Wind Energy

Prof. Ashoke De

Department of Aerospace Engineering, IIT Kanpur

Lecture 29: Blade design with Momentum theory

Welcome come back so we'll now continue our discussion on again we'll go back and look at the turbine calculations so earlier what we have done is that we have looked at those momentum theory um blade element momentum theory so now we'll those things to connect with those calculations, how one can do some sort of design calculations and things like that. So, that's the primary motivation here that we would like to continue with our calculations and some kind of a design procedure. This is very important because you need to see these equations which are there. And that's how you have the...

Now, what we'll do, we would try to see what we have done earlier with momentum theory and blade element theory. So, we'll actually recall those things and then from there we'll continue the calculations from there. So, what we have earlier done, we have done the linear momentum theory and also the blade element, So, this we have done. So, analysis here, it is going to, what we are going to do, the analysis at this moment, this is going to use these two particular calculations at this moment.

So, what we would be doing, essentially if, you look at the momentum theory that uses kind of a control volume analysis of the forces at the blade based on the conservation of linear and angular momentum. Whether blade element theory this refers to an analysis of forces at a section of the blade as a function of blade geometry. So, now the result of these two, so, basically one can say that instead of this we can say the blade element theory and of these approaches can be combined which is known as blade element momentum theory. So, combining this is known as blade element momentum theory, BEM theory, which is also we have. So, that theory we have also seen that is used to rotate blade shape of the rotor stability and extract to it.

So, essentially that uses that weak rotation. So, that BEM theory, blade element momentum theory uses the momentum theory plus blade element theory. Okay, so, the simplest optimum blade design with an infinite number of blades and no weak rotation, then you can have performance characteristics which is forces, rotor air flow characteristics, power coefficients and also you can have simple optimum blade design including weak rotation and a finite number of blades. Now, what we have from the momentum theory we have is that so, momentum theory gives us the thrust or the differential thrust rather differential thrust that you have in terms of dt which is rho u square 4 a 1 minus a pi r dr so, it's essentially you think about that you have a turbine and this is the air coming in to the turbine which is at U and this kind of a mount and this is your down state location so that's your equation number one and the torque which is again the differential torque due to so, the differential torque is essentially due to the tangential force so that is comes because of B into r into dFt So, that here you can have 4 A prime 1 minus A rho U pi R cube omega DR. So that is what you get.

So, from this momentum theory one gets two equations. An equation 1 and 2 that can define the thrust and the torque of an angular section. Now if we look at the blade element theory, what we have already got, the forces on the blades of wind turbine which we can calculate it by expressing there in terms of lift and drag coefficients, angle of attack. There are some assumptions. One of the assumptions is that there is no aerodynamic interaction between the elements.

Thrust (differential) $dT = \rho U^2 4a(1-a)\pi r\, dr \quad \ldots (1)$

Torque (") $dQ = B \cdot r \cdot dF_T$

$= 4a'(1-a)\rho U \pi r^3 \Omega\, dr \quad \ldots (2)$

There is no radial flow as such. And the forces of the blades are determined solely by the lift and drag characteristics of the aircraft that we have already done. Obviously, what you need to consider is that the wind, lift, drag, perpendicular forces, parallel forces, effective relative wind, all these come into the system. and you can basically can have that omega r plus omega a prime r which is induced this is your let's say induced angular velocity or one can say that tangential velocity which is omega r into one plus a pi that is your induced tangential velocity or angular now here we had we have already i think earlier we have used the blade pitch angle so let's say we say blade pitch angle which is theta P and if you have blade pitch angle at tip is theta P0 then the blade twist angle is theta t which is theta p minus theta p zero however for angle of attack which is you can write phi which is p plus alpha if angle of attack is alpha so where this is flow angle So, that's you can say that these are all equation four that you have. So, essentially, these are all the nomenclature that we have already used earlier.

