Wind Energy

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Lecture 33: HAWT performance calculation

So welcome come back and now we'll continue the discussion that we kind of stopped in the last class where we're looking at this. I think this is where we stopped, where we're talking about finding the Cln alpha using method one, where we start with some, I mean, initial guess of FI and then finding the values and then change the, So the other ways to iteratively solve. So, that is what we call it a method method to where you solve. So, this is where you solve A and A prime iteratively. Okay. This is a method to solve this.

so, here we are trying to do iterating to find so since as the name suggests that we are trying to solve iteratively so here we iterate ping to find out the axiolar tangential momentum factor so here is the axial induction factor a prime is the tangential induction factor. So, iteratively we try to solve that. But obviously, when you iteratively solve, you need some initial guess. Okay.

So, to find initial values, we start with the values from an adjacent blade section and values from the various or previous blade design in the iterative rooted design procedure or use an estimate based on the designer. Something that we have to use. So, importantly, what it is required is that we kind of start with an initial guess and these guesses can be values from the, as I said, the adjacent break section. This could be from previous design. in the iterative rotor design process that could be method 1 or use an estimate based on the design values from optimum blade design.

So, what we get let's say what we start with P i1 is 2 3rd tan inverse 1 by lambda ri which is then we get a i1 1 by 1 plus 4 sine squared phi i1 sigma prime i design cl design cos phi i1. So, this is what you get and then A prime I1 1 minus 3 I1 for A I1 minus 1. Okay. So, this is what would allow you to get the t flow angle xl induction factor ai and a prime ai so now we use some guess value for that and move to the jth iteration for using guess values we move on jth iteration where j equals to 1 is first iteration. So, these are all we kind of getting these are all values are for first iteration.

So, what we calculate the angle of relative wind so that means 10 phi ij which is one minus a ij by omega r one plus a prime ij or it is one minus a ij one plus a prime ij lambda ri so that means if j equals to one That is the initial values of a prime i1 that can be used and we can move on j2 like that. Similarly, the t plus factor if ij, it would be 2 by pi cos inverse exponential b by 2 1 minus ri by r and R i by R sine phi i j. So that's what we get in as in F i j. Now what we need to estimate is the CLIJ and CDIJ from airfoil lift and dead data. So, what we can determine CLIJ and CDIJ from airfoil lift and drag data.

using guess values, we move on jth iteration. where j=1 in first iteration

$$\varphi_{i,1} = \left(\frac{2}{3}\right)\tan^{-1}\left(\frac{1}{\lambda_{r,i}}\right) \quad \ldots (75)$$

$$a_{i,1} = \frac{1}{\left[1+\dfrac{4\sin^2(\varphi_{i,1})}{\sigma'_{i,design}\, C_{l,design}\cos\varphi_{i,1}}\right]} \quad \ldots (76)$$

$$a'_{i,1} = \frac{1-3a_{i,1}}{(4a_{i,1}-1)} \quad \ldots (77)$$

$$\tan\varphi_{i,j} = \frac{U(1-a_{i,j})}{\Omega r(1+a'_{i,j})} = \frac{1-a_{i,j}}{(1+a'_{i,j})\lambda_{r,i}} \quad \ldots (78)$$

$$F_{i,j} = \left(\frac{2}{\pi}\right)\cos^{-1}\left[\exp\left(-\left\{\frac{(B/2)\left[1-(r_i/R)\right]}{(r_i/R)\sin\varphi_{i,j}}\right\}\right)\right] \quad \ldots (79)$$

So, where we can use alpha ij equals to pij minus theta pij which is essentially the angle of attack with the flow angle and pitch angle. So, now we can calculate local thrust coefficient local first coefficient which is CT ij or CTr ij sigma e prime 1 minus a ij square Cl ij cos phi ij Cd ij so we can update a and a prime for the next iteration if local thrust coefficient CTr ij less than 0.96. Then we can use Ai j plus one equals to one by one plus four sine square phi ij. We have sigma prime i Cl ij cos phi ij so that is what we so it depends on the local first coefficient so if it is less than 0.

96 that means it is in that area of the curve because ct versus a varies like this so this is 1 this is 1. So, it is this side where windmill operation and this is the turbulent side. So, and if CTr ij greater than 0.96, we have aij 1 by fij 0.

