Wind Energy

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Lecture 03: Fluid Mechanics- basics

Welcome back this particular discussion. So now what we have discussed is the renewable energy, types of renewable energy, national status, some of the worldwide status and all these things. So, now what we are planning is that we'll discuss now slowly go towards discussion of the wind energy in more detail. So, when we go in detailed discussion of the wind energy, one of the fundamental things that is required or the knowledge that one should have in the fluid mechanics. Because when you talk about, as I've been talking about this, that mostly what we are going to look at the aerodynamics perspective of this wind energy obviously later stage we will touch upon some of these design related constant and things like that, but primarily the discussion would be confined in the context of the aerodynamics and all these things so, obviously to do that the fundamental things for doing such things is the fluid mechanics so what would go about it, will have our this fluid mechanics that will do some review and then we'll move to the aerodynamics so that one will have some knowledge about basic fluid mechanics and then we can easily discuss about fluid mechanics and things like that okay. so first point which comes is what is fluid so you can see this nice picture here where some of these water drops which are falling in a pool of water so water is definitely fluid.

so unlike other things fluid is also made of discrete molecules and obviously whatever we will do in this particular course it will be treated as continuum. So, we are going to consider everything under continuum hypothesis that means nothing is going to violate this continuum hypothesis as of now. So, that means When you talk about that, which means if you look at the continuum approximation, so small fluid volume, sometimes we call it a fluid particle, contains many, many atoms per molecule. So, that means these fluid particles are going to contain small atoms per molecule.

So, that is one of the continuum approximation. Obviously, alternative way one can think about, it's a length scale is very, very larger than the molecular length scale. Obviously, that brings to some kind of a number, the Knudsen number. That defines the thing. Obviously, these continuum scales are preferred choice for any kind of engineering applications because most of the applications we are dealing with are in nature in continuum.

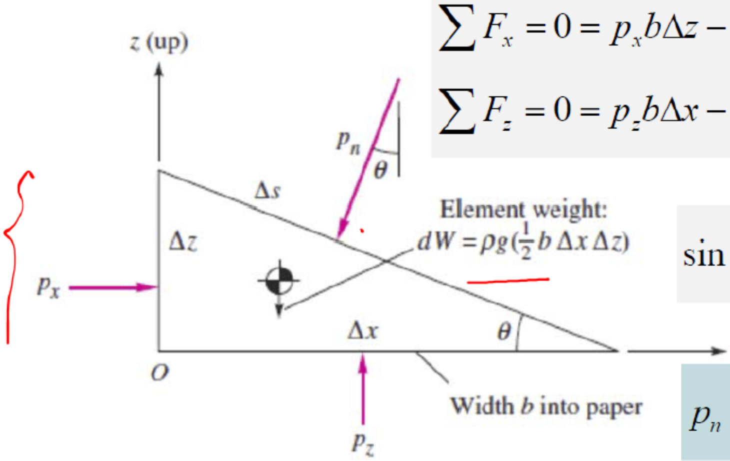
So, that's what. Now, the first thing when you talk about fluid mechanics, obviously one has to, I mean, one can go through the textbook of the fluid mechanics to review their knowledge and all these things. What here we are planning to do, we are going to touch upon the basics of fluid mechanics and then we would move through the aerodynamics of the wind turbines and things like that. So, the point here is that. Different subtopics that we would discuss here or touch upon.

Some detailed discussion or information if someone has read it, then you are always requested to refer to the text. Now, coming back to that, the first thing that we will talk about is fluid statics. So, again the question comes back is that what is fluid? The fluid, the material that flows or rather deforms continuously under shear stress. So, here is a picture which you can see this could be solid or fluid, this could be solid or fluid, this is fluid only because this continuously deforms. From this picture, when there is a force which is applied in the top surface, You can see the deformation that takes place in these two pictures.

So, fluid is something which continuously deforms under shear stress. So, which other way one can say that under static condition fluid sustains only normal stress. While solid sustains to deform shear stress, fluid flows under shear. So, zero shear here prevails in static fluid. This is kind of a free surface, this is the liquid and this is the force diagram which is going to happen.

This is obviously connected with the solid, because solid would deform to some loading but it will try to sustain the field; whereas fluids try to flow okay. now if we carry forward the same logic or the same statement here the, we can derive the equation for fluid static so this is an infinitesimal fluid element here. obviously these are the coordinate direction so x and z then this direction normal pressure p x this is p z and weight of the element which is calculated. this is -this hypotenuse normal pressure at the angle so we can do force balance so x direction force balance z direction force balance and obviously both the directions we can do the force balances and we can get this sine theta cos theta - essentially what we get is that px equals to pz minus half rho dz and if the infinitesimal fluid element delta z goes to zero then these are all same. so pressure at a particular point is independent of direction so that becomes independent of scalar.