Blade Element theory

Induced angular vel. $\Rightarrow \Omega r + \Omega a' r = \Omega r(1+a') \quad \ldots (3)$

Blade pitch angle (θ_p), blade pitch angle at tip (θ_{p0})

blade twist angle: $\theta_T = \theta_p - \theta_{p0}$, angle of attack (α), $\quad \ldots (4)$

Flow angle: $\phi = \theta_p + \alpha$

Kanpur

Also, you have all the relationship like tan phi, which is one minus a by one plus a prime lambda r is five, then you have U relative is U into 1 minus A by sine phi, which is 6. Then you have dFl. So, we are recalling those expression again, Cl half rho U relative square Cdr. And then you have dFd, Cd half rho U real square into cdr then you have f normal force which is essentially dfa axial force dfl cos phi plus dFd sine phi and you have dFd which is dFl sine phi minus dFd cos p. So that's what we have already done.

So, if the rotor blade has b number of blades and total normal force on the section at a distance r so that we get dFn equals to b into half rho u relative square Cl cos p plus C D sine phi. Into. Cdr. Okay. So, we are recalling those.

Equations. And. Try to. So, the. Obviously. Our differential.

Torque. Which is dQ. That would be B r into dF. Okay, so our dq becomes b half rho u rel square cl sine phi minus cd cos p cr dr. Here you can note that the effect of drag is to decrease torque hence power but to increase the thrust loading. So blade element theory one also obtained two equations which are essentially 11 and 12 or 13 whatever I mean 11 and 13.

$$\tan\varphi = \frac{1-a}{(1+a')\lambda_r} \quad \cdots (5) \qquad U_{rel} = \frac{U(1-a)}{\sin\varphi} \quad \cdots (6)$$

$$dF_L = C_l \frac{1}{2}\rho U_{rel}^2\, c\, dr \quad \cdots (7) \qquad dF_D = C_d \frac{1}{2}\rho U_{rel}^2 \cdot c\, dr \quad \cdots (8)$$

$$dF_N\,(dF_A) = dF_L \cos\varphi + dF_D \sin\varphi \quad \cdots (9)$$

$$dF_T = dF_L \sin\varphi - dF_D \cos\varphi \quad \cdots (10)$$

$$dF_N = B \cdot \frac{1}{2}\rho U_{rel}^2 (C_l \cos\varphi + C_d \sin\varphi) \cdot c\, dr \quad \cdots (11)$$

Differential torque: $(dQ) = B r \cdot dF_T \quad \cdots (12)$

$$dQ = B \cdot \frac{1}{2}\rho U_{rel}^2 (C_l \sin\varphi - C_d \cos\varphi)\, c\, r\, dr \quad \cdot (13)$$

So, these two equations that you get. So, which define the normal force or the thrust force and the tangential force. force of the torque on the angular rotor section. So, these equations would be used. So, the relative velocity is effectively that wind which we used earlier probably W to determine ideal blade shapes of optimum performance and all this.

Now, what we want to now look at is we look at blade shape for ideal rotor okay this case we say that no quick rotation so we try to so we can combine the momentum theory relations with the from the blade element theory to relate blade shape to performance now since the algebra can get complex a simple but useful example can be discussed. So, what we can do, we have some assumption for the simple analysis, we'll have some assumption. So, no wake rotation, which means a prime is zero, no drag. So, that means CD is zero. and no losses due to finite number of blades so that means no tip loss and then the axial induction factor which is a equals to one three in each annular strip G.

We have this. So, we have those basic definition of the speed ratio, number of blades B. So you have all this lambda B, R, careful with no lift and drag coefficients. So, you can have maximize I mean CL by CD then choose we can choice of twist for distributions and all these things. So, what we write for A equals to 1 by 3 from equation 1 we get DT equals to row u square 4 1 by 3 1 by 1 by 3 pi r dr so that becomes rho u square 8 by 9 pi r dr which is okay now uh from equation 11 which is the blade element theory with cd equals to zero you get dfn b into half rho u relative square cl cos p cdr okay now another equation is the third equation the equation 6 can be used to express u relative in terms of known variables. So, what from there we can write u relative equals to u into 1 minus a by sin phi which is 2 u by 3 sin phi.

