Wind Energy

Prof. Ashoke De

Department of Aerospace Engineering, IIT Kanpur

Lecture 30: Rotor blade design

Welcome back. So, we continue our discussion on this particular session of the wind energy. So, what we have obtained in our earlier session is that combining both the momentum theory and the blade element theory, and then for no-wick rotation or with-wick rotation, the equation system. For the design purposes, for an ideal rotor that we would like to calculate this a prime and all this. So, this is what what we got is this the different equations that you can see. And, now there could be some I mean, the way we can solve these things is that like there could be two solution methods so, one could be typically numerical i mean measured airfoil characteristics and the equations for a solve directly c and a This could be solved numerically, and other approach can use some kind of flow conditions, blade, and all these things.

And then second solution could be iterative numerical approach that is most easily extended for flow conditions with large axial induction factors and things like that. So, we can adopt one of the solution methodology. And then we can see how this could be solved for design purposes. So, now we have two solution methods, essentially.

Now solution methods. One case, what you do, let's say the method one, what you do, is that the solving for CL and A. Since you have P equals to alpha plus theta P for a given blade geometry and operating condition. So, there are two unknowns that you have. Okay.

So, there are two unknowns that you have. which you can actually uh so so this is you have for a given blade geometry and operating conditions so that you have uh so there are two unknowns in equation 27. So, that is Cl and alpha at each section. So, to find these values, one can use empirical. So, to find these values, you can use spherical Cl versus alpha curve okay for airfoil chosen airfoil and then one can find out Cl I mean this Cl and alpha to satisfy equation 27, so, this could be either done numerically or graphically okay once cl and alpha have been found so once cl and alpha found then one can find out a prime a using equation 28 and 31.

Now, then one can verify that then axial induction factor at the intersection point of the curve. So, one has to verify A prime at the intersection point is less than 0.5. to ensure that the results are valid. So, that's what one can do.

Essentially, you use the typical CL versus alpha curve For a given aerofoil, this is your stall point. So, we use that these are standard Cl versus alpha. So, you have standard Cl versus alpha curve, which one can use. And then from there, you find out this and once Cl and alpha is fixed, then what you can do, use that to find out A prime and A using this equation. So, this is an solving for the Cl and alpha.

Method 1: Solving for C_l & a

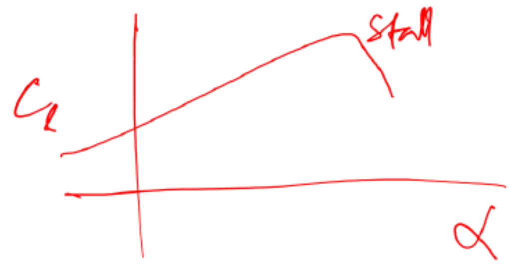
Since: $\phi = \alpha + \theta_p$ — for a given blade geometry & operating conditions

2 unknowns — in eq-(27) → C_l & α at each section.

Empirical C_l Vs α Curve → C_l, α → to satisfy eq. (27)

Once, C_l & α, → found → a', a (using eq- 28, 31)

Verify: a' at the intersection point < 0.5 to ensure that the results are valid.



(Standard C_l Vs α)

Now, the second method would be iterative solution for A and A prime. Okay. So, this is an iterative process so obviously one has to solve numerically. So, this starts with guesses of A and A prime from which flow conditions and new induction factors can be calculated. New induction factors can be calculated.

So, the process starts like gives values of A and A prime to One can calculate the angle of the relative wind. So, one can use equation five. Then one can calculate the alpha. from theta equals to alpha plus theta p and then so from p equals to alpha plus theta p sorry and then cl and ed okay then what can you can update A and A prime from equations 25, 26, 30 and 31. So, these are the equations you can use, do them.

So, this process is repeated until the newly calculated induction factors are within some acceptable tolerance of previous one. This is a kind of an iterative process. until new induction factor is found to be within tolerance limits. This method is especially very useful for highly loaded rotor condition. very useful method for highly loaded rotor condition.

Now, what we will do, we can calculate the power coefficients. so, calculation of power coefficient so once a is obtained for from each section then the overall rotor power coefficient may be calculated. Then the overall rotor power coefficient may be calculated from the below equation, which is Cp, eight by lambda square, lambda H two lambda, lambda R cube, one minus A, Cd by Cl, So, this is how you kind of get this. So, here lambda is the local speed ratio at the hub. So, the equivalently what we can write Cp equals to 8 by lambda square lambda h to lambda sine square phi cos phi minus lambda r sine phi sin phi plus lambda r cos phi into 1 minus cd by cl cot phi lambda r square d lambda r.

Usually these equations are solved numerically. As we will discuss it little later. This would be solved, how we can solve this thing numerically. Here also even we can determine the axial induction factor assuming Cd equals to zero. Anyway the drag has been included in the power coefficient calculation.

