Wind Energy

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Lecture 32: Blade shape optimization

Welcome back. So, we continue the discussion on this issues on the design. Now, what we'll talk about blade shape optimization. So, blade shape optimization. or rather blade shape i mean this is for one can think about optimum rotor with weak rotation, okay so, essentially what we'll talk that the blade shape for an ideal rotor which includes the effect of weak rotation that we can determine using the analysis that can be developed for general rotor. Now, the optimization actually includes weak rotation but ignores so essentially the optimization that includes weak rotation rotation but ignores Cd which is essentially considered to be zero that means the drag and t plus for which the f factor would be one now what one can do one can i mean how this uh let's say can be determined for optimum rotor with weak rotation.

So, here the procedure that we will discuss, we will discuss by considering the weak rotation, but at the same time we consider there is no tip loss, there is no drag. So, you can find out the optimum one by taking the partial derivative of the part of the integral of the CP, which is the power coefficient. Essentially, the Cp is function of angle of relative wind, flow angle, and other things. We can now find out that system.

by taking this derivative or so. So, what can do del del phi of sin square phi cos phi minus lambda r sin phi sin phi plus lambda r cos phi which is 0. So, this yields that lambda r equals to sine phi 2 cos phi minus 1 divided by 1 minus cos phi 2 cos phi plus 1. This is what you get. So, some more if you do doing some more algebra what we have p we get two-third tan inverse 1 by lambda r and we get quad is h pi r bcl 1 minus cos p Okay.

So, that is what you get. And also, we can calculate, we can calculate a from equation 30. Okay. And, other relevant equation. So, which is 1 divided by 1 plus 4 sine square phi sigma prime c l cos phi so that's what you get when you get a prime equals to 1 minus 3 a 4 a minus 1.

$$\frac{\partial}{\partial\varphi}\left[\sin^2\varphi\,(\cos\varphi - \lambda_r\sin\varphi)(\sin\varphi + \lambda_r\cos\varphi)\right] = 0 \quad \cdots \quad (54)$$

$$\Rightarrow \quad \lambda_r = \sin\varphi\,(2\cos\varphi - 1)/\left[(1-\cos\varphi)(2\cos\varphi+1)\right] \quad \cdots \quad (55)$$

doing some more algebra, $\varphi = \left(\frac{2}{3}\right)\tan^{-1}\left(\frac{1}{\lambda_r}\right) \quad \cdots \quad (56)$

$$C = \frac{8\pi r}{BC_l}(1-\cos\varphi) \quad \cdots \quad (57)$$

We can calculate 'a' from eq. 30 -

$$a = 1/\left[1 + 4\sin^2\varphi/(\sigma' C_l\cos\varphi)\right]$$

$$a' = \frac{1-3a}{4a-1}$$

Can compare with the results for an ideal blade without wake rotation.

for eq (20) & (21) $\quad \varphi = \tan^{-1}\left(\frac{2}{3}\cdot\frac{1}{\lambda_r}\right), \quad C = \frac{8\pi r}{BC_l}\left(\frac{\sin\varphi}{3\lambda_r}\right)$

so these results one can compare with the results for an ideal blade without work rotation. So, which you get from essentially from equation 20 and 2, you have Phi equals to tan inverse 2 by 3 by lambda r and C equals to 8 pi r by BCl sine phi by C lambda r. Now one has to note that the optimum values for Phi and C including rotation are often similar to but could be significantly significantly different from those obtained without assuming weight rotation. So, this optimum value including weight rotation can be significantly different from values without which is quite obvious because the wave rotation takes into account a lot of physical processes that usually takes place in realistic scenario. Also, as before we have done, we can select alpha where Cd by Cl is Now, solidity again is the ratio of that platform area of the blades to the surface area.

So, what we get the solidity sigma 1 by pi r square r is to r here dr. Now, again one can find out the optimum blade rotor solidity. And, when the blade is modeled at set of in blade section of equal span, so the blade is modeled as in blade sections of equal span. then we can have solidity defined at B by n pi summation of i equals to 1 Ci by R, so, that's how you can also find out the optimum blade rotor solidity okay now blade shapes or sample optimum, I mean, for different conditions would be obtained. I mean, assuming different lambda value, different V values.

Solidity: $\sigma = \frac{1}{\pi R^2}\int_{r_h}^{R} c\, dr$ . . . (58)

Blade is modeled as 'N' blade sections of equal span,

$$\sigma \cong \frac{B}{N\pi}\left(\sum_{i=1}^{N} c_i/R\right) \quad . . . (59)$$

So, that includes the weak rotation and obviously, but that's how you can use these parameters to find out those things, but one has to kind of consider that would show different kind of variation. I mean, one of the variation that one can have that the slow, I mean any particular slow 12 bladed for example machine would have blades that had a roughly constant chord over outer half of the blade and smaller chords closer to the half. The blades can have significant twist. Also, faster machines would have blades with increasing cord as one went from the tip to hub. Blade twist could be significant for different things.