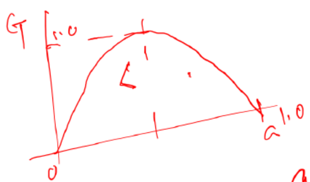
143 0203 0.889 minus ij okay and here we'll update a prime ij plus 1 equals to 1 by 4 Fij cos Pij minus 1 divided by sigma prime Clij. So, that gives us obviously sigma prime it has to be i. So, this is what you get depends on the local thrust coefficient that you get to estimate these things. Now, what one has to check whatever the new induction factor that it has been obtained. These are acceptable tolerance limit or within the acceptable tolerance limit.

Determine, $C_{l,i,j}$ & $C_{d,i,j}$ from airfoil lift & drag data,

Use, $\alpha_{i,j} = \phi_{i,j} - \theta_{p,i}$ ... (80)

Local thrust coeff: $$C_{T_{r,i,j}} = \frac{\sigma_i'(1-a_{i,j})^2 (C_{l,i,j}\cos\phi_{i,j} + C_{d,i,j}\sin\phi_{i,j})}{\sin^2\phi_{i,j}} \quad \ldots (81)$$

Update a & a' for the next iteration. If $C_{T_{r,i,j}} < 0.96$,



$$a_{i,j+1} = \frac{1}{\left[1 + \frac{4F_{i,j}\sin^2\phi_{i,j}}{\sigma_i' C_{l,i,j}\cos\phi_{i,j}}\right]} \quad \ldots (82)$$

If $C_{T_{r,i,j}} > 0.96$

$$a_{i,j} = \frac{1}{F_{i,j}}\left[0.143 + \sqrt{0.0203 - 0.6427(0.889 - C_{T,i,j})}\right] \quad \ldots (83)$$

$$a'_{i,j+1} = \frac{1}{\frac{4F_{i,j}\cos\phi_{i,j}}{\sigma_i' C_{l,i,j}} - 1} \quad \ldots (84)$$

Then the others parameter can be calculated and if not then the procedure starts again with the again starts with equation so so if converged then stop or start the procedure equation that means again we go back there and start with the situation process now now once we have solved the equation for performance so once performance at each blade section blade section or element is solved, then the power coefficient we can find out the power coefficient which is CP summation of I equals to one to N lambda R by lambda square FI sine squared PI cos phi i lambda r i sine phi i sine phi i i cos phi i into 1 minus cd by cl cot phi i into lambda square ri so if the total length of the hub and blade is assumed to be divided into n equal blade elements so basically the total length of the hub and blade by n equal length what we have filled r is i minus lambda r i which is lambda by n so what we can get that cp is 8 by lambda n summation of i equals to k to n fi and square phi cos phi r i phi sin phi cos phi 1 minus Cd by Cl, cot Ci, lambda Ri square. So, where A is the index of the first blade section consisting actual blade therefore so you need to use the essentially you need to have the actual blade section which will have this data.

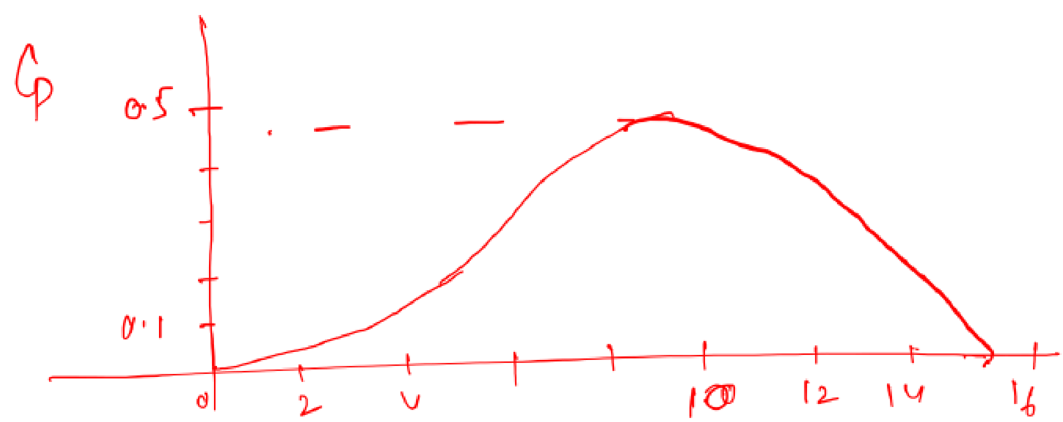
total length of the hub & blade is divided by N equal ... (85)
length, $\Delta\lambda_r = \lambda_{ri} - \lambda_{r(i-1)} = \lambda/N$ ... (86)

$$C_p = \frac{8}{\lambda N}\sum_{i=K}^{N} F_i \sin^2\phi_i \left(\cos\phi_i - \lambda_{ri}\sin\phi_i\right)\left(\sin\phi_i + \lambda_{ri}\cos\phi_i\right)\left[1-\left(\frac{C_d}{C_l}\right)\cot\phi_i\right]\lambda_{ri}^2 \quad \ldots (87)$$

'K' → index of the first 'blade' section consisting actual blade airfoil.