So, we can see, I mean basically the derivation of this equation, the derivation of equation 32. So, we can see how we can derive that. So, let us say the power contribution from each annulus is dp omega dq. So, here omega is the rotor rotational speed. So, the total power from rotor is P which is RH to R dP that is RH to R omega dQ that is 35.

$$C_p = \frac{8}{\lambda^2} \int_{\lambda_h}^{\lambda} \lambda_r^3 a'(1-a)\left[1 - (C_d/C_l)\cot\varphi\right] d\lambda_r \quad \cdots (32)$$

λ_h = local speed ratio at the hub.

equivalently:

$$C_p = \frac{8}{\lambda^2} \int_{\lambda_h}^{\lambda} \sin^2\varphi\,(\cos\varphi - \lambda_r \sin\varphi)(\sin\varphi + \lambda_r\cos\varphi) \times \left[1 - \left(\frac{C_d}{C_l}\right)\cot\varphi\right] \lambda_r^2\, d\lambda_r \quad \cdots (33)$$

— Derivation of eq-32

The power contribution from each annulus is $dP = \Omega\, dQ \quad \cdots (34)$

Ω = rotor rotational speed.

Total power from rotor is: $P = \int_{r_h}^{R} dP = \int_{r_h}^{R} \Omega\, dQ \quad \cdots (35)$

Now rh is rotor radius at hub of the blade and the power coefficient then we can have the power coefficient Cp, P by P wind. So we have rh toR omega d omega of rho pi R square U cube. Now, we can use the differential torque from equation 23 and the local speed ratio. So, these two definitions we can use. and what we get is Cp equals to 2 by lambda square lambda H2 lambda sigma prime Cl 1 minus 1 by sin phi 1 minus Cd by Cl cot phi lambda r squared d lambda r now where again lambda it is the local tip speed ratio at the hub what we have from equation 26 and 29 we have sigma prime Cl one minus a four a sine squared phi by cos phi which is set it.

So, we have A tan phi A prime lambda r. Now we substituting this you these in equation 37. So, what we get is essentially equation 32. That is a Cp equals to eight lambda square lambda h lambda lambda r cube a prime 1 minus Cd by C A cot phi. So, this is what you get which is your equation.

The power coefficient $(C_p) = \frac{P}{P_{wind}} = \frac{\int_{r_h}^{R} \Omega \, dQ}{\frac{1}{2}\rho \pi R^2 U^3}$ ... (36)

Use the differential torque from eq-(23) & local tip speed ratio

$$C_p = \frac{2}{\lambda^2}\int_{\lambda_h}^{\lambda} \sigma' C_l (1-a)^2 \left(\frac{1}{\sin\varphi}\right)\left[1-\left(\frac{C_d}{C_l}\right)\cot\varphi\right]\lambda_r^2 \, d\lambda_r \quad \ldots (37)$$

From, eqs-(26) & (29) $\quad \sigma' C_l (1-a) = 4a \sin^2\varphi / \cos\varphi$ ... (38)

$a \tan\varphi = a' \lambda_r$ ... (39)

Substitute these in eq-(37), we get $\equiv$ eq-(32)

$$C_p = \frac{8}{\lambda^2}\int_{\lambda_h}^{\lambda} \lambda_r^3 \, a'(1-a)\left[1-\left(\frac{C_d}{C_l}\right)\cot\varphi\right] d\lambda_r$$

So, when you have Cd equals to 0, the Cp is the same as 1 derived from the momentum theory including weight rotation.

When $C_d = 0$, $\quad C_p$ – as obtained using mom. theory with wake rotation.

So, Cp as obtained using momentum theory with rotation. So, obviously, if someone tries to derive these things, so that is absolutely kind of possible so that means you can see how this equations here that we got for 32 and we can i mea obviously 33 is little bit more involved but yes one can find once you get the 32 so how we get obtained did i 32 and this is a special case when cd is 0 then you can get the cp Obviously this second term in this expression here that goes off. So, it remains a lambda r cube a prime minus by one minus a which is d lambda and then you can have integration of lambda r cube which is lambda to the power four by four and you get the same Cp as you have obtained using the momentum theory with quick rotation. so, that's how you obtain that now we try to look at the other factor that is tip loss that said we try to look at the tip loss so, that means what would be the effect of tip loss on power coefficient so the, what happens is that pressure on the suction side of the blade is lower than on the pressure side and air tends to flow around the tip from the lower to upper surface and that's why they are reducing the lift and hence power production near the tip.

This effect is most noticeable essentially if you look at the blade aerofoil this is the pressure side and this is the suction side so once the flow goes around the tip of the finite blades so that is if you see this is a finite blade and at some point of time you have this tip so this is the tip when this glows around the tip from the lower to upper surface, that means it goes from like that, that reduces the lift. And obviously, whatever power you get at the tip. So, if you have fewer and wider blades, so this effect is noticeable okay so that means if you have that then your effect is very much so there are number of methods which has been suggested for including the effect of tip loss so the most straightforward approach to use by Prandtl so consider tip loss in calculation. So, what Prandtl has suggested a correction factor of if. So, correction factor of if to be introduced and this correction factor is a function of number of blades.