So essentially, one can use this to find out the optimum blade tip for that. Now, what we'll talk about now we can talk about in general some rotor design procedure which is in general way we will talk about that. So, the blade optimum or the other factor that we have talked so far those can be used in generalized this rotor design procedure. This procedure begins with the choice of various rotor parameters and the choice of airfoil. Obviously, this includes the choice of airfoil and other associated parameters.

Okay. so, obviously one can assume or one can kind of determine the initial blade shape by assuming weak rotation considering weak rotation and then the final blade shape and performance can be determined iteratively considering so initial design or shape can be considered by including weak rotation and then the final shape can be determined by considering drag, tip loss, other manufacturing factors etc. That means what you can do, the analysis that we have so far discussed upon or talked about. So, the idea here is that you start with some initial shape or the blade and that you determine by considering the equations that we have obtained. But, obviously you consider the weak rotations. And then when you move on to really optimize the thing, then the final shape you can optimize or design iteratively, obviously.

That time you consider all your drag loss, tip loss, other manufacturing constant, so that how much twist or the quad at different sections could be allowed to have proper tolerance and all this. So, there are two-stage procedure. you have some initial design and then you consider each of these issues in practice that you encounter and then go to the final set. So, the first stage could be let's say the determination of basic rotor, So, that would be the first test to do. I mean, once you get the basic parameters, you can find the final parameters through iteration.

To begin with, one can decide what power is needed at a particular wind velocity, u. One can include the effects of probable Cp and efficiencies η for components such as the gearbox, generator, and pump, etc. So then we can have P = Cp η (1/2) ρ π r² u³. So, that means you are considering some power coefficients and some efficiencies of mechanical components like the gearbox, generator, pumps, and all these things, and then, based on the wind speed and the type of application, you have to choose the tip speed threshold, lambda. Let's say lambda a depends on the type of applications; for example, if you are doing water pumping with a windmill, then you need greater power.

So, your lambda should be between one and three for electric power generation. You can use Lambda between four and ten times. Okay. So, if you have a higher-speed machine that uses less material in the blades and has smaller gearboxes, it requires a more sophisticated aircraft. So, what you can say is that higher-speed machines use less material in the blades and have smaller gearboxes but require more sophisticated gear systems.

1 Determination of Basic Rotor parameters

(a) $P = C_p \eta \left(\frac{1}{2}\right) \rho \pi R^2 U^3$ ... (60)

(b) $\lambda \longrightarrow$ depends on type of application.

For : Water pumping windmill — greater torque needed. $1 < \lambda < 3$

Electric power generation — $4 < \lambda < 10$

Higher speed machines use less material in the blades & have smaller gearbox, but require more sophisticated airfoils.

Now, you need to choose a number of blades, which is B, so if three blades are selected, then there are a number of structural dynamics problems that must be considered in half design. One possible solution would be to use a fitted half; typically, three blades are used. And if it is something less than three blades, then one must now select an airfoil; if your lambda is less than three, then carb plates can be used; if lambda is greater than three, use a more aerodynamic step. Okay. What you can do now you can so that was our basic rotor parameter.

Now, what we will do is define the shape of the blade. So, here we can obtain and examine the empirical curves for the aerodynamic properties of the airfoil in each section. Essentially, what we need are the airfoil curves, which means Cl versus alpha and Cd versus alpha, and then we need to choose the design conditions, which means Cl design and Cd design, such that Cd design divided by Cl design is minimized at each blade section. So, this is something very unique because one has to use these empirical curves of the aerodynamic properties of the airfoil in each section. So, essentially one has to use this for each section.

So, that means there are different airfalls can be. Then, we can divide the blade into a number of elements and use optimal rotor theory to estimate the shape of each blade. So, we can divide the blade into n elements and estimate the shape of the blade's height using optimal rotor theory. okay! so, that means we can have lambda r i equals to lambda ri by R that is one then we have phi i two-third tan inverse one by lambda ri 62, we have c i 8 pi ri B Cl design i1 minus cos phi 63, then we have theta T i theta p i minus theta p 0 which is 64 so this is the twist angle and then p i Theta p i plus alpha design i, so that's what you get here. Alpha is the angle of attack; this is the pitch angle, so this is the initial pitch angle, or I mean the pitch angle at the top.

$$\lambda_{r,i} = \lambda (r_i/R) \quad \cdots (61) \qquad \varphi_i = \left(\frac{2}{3}\right) \tan^{-1}\left(\frac{1}{\lambda_{r,i}}\right) \quad \cdots (62)$$

$$c_i = \frac{8\pi r_i}{B\, C_{l,design,i}} (1 - \cos\varphi_i) \quad \cdots (63) \qquad \theta_{T,i} = \theta_{p,i} - \theta_{p,0} \quad \cdots (64)$$

$$\varphi_i = \theta_{p,i} + \alpha_{design,i} \quad \cdots (65)$$

So you get all this at the shape parameters of the blade for that. Now, using this optimal blade shape as an initial guide, we can select a blade shape that promises to be a good approximation. And also, for each fabrication, some linear variations of the cord, thickness, and twist might be chosen. For example, what we can say is that now, using the optimum blade initial data, we can select a blade shape that promises a good approximation; for ease of application, some linear variations of pod thickness twist are chosen. For example, if a1, b1, and a2 are coefficients for the chosen quad and twist distribution coefficients, then ci is a1 * ri plus b1 * theta i plus a2 * (r - ri).