Equilibrium of a fluid element



$$\sum F_x = 0 = p_x b\Delta z - p_n b\Delta s \sin\theta$$

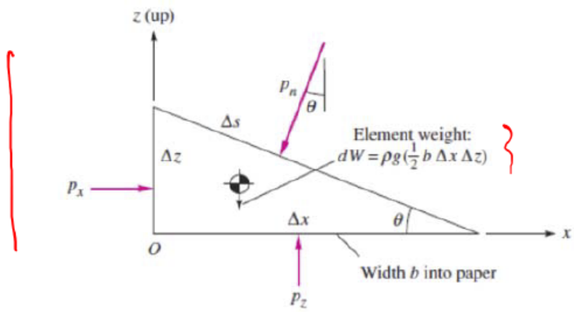
$$\sum F_z = 0 = p_z b\Delta x - p_n b\Delta s \cos\theta - \frac{1}{2}\rho g b\Delta x\Delta z$$

$$\sin\theta = \frac{\Delta z}{\Delta s};\cos\theta = \frac{\Delta x}{\Delta s}$$

$$p_n = p_x = p_z - \frac{1}{2}\rho g\Delta z$$

For an infinitesimal fluid element $\Delta z \to 0 \Rightarrow p_n = p_x = p_z$

Static equilibrium of a fluid element



Body force

Surface force

$$p_x(b\Delta z) - p_n(b\Delta s\sin\theta) = 0 = p_z(b\Delta x) - p_n(b\Delta s\cos\theta) - \rho g\left(\frac{1}{2}b\Delta x\Delta z\right)$$

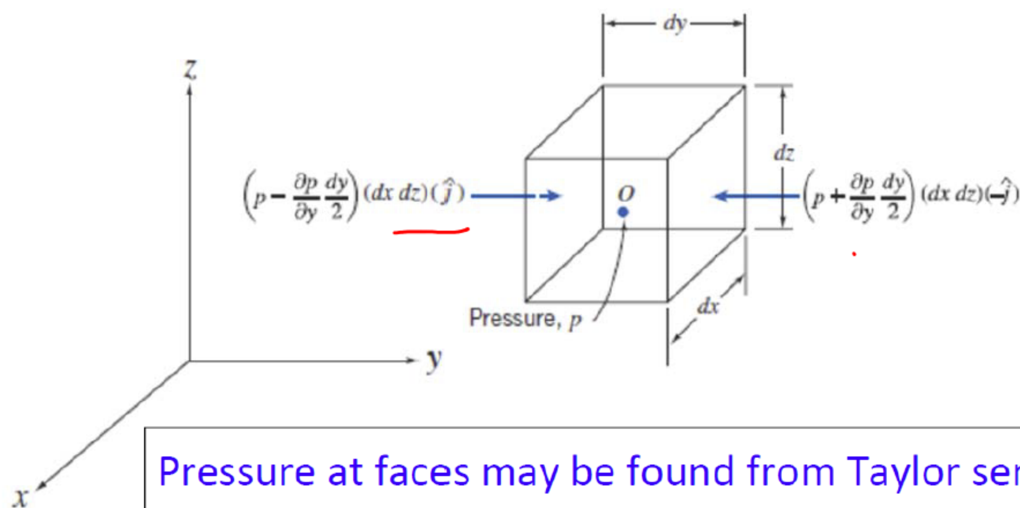
Static equilibrium: Σ(surface forces + body forces) = 0

so this was already kind of achieved by this famous experiments which is known as Pascal's barrel buster experiment. So it was conducted in 1646. So where, I mean, obviously, you can see this picture, which is available in the internet, then you can see how somebody is getting the top and you have this. So this is a nice experiment where we get to know about the pressure and all these things but this is a famous experiments in the fluid mechanics. now coming back to this particular fluid element and having this force distribution so the static equilibrium of this fluid element what we can say that if you write down the forces and you can identify all these forces are surface force whereas this is the body force which is coming from this component.

So, when you talk about the fluid element in static equilibrium, then the surface forces and the body forces collectively or the sum of that has to be zero. So obviously, one can have an understanding that as the size of the fluid element reduces, the body force drops faster than the surface force. That's how the connectivity between the surface force and the body force that one can figure that out. But this is what happens when the fluid elements are kind of in equilibrium.