So, this is function of number of blitz and the angle of relative wind and the position of blitz. based on Prandtl method what you have this correction factor f is given by 2 pi pi cos inverse exponential minus b by 2 1 by r by r r by r sine phi. So, that's what it is. Here the angle that is this angle so one can equivalently write that 2 by pi cos inverse something called let's say some m. So this angle m that is coming from this inverse cosine function is in radian i mean it assumed to be in radian if this inverse cosine function is in degrees then the initial factor 2 by pi is to be replaced by 1 by 90.

so in this expression this is in in radian if it has to be in then that 2 by pi has to go by 1 by 90. So, the interesting part F is always belongs to 0 to 1 and that the tip loss correction factor characterizes the reduction in the forces at radius r along the blade and due to that tip loss at the end of the blade. So, obviously This tip loss is going to affect the forces which we have obtained from the momentum theory. So, that can be rewrite as, so which is equation 1 and 2, we can write that dt equals to f rho u square 4a 1 minus a pi r dr. So, this incorporates the tip loss.

Based on ~~Rao~~ Prandtl's method:

$$F = \left(\frac{2}{\pi}\right) \cos^{-1}\left[\exp\left(-\left\{\frac{(B/2)[1-(r/R)]}{(r/R)\sin\phi}\right\}\right)\right] \quad \cdots (40)$$

$$\equiv \left(\frac{2}{\pi}\right) \cos^{-1}(M)$$

Angle $\to M \longrightarrow$ assumed to be in radian.

$F \in [0,1]$

4F a prime 1 minus a rho u pi r cube omega dr so these two equations here the new equation so the you have a t plus correction factor which comes into the picture so the equations which are based on the definition of the forces used in the blade element theory remain unchanged so that means all equation in blade element theory remains unchanged, okay! so, when the forces from momentum theory and the blade element 3d are set equal using methods of this strip theory so the flow conditions derivations is going to be changed. One can find this so and obviously what one has to do, one has to take into consideration this tip loss effect into those derivations. So, what one needs to consider this tip loss factor, consider this tip loss factor in derivations or equations so how they are going to be changed one can have like a prime by 1 minus a equals to sigma prime cl by 4 f lambda r sine phi this is my new equation 43 a by 1 minus a equals to sigma prime cl cos phi 4F sin square phi 44 Cl 4f sin phi cos phi minus lambda r sin phi sigma phi sin phi plus lambda r cos phi. this is 45 then you have a prime by 1 plus a prime sigma prime cl 4 f cos p which is i mean essentially both of them are clubbed together in 45 a and b then we have a equals to 1 by 1 plus 4f times square phi divided by sigma Tl cos phi.

So, that is 46. Then you have a prime. a prime equals to uh 1 by 4f cos p divided by sigma by cl minus 1 47 and then finally what you have you have u rel equals to u into 1 minus a by sign fee which is u by sigma prime cl by 4 f what phi plus sign field okay so which uh so obviously the power coefficients cp can be calculated lambda h to lambda f lambda r cube a prime one minus a one minus c d by c l for theta into d lambda r or you can write c p equals to eight by lambda square lambda h into lambda f sine square phi cos phi minus lambda r sin phi sin phi lambda r cos phi 1 minus Cd by Cl cot phi lambda r square d lambda r. So these are the equations which actually kind of takes into account the T plus correction factor. So, this T plus correction factor would be incorporated. So, all these equations we have earlier looked at it, but here we are considering or incorporating this T plus factor.

Mom. theory (eq. 1 & 2):

$$dT = F \rho U^2 4a(1-a)\,\pi r\,dr \quad \cdots (41)$$

$$dQ = 4F a'(1-a)\,\rho U \pi r^3 \Omega\,dr \quad \cdots (42)$$

All eq. in Blade element theory remains unchanged.

Consider tip losses factor in derivations / eq.

$$\frac{a'}{1-a} = \frac{\sigma' C_l}{4F \lambda_r \sin\varphi} \quad \cdots (43)$$

$$\frac{a}{1-a} = \sigma' C_l \frac{\cos\varphi}{4F \sin^2\varphi} \quad \cdots (44)$$

$$C_l = 4F \sin\varphi \frac{(\cos\varphi - \lambda_r \sin\varphi)}{\sigma'(\sin\varphi + \lambda_r \cos\varphi)} \quad \cdots (45a)$$

$$\frac{a'}{1+a'} = \frac{\sigma' C_l}{4F\cos\varphi} \quad \cdots (45b)$$

$$a = \frac{1}{\left[1 + 4F \sin^2\varphi / (\sigma' C_l \cos\varphi)\right]} \quad \cdots (46)$$

$$a' = \frac{1}{\left[4F\cos\varphi / (\sigma' C_l) - 1\right]} \quad \cdots (47)$$

$$U_{rel} = \frac{U(1-a)}{\sin\varphi} = \frac{U}{(\sigma' C_l / 4F)\cot\varphi + \sin\varphi} \quad \cdots (48)$$

So, this is what happens in step by step, one of the other factor that you need to take into consideration while doing the design. Okay, we'll talk about all these in more details in the next section. Thank you.