, L is lift coefficient. So, this is what you have. Okay. Now, what you can do using this last two equation that is equation 9 and 10 so by equating equation 9 and 10 what we get is that a into 1 minus a 1 by 8 BC by R 1 by 2 pi 0 to 2 pi U rel by U square cl cos alpha plus pi d phi okay s that is number 11.

The change in velocity: $\Delta U = U - U(1-2a) = 2aU$ ... (6)

Force / height $(\tilde{F}_D)$ in the wind direction: $\tilde{F}_D = \tilde{\dot{m}} \Delta U$ ... (7)

mass flow rate/height

$\tilde{\dot{m}} = \rho 2RU(1-a)$ .... (8)

R = radius of rotor

ρ = den. of air

the force: $\tilde{F}_D = 4R\rho a(1-a)U^2$ -- (9)

By integrating around the circle & using blade element theory,

$\tilde{F}_D = \frac{B}{2\pi}\int_0^{2\pi} \frac{1}{2}\rho U_{rel}^2 c C_l \cos(\alpha+\varphi)\, d\varphi$ ... (10)

B = # of blades, C_l = lift coefficient

So, that's what you have. So now, what we have is equation which this particular equation, you cannot solve directly. So, you need iterations. And to do this, to solve iteratively let y equals to lambda by 1 minus a and then we divide both sides of equation 11 by 1 minus a square so equation 11 divided by 1 minus a whole square and then also you use equation 4 sine phi whole square plus cos square phi plus sin phi d phi. So, this is little bit complicated in that sense that one could expect.

Now, if you have for a given geometry BC by R is fixed then you can have this value of y can be assumed and 1 minus a can be calculated and from there the corresponding so the corresponding lambda is y into 1 minus a. So this calculation is repeated until the desired tip speed ratio is found. So, this is what you do the iteration. So, once you have obtained the proper tip speed ratio then you can do calculation for the power. So, typically the power would be average torque into rotational speed.

Now, the torque varies with angular position. What we write P equals to omega 1 by 2 pi 0 to 2 pi Q d phi. Now, torque is also the product of the torque is also product of tangential force into radius. Then what we have tangential force per unit length on each blade you have 1 by rho u real square v into Cl sin alpha Cd cos alpha. Now, the if you assume the V number of blades then at a rotor height of H.

So, if you say that rotor height is h, b is number of blades, then I have q which is bh. Then the average rotor power we can calculate, average rotor power over one revolution would be P omega R +1 BC by 2 pi half rho 0 to 2 pi UL square Cl sin alpha CD cos alpha d phi. So, then we can find out the power coefficient as well which is power divided by power in the wind passing through that area, projected area of the rotor. So, the projected area of rotor would be 2RH. So, the power coefficient that you can have for second which is Cp that you get P by half rho 2 Rh u cube.

by equating eq. (9) & (10) --- we get,

$$a(1-a) = \frac{1}{8}\frac{BC}{R}\frac{1}{2\pi}\int_0^{2\pi}\left[\frac{U_{rel}}{U}\right]^2 C_l \cos(\alpha+\varphi)\, d\varphi \quad \cdots \quad (11)$$

To solve iteratively, let $y = \frac{\lambda}{1-a}$, eq. (11) $\div (1-a)^2$

$$\frac{1}{1-a} = 1 + \frac{1}{8}\frac{BC}{R}\frac{1}{2\pi}\int_0^{2\pi}\left\{(y+\sin\varphi)^2 + \cos^2\varphi\right\} C_l \cos\left\{\varphi + \tan^{-1}\left(\frac{\cos\varphi}{y+\sin\varphi}\right)\right\} d\varphi \quad \cdots \quad (12)$$

For a given geometry, $\frac{BC}{R}$ is fixed. corresponding $\lambda = y(1-a)$ $\cdots$ (13)

Power = average torque $\times$ rotational speed

$$P = \Omega \frac{1}{2\pi}\int_0^{2\pi} Q\, d\varphi \quad \cdots \quad (14)$$

So, that is so you get average power per revolution you get what you called. So, what we can write for that is that therefore what we have this Cp lambda by 4 pi BC by R 0 to 2 pi U rel by U square Cl sin alpha 1 minus Cd by cl tan alpha d phi, so, that's what you have that you get your cp in this particular form so you get the power per revolution and and, cp.

tangential force / length on each blade: $\tilde{F}_T = \frac{1}{2}\rho V_{rel}^2 c \left(C_L \sin\alpha - C_d \cos\alpha\right)$ ... (15)

rotor height = H, B = # of blades, $Q = BHF_T$ ... (16)

average rotor power over one revolution:

$$P = \Omega R H \frac{BC}{2\pi} \frac{1}{2}\rho \int_0^{2\pi} U_{rel}^2 \left(C_L \sin\alpha - C_d \cos\alpha\right) d\phi \quad \cdots (17)$$

Projected area of Rotor: 2RH

$$\text{Power coefficient } (C_p) = \frac{P}{\frac{1}{2}\rho\, 2RHU^3} \quad \cdots (18)$$

Therefore,

$$C_p = \frac{\lambda}{4\pi} \frac{BC}{R} \int_0^{2\pi} \left[\frac{U_{rel}}{U}\right]^2 C_L \sin\alpha \left[1 - \frac{C_d}{C_L \tan\alpha}\right] d\phi \quad \cdots (19)$$

So obviously, i mean you can see this particular integration this can be solved numerically but, we can so, this is the analysis that we would like to establish from basic momentum theory as we have done in horizontal axis winter we'll talk about this in more details in the next session. Thank you