⇒ If Converged, STOP here — or we need to repeat

So now, if the design procedure is kind of complete, if all the parameters are within the acceptable tolerance limit, if the iterations have converged, then we can, so we can put like this, if converged, we can stop here or we can, or we need to Repeat. We need to repeat this design calculations once again so that you get this design data of this. But this is how, I mean, you can have an idea how this rotor blade design procedure works and how to estimate this parameter so primarily you have to select the airfoil you need to select some kind of a twist you need to see the variation of the chord i mean obviously once you select the airfoil at different section then the chord variations from hub to tip And then considering those losses and factors so that the induction factors is important, and then cell induction factor is important. And obviously, while doing that, you need those, I mean, the airfoil carb, like CL versus alpha carb, CD versus alpha carb, and the rest of the procedure can follow accordingly.



So now, what we can move on and look at I mean essentially now one can see this CP versus lambda curve. So, once the blade has been designed I mean once the blade has been designed so this curve we can plot zero one so this is lambda this is oh i have between here it goes like this so once the blade has been designed for optimum operation at specified design speed ratio the performance of the rotor over all expected speed ratios can be determined so essentially the optimum blade now use for various lambda. For each speed ratio, the aerodynamic condition at each blade section can be determined. And this from there, you can plot this kind of curve. And these curves are used in turbine design.

to determine so this CP versus lambda curves are used in wind turbine design to determine rotor power for any combination of wind and rotor speed. So, these curves are essentially important. So, what they can do, they provide immediate information on the maximum rotor power coefficient and optimum tip speed ratio. So, these curves provide maximum Cp and lambda optimum okay but obviously one has to be careful while finding these things but variations in airfoil lift or drag coefficients depend on the airfoil and the Reynolds number, so, all this can be needs to be considered in this particular procedure okay what we can look at it we can look at some horizontal axis wind turbine performance calculation procedure, which is a simplified one, simplified version of that. So, this was proposed by essentially proposed by Maxwell in 1990.

So, he proposed a simplified calculation for performing this horizontal axis wind turbine rotor, which is particularly applicable for an un-stalled rotor. If the rotor is getting stalled, then this analysis cannot be cannot be applied. But, also it could be useful for certain stall conditions. Obviously this method also uses the blade element theory that we have talked about and also incorporate this analytical method for finding the angle of attack, Cl, induction factors. Now depending on the t plus whether included few or no iterations are required if you include t plus definitely it goes in some iterative procedure if you do not include the t plus then this doesn't include energy so obviously, there are two conditions which it applies number one the airfoil section the airfoil section lift coefficient versus alpha relation must be linear in the region of interest which is simply tells that you stay within that linear variation of the Cl versus alpha curve, which is before stall.

Okay. Two, the alpha is small enough that that small angle approximation may be used, which is sine of alpha would be equivalent to alpha. So, that is very, very important. That means one is that you stay within the linear variation. Okay, you stay within that linear variation of the Cl versus alpha. And, then you kind of also say that the alpha is small enough.

And, then you can use this small angle. So, one can immediately, those who are familiar with the airfoil theory or airfoil aerodynamics, this clearly indicates this is applicable for unstalled region. I mean, this essentially you talk about L versus alpha curve and this is my stall then you are trying to restrict yourself in this particular zone. So, that means you try to stay within the unstalled. But as I have said that it can be applied to some stalled conditions for moderate angle of attack if the lift curve can be linearized.

Here, this simplified method is same as method one, which we have already talked about. But obviously there are some exceptions. I mean with exception of a simplification for determining alpha and Cl for each blade section. The essence of this simplified method is the use of an analytical or closed form expression for finding the angle of attack of the relative wind at each blade element. So, what it starts, so obviously, this is with a small variation in the small variation in the angle of attack what you can write the lift and drag curve so the lift and drag curve can be approximated as Cl equal to Cl naught plus Cl alpha into alpha.