Okay. From here, one can derive the equation for the fluid static. So, we've taken this kind of small cuboid. Then we write down the pressure on different surfaces. Obviously, you can write this pressure using simple Taylor series expansion. okay and then neglect the higher order term so you could establish that thing.

Governing Equation of fluid statics



Pressure at faces may be found from Taylor series expansion (neglecting higher order terms)

$$p_{y \pm dy/2} = p \pm \frac{\partial p}{\partial y}\frac{dy}{2} + \text{...higher order terms}$$

So, what we can do let's say if i try to write or net surface force in the y direction that means this is the direction we are talking about okay then we write the net surface force in term of pressure this is coming from this then this which will get me $-\frac{\partial p}{\partial y}$. now if i consider all three directional forces then this is what i get the surface force in all three directions okay. then collectively i'll have that now the second thing comes as the body force. so which is weight of the fluid element so this is straight forward so which will be ***ρ***, g, v, i mean obviously double kind of an one can think about mg actually okay. now the total force on that particular fluid element would be the surface force and body force this is what it is now.

$$d\vec{F}_{sy} = \left(p - \frac{\partial p}{\partial y}\frac{dy}{2} \right) dxdz\left(\hat{j}\right) + \left(p + \frac{\partial p}{\partial y}\frac{dy}{2} \right) dxdz\left(-\hat{j}\right) = -\frac{\partial p}{\partial y}\hat{j}d\forall$$

Total surface force, considering all three directions:

$$d\vec{F}_s = -\left(\frac{\partial p}{\partial y}\hat{i} + \frac{\partial p}{\partial y}\hat{j} + \frac{\partial p}{\partial y}\hat{k} \right) d\forall = -\nabla p d\forall$$

For a static fluid element $d\vec{F} = 0 \Rightarrow -\nabla p + \rho\vec{g} = 0$

Governing Equation of fluid statics $\boxed{\nabla p = \rho\vec{g}}$

If the static fluid element the total force is zero so this goes to zero so what i get is that ∂p is $\overline{\rho g}$. So, obviously this concept can be extended for the rigid body motion as well, but we will not go in discussion of that. I mean those who are interested, they can see through the textbooks and all these things that how that thing can be achieved. Now coming to the fluid part, second is that, we have got that basic equilibrium of the fluid statics we can talk about kinematics so which will describe the fluid motion. so how to describe the fluid motion okay.

so there are two ways one can describe one is the lagrangian other is the eulerian so which is again material or total derivative or the partial derivative so all this would come into the picture. so now, first let's look at the lagrangian derivative part or description part so here what we do that here we our primary attention is fixed to a particular mass of fluid as it flows so what we try to do this is an example of a billiard boards where we try to look at this each of these individual balls and their motions that means, we fix each of these particles and try to track them. So that means you track their position vector, velocity vector of each fluid particle. Obviously, any fixed mass system is a function of time. So you try to locate their positions, velocity, location.

So essentially, the individual fluid particles are always tracked in this lagrangian description, while we talk about the Eulerian description we focus on some control volume which could be imaginary usually fixed in space and then fluid flows in and out of the control volume for example this is the control volume this whole thing is the control volume and the surface which are doing that that is called control surface and here the fluid goes in it goes out so usually in the Eulerian concept we do not track each particle instead we rely on the field variable that means - Something coming in, going out. Obviously Lagrangian description helps in deriving the equation. Eulerian frame is more often used for the solution. So here we have a fixed control volume and then you allow the flows or the fluids to flow in and flow out of the system. Now, we can have Eulerian representation.

So, here the important flow variables are the velocity vector, pressure, So, they are the essentially their function of space, time and so on. When you have fluid particles which usually contains large number of molecules at a particular point, it attains the value of the field variables assigned at that point. Now, the flow field can be said steady if the time derivative to be 0 and steady otherwise. Tangent fields evolve into steady when periodic state and all these things. Dimensionality which is primarily dependent on the spatial coordinate system so which could be three because we are having those kind of dimensions.

So here is an example in terms of looking at the lagrangian description here the fluid particle passes so these are the different point or the different time where this particle is tracked and we try to track the particle speed. That means at different time instance, so it is a single individual particle when it is passes through these points. So, at different time instant their velocity has been tracked. Is this is a steady or unsteady flow. Now, if I look at the Eulerian description that means the flow comes here, flow goes down.