This is B1. 67. So if A1, B1, and A2 are the coefficients for a chosen quadrant twist, then the quadrant twist can be executed. So, these are some examples of how this linear variation could be. Now, the final part of that would be to calculate the router's performance and modify the design accordingly. So, here I mean what one has to do. I mean, now we can use the method that we have already talked about, either method one or method two, and then consider, I mean calculate the performance, and then accordingly, you can modify the design of the blade.

For example, if a_1, b_1, a_2 are coeff.s

$$C_i = a_1 r_i + b_1 \quad \ldots \quad (66)$$

$$\theta_{T,i} = a_2(R - r_i) \quad \ldots \quad (67)$$

Okay, so what we can do is solve for our method one again. L and an alpha. So, we can find the actual angle of attack and lift coefficients for the center of each element by using the equations and empirical curves. So, like C_L equals 4 F_i sine phi_i, we have cos phi_i minus lambda r_i sine phi_i, and sigma prime_i sine phi_i plus lambda R_i. So that may be what we are trying to solve for C_L and alpha, but the actual angles of attack and lift coefficients are for the center of each element.

So, here sigma prime i is BCi by 2 pi ri, and we have Cl equal to alpha i, which includes the initial pitch angle, twist angle, and all of this, and our fi is 2 by pi cos inverse exponential b by 2, 1 minus ri by r. Then we have r. Now, the lift coefficient and angle of attack can be found by iteration or graphically. Typically, the graphs show that one can have a variation of alpha, which is CL; it goes like this, and there is a curve.

③ Calculate Rotor performance & modify design of blade

Method 1: Solve for C_ℓ & α

$$C_{l,i} = 4F_i \sin\varphi_i \frac{(\cos\varphi_i - \lambda_{r,i}\sin\varphi_i)}{\sigma_i'(\sin\varphi_i + \lambda_{r,i}\cos\varphi_i)} \quad \ldots \quad (68)$$

$$\sigma_i' = \frac{BC_i}{2\pi r_i} \quad \ldots \quad (69)$$

So, this is alpha I, this is CLI. So Cl and alpha are found either iteratively or graphically from the CL versus α curve. But obviously, if someone wants to do it iteratively, it requires an initial estimate or guess. So, guess the tip-loss factor. To find a starting FI, we can start with an estimate for the angle of relative wind of π/1.

2, using tan inverse(1/λRi). And then, in subsequent iterations, we can find f_i using y_{ij+1}, p_i, and alpha_{ij}. So here is the number of iterations; it basically proceeds iteratively. Usually, only a few iterations are needed to get a converged solution, and then finally, we calculate the axial induction factor, which is ai, by dividing 1 plus 4 sin squared phi i by sigma i cli. So, this is the axial induction vector, and then you need to check if Ai is greater than 0.

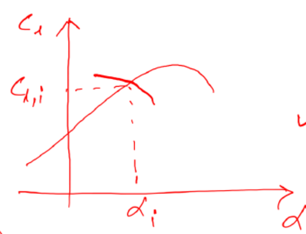
4. If it is greater than 0.4, then we cannot use this design consideration; we have to use Method 2 instead. Discussed earlier, but we will have that discussion here as well. Essentially, this method involves trying to solve for the CL, alpha, and the initial angle of attack. You calculate the CL using equation 68, and you also calculate the solidities, then calculate the phi and fi, and all these things.

$$\phi_i = \alpha_i + \theta_{T,i} + \theta_{p,0} \quad \ldots \quad (70)$$

$$F_i = \left(\frac{2}{\pi}\right)\cos^{-1}\left[\exp\left(-\left\{\frac{(B/2)[1-(r_i/R)]}{(r_i/R)\sin\phi_i}\right\}\right)\right] \quad \ldots \quad (71)$$

$C_l, \alpha \longrightarrow$ interatively / Graphically

requires some initial estimate/guess of tip loss factor.



To find startin F_i,

$$\phi_{i,1} = \left(\frac{2}{3}\right)\tan^{-1}\left(\frac{1}{\lambda_{r,i}}\right) \quad \ldots \quad (72)$$

in subsequent iterations, find F_i using $\phi_{i,j+1} = \theta_{p,i} + \alpha_{i,j}$ .. (73)

j = no. of iterations.

Finally, $a_i = 1/[1 + 4\sin^2\phi_i/(\sigma_i C_{l,i}\cos\phi_i)]$ .. (74)

check if $a_i > 0.4$

Then, you try to... Get the Cl and alpha through an iterative process or graphical process like this, where you have the plots, and then in each section, you use some graphical method. However, the iterative method is slightly more robust in that you start with some initial guessed factor of phi and then calculate phi i. Once you converge to phi i, you then get the axial induction factor. So, that's how you find the Cl and alpha and all these things, but once you find ei. We check whether this is valid or not and then move to the other one, which we'll talk about in the next session.