And Cd equals to Cd naught plus Cd alpha 1 alpha Cd alpha 2 alpha square. When the Lift curve is linear and small angle approximations is used. So, what we can show that alpha is minus q2 plus minus q2 square minus 4 q1 q2 divided by 2 q3. So, where you have q1 equals to Cl0 into d2, 4F by sigma prime, d1 sine theta p. You have q2 equals to Cl alpha, d2 plus d1 Cl naught minus 4f by sigma prime d1 cos theta p d2 by theta p.

Lift & Drag curve can be approximated as:

$$C_l = C_{l,0} + C_{l,\alpha}\alpha \quad \ldots (88)$$

$$C_d = C_{d,0} + C_{d,\alpha_1}\alpha + C_{d,\alpha_2}\alpha^2 \quad \ldots (89)$$

We can show:

$$\alpha = \frac{-q_2 \pm \sqrt{q_2^2 - 4q_1q_2}}{2q_3} \quad \ldots (90)$$

Where,

$$q_1 = C_{l,0}d_2 - \frac{4F}{\sigma'}d_1\sin\theta_p \quad \ldots (91)$$

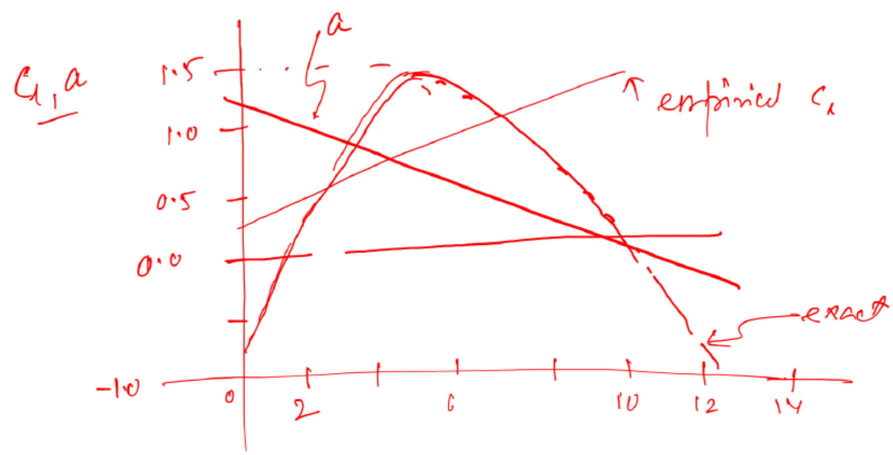
$$q_2 = C_{l,\alpha}d_2 + d_1C_{l,0} - \frac{4F}{\sigma'}(d_1\cos\theta_p - d_2\sin\theta_p) \quad \ldots (92)$$

$$q_3 = C_{l,\alpha}d_1 + \frac{4F}{\sigma'}d_2\cos\theta_p \quad \ldots (93)$$

Okay! And you have q3 which is Cl alpha d1 4f by sigma prime d2 cos theta p d1 equals to cos theta p minus lambda r sine theta p and d2 equals to sine theta p plus lambda r cos theta p what one so what you can do So, basically if I plot this 0 to 10. So, then this is minus 1, 1. so, what can i can have one curve going like that and coming like that so this is one then i have zero line then i have this line then i have this line so this is exact variation so this side is your variation of cl axial induction factor this is empirical Cl this is a how this varies so what happens is that using this approach the angle of attack can be calculated using the given equations once the initial estimate that you have then you can calculate the t plus factor also and then again you can use the this equations to calculate the cl drag coefficients and lift coefficients, all this. So, this simplified methods provides an angle of attack very close of those two more detailed method for operating the condition. So, this was actually, though we call it a simplified method, but this can very closely predict the behavior of actual conditions, obviously, depends on the airfoil and the flow conditions as we have talked about the applicability of this simplified method is essentially in this particular range where you are away from the stalled region but for certain high angle of attack stall condition this can be used also.

$$a_1 = \cos\theta_p - \lambda_r \sin\theta_p \quad \ldots (94)$$

$$a_2 = \sin\theta_p + \lambda_r \cos\theta_p \quad \ldots (95)$$



So, this is what how the design procedure work can but we will see other parameters effect in other session. Thank you.