So, at different time instant A and B they would show Same amount of velocity because it's a fluid which comes in and goes out. So, these are the differences. So now you can interpret physically what is del by dt, dv/dt. So dv by dt is also written capital dv/dt. So how do we relate this rate of change of B, a fluid particle located at a particular point at a given time instant to the rate of change of B to the point space.

Total (material) derivative

Partial (local) derivative

$$\frac{dB}{dt}=\frac{\partial B}{\partial t}+\left(\vec{u}.\nabla\right)B$$

convective derivative

Steadiness requires partial derivatives to be zero for all flow variables

so this is where you get the total derivative which is also same thing one can write dB by dt is having a component called partial derivative and then the convective derivative so when you talk about the flow to be steady then this partial derivative has to be zero for all flow variable then you can say that that is steady i mean, one can find out this expression i mean, simply by using so db/dt is essentially i can write in terms of limit so at a particular i mean, if we have a t and x then we can have t + ∂t and x plus delta t minus this so we segregate these things by doing little bit of algebra so this gets you ∂B/∂t and this is u∂B by ∂x assuming ∂x so similar way one can extend that to three dimensions in general one can write okay so that's how you connect your total derivative with your partial derivative and the convective derivative. So that's the connectivity of that.

$$\frac{dB(t,x)}{dt}=\lim_{\Delta t\to 0}\left[\frac{B(t+\Delta t,x+\Delta x)-B(t,x)}{\Delta t}\right]$$

$$=\lim_{\Delta t\to 0}\left[\frac{B(t+\Delta t,x)-B(t,x)}{\Delta t}\right]+\lim_{\Delta t\to 0}\left[\frac{B(t+\Delta t,x+\Delta x)-B(t+\Delta t,x)}{\Delta t}\right]$$

$$=\frac{\partial B}{\partial t}+\lim_{\Delta t\to 0}\left[\frac{B(t+\Delta t,x+\Delta x)-B(t+\Delta t,x)}{\Delta x}\frac{\Delta x}{\Delta t}\right]$$

$$=\frac{\partial B}{\partial t}+u\frac{\partial B}{\partial x}\quad(\text{assuming }\Delta x\to 0)$$

$$\frac{dB}{dt} = \frac{\partial B}{\partial t} + \left(\vec{u}.\nabla\right) B \quad \text{where } \vec{u}.\nabla = u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

Again, we can look at this particular example. Here you can see there is a cold fluid coming in, goes through some pipe here and then finally goes out as a hot fluid. So the temperature what is coming in and when it passes through these different channels, it's higher than.

So, this is a steady state operation of the cold heater. The fluid gets heated up inside the heater. So ∂T /∂t of any fluid particle is zero, but dT/dt is not zero because there is a total change. So convective derivative of T is not zero. Now here is an example where fluid comes in, you have a profile like that, then it goes through the contraction, so it increases, then again it, so it is, so it is again steady state, uniform flow in convergent-divergent nozzle.

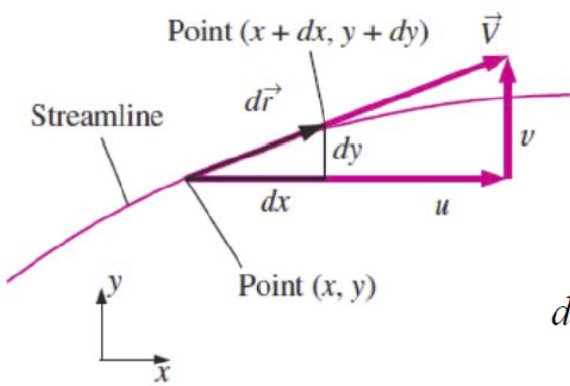
So it first accelerates and then decelerates because there is a change in area. So ∂u/∂t of a fluid particle is 0, but du/dt is not 0. Again the convective derivative part is not 0, so that gives you a example so similarly you can see this particular example but the flow goes through pipe obviously pipe has this constant area then diverging area so here it's in one dimensional flow then it becomes two dimensional here find it it becomes bit independent of what you can see here your 3d flow field shown by cigarette smoke okay. so this particular picture you can see when it comes out then how this structure looks like obviously it's quite complicated and important structure but say. So now, given a flow field, one can or should be able to visualize the velocity acceleration of the flow field.

So, if you are given a flow field like this, then the acceleration of the particle is given by this. So here, the pink arrow shows the acceleration vector, magnitude and direction at each point, black line. streamlines, the figure shows velocity and acceleration fields simultaneously. So one can calculate this. Obviously, as soon as we say that, the question which brings in or comes up is that what is streamline? Streamline is an imaginary line in the flow field.

The tangent at any point of the flow field indicates the velocity direction. That means if this is my streamline, then the tangent here that would indicate the focal. So, if I have a two dimensional velocity field, so that will have two component. Then if the steam line segment, that would be DR, this will have two segments.

The DR is parallel to V. Then I can get DX/U equals two. So that is the equation of steam line. And if I use it in 3D, then this is how it is. Okay. So that's purely, I mean, one hand we are trying to define the basic fluid behavior, but at the same time, one can think about we are primarily doing some kind of a vector calculus to find out this thing.

Now, streamlines are imaginary lines in the flow field which are tangent at any point, indicates the velocity direction. So, it see these streamlines and the tangent shows the velocity direction but one important thing two streamlines must not cut each other they never cut each other because these are given by these particular equations where the velocity is okay so again fluid particle cannot cross the streamline because the streamlines cannot intersect and when you have a series of streamlines and all these things then it can make them steam tube which are made of steam line. So, if any fluid particles enters here inside the steam tube, it remains within the steam tube. It cannot go out of the steam tube because the fluid particle cannot cross steam line.



In a two-dimensional (2-D) velocity field $\vec{V} = u\vec{i} + v\vec{j}$

Streamline segment: $d\vec{r} = dx\vec{i} + dy\vec{j}$

$d\vec{r}$ is parallel to $\vec{V} \Rightarrow \dfrac{dx}{u} = \dfrac{dy}{v}$

Equation of a streamline

In 3-D, Equation of a streamline is given by $\dfrac{dx}{u} = \dfrac{dy}{v} = \dfrac{dz}{w}$

This is the important thing. And the steam tube is made up of steam lines only. So, for example, we can look at this is a given a velocity field. So, you can find out the equation of the steam line. So, you can use this simple equation dx/u. and then do the integration so this is what you do so obviously as you change the value of c you will get different family of streamlines and things like that okay.

now, we have decline which is locus of fluid particle that have passed sequentially through a prescribed point so stick line is one can look at this kind of die visualization and to get this. It's an individual fluid particle. You try to find out the locus of that. So if you take a fluid particle, it comes here, it passes through this point. So that locus is going to give you the stick line.

That means it's individual stream line that goes through. And then you have another terminology or definition is called the path line. So that is the actual path traveled by any individual fluid particle over a prescribed period of time. So if the fluid particle at the starting here, then go like this in medial and then finally end there.

So this is the path line of the fluid particle. You can see this wave in a free surface flow where the path lines are captured along specific protocol. So we have to summarize what we have. We have stream line. we have streak line, we have path line, we have time line. So, steam line is imaginary line in the flow field where tangent at any point of that gives the velocity direction.

Streak line is the locus of the fluid particle that pass sequentially through the space. Path line is the actual path of the fluid particle which are traveled or which are traveled over a, which has traveled over a period of time. And timeline is adjacent three particles that are marked in the same instant. So essentially, you get all these definitions, which are important to know. what we talk now, we talk about some kind of an system which is called the control mass system or flow system or we talk about control volume or open system so what you say the system is essentially in the collection of matter which fits i mean it doesn't change mass with surroundings then you have control volume which is again a geometric entity it could be fixed or moving rigid or deformable in space with fluid flow.

then the control surface so if you look at this is your control volume so this is a boundary which is called control surface, this is the moving boundary of this so you can have this kind of system so what is Important here is that we can apply the conservation of mass to a particular system. So this particular system you see this is a kind of a system boundary and this is a control volume. So how do we use the above equation to find equation of mass conservation in control volume? So you can define a control volume like this. From point 1 fluid comes in, point 3 goes out. So we can say that the change in the mass in the control volume would be the whatever mass is coming in and whatever mass is going out.

So, that's how, is essentially the change in the mass, that's what you try to find it out. So obviously when you try to do that, the other important things one has to define is the extensive versus intensive property. This is the mass, B is the intensity, small beta is the intensity, B is the extent. So in general, the intensive and extensive property is only thing to find out that what is dependent on mass and what is independent of mass.

So, that is what you talk about these different properties. So now what we're going to look at is we're going to look at these equations, how one can define or derive from the control volume system, but we'll stop the discussion here. and take the discussion further from here.