So, that is Now the blade animal momentum theory or steep theory refers to the determination of wind turbine blade performance while combining the equations from the momentum theory and blade animal theory. Now using equation 14, 15 and 16 what we get C L B C by 4 pi r equals to tan phi sin phi. Now we can use equation 5 to relate A, A prime phi based on the geometrical consideration and can be used to solve the blade shape. So, equation five with A prime zero and A equals to one third, that becomes tan phi equals to two third by lambda R.

For $a=\frac{1}{3}$, From eq-(1) $\quad dT = \rho U^2 4\left(\frac{1}{3}\right)\left(1-\frac{1}{3}\right)\pi r\,dr = \rho U^2 \frac{8}{9}\pi r\,dr \quad \cdots (14)$

From eq (11), with $C_d \approx 0$, $\quad dF_N = B \cdot \frac{1}{2}\rho U_{rel}^2 \left(C_l \cos\varphi\right) c\,dr \quad \cdots (15)$

eq-(6) $\rightarrow \quad U_{rel} = U(1-a)/\sin\varphi = \frac{2U}{3\sin\varphi} \quad \cdots (16)$

Using eq-(14), (15), (16) $\quad \frac{C_l B c}{4\pi r} = \tan\varphi \sin\varphi \quad \cdots (17)$

Use eq-(5) with $a'=0$ & $a=1/3$, $\quad \tan\varphi = \frac{2}{3\lambda_r} \quad \cdots (18)$

Okay. So, what we have, therefore, we have Cl Bc four pi R 2 by 3 lambda r in sine phi. So now if you rearrange if you rearrange and use lambda r equals to lambda r by r so one can determine the angle of relative wind So, angle of relative wind and the chord of the blade for each section. So, ideal rotor. So, angle of relative wind and chord of blade for each section of the ideal rotor.

Okay. So, what we get? P equals to tan inverse 2 by 3 lambda r. And from here we get quad equals to 8 pi r, sin phi, Pb, Pl, lambda. So, now these relations that you have can be used to find the chord and twist distribution of the optimized blade. I mean, obviously, the beige optimal blade, because we have considered the axial induction factor equals to 1 third. So, you can take an example of, let's say, this lambda equals to 7 and carry out this calculation to have all this.

Therefore, $\quad \frac{C_l B c}{4\pi r} = \left(\frac{2}{3\lambda_r}\right)\sin\varphi \quad \cdots (19)$

Rearrange, and use $\lambda_r = \lambda\left(\frac{r}{R}\right)$, angle of relative wind & chord of blade for each section of the ideal rotor

$$\varphi = \tan^{-1}\left(\frac{2}{3\lambda_r}\right) \quad \cdots (20)$$

$$c = \frac{8\pi r \sin\varphi}{3 B C_l \lambda_r} \quad \cdots (21)$$

Now, what one can see is that the blades designed for optimal power production have an increasingly large chord and twist angle as one gets closer to the blade root. So, one consideration in blade design so blade design is cost and difficulty of blade difficulty of fabricating the blade and optimum blade would be very difficult to manufacture at a reasonable cost but the design provides insight into the blade shape that might be desired for a wind turbine so, this is what you get from the blade element momentum theory which kinds of combine the which combines the uh basic momentum theory and now what we do we'll move to general rotor blade shape performance prediction In general, the rotor is not of the optimum shape because of fabrication difficulties. So, when an optimum blade is run at different speed ratio, then one for it is designed, it is no longer optimum, which is obvious. that's why the blade shape must be designed for easy fabrication and overall performance over the range of wind and range of wind and rotor speed that so so the So, considering non-optimum blades, one generally uses an iterative approach. One can assume, I mean, essentially assume a blade shape and predict its performance, then try another blade shape.

So, these are your iterative process. So, for blade shape for an ideal rotor work without wake rotation has been considered. Now, what we are going to analysis of, here we are going to analyze arbitrary blade shape. Obviously, this includes wake rotation losses due to finite number of blades of design performance. So, these are the things which we will consider here.

So, now we would again recall those equations which you have from momentum theory. One is that dt equals to rho u square for a one minus a pi r dr and dq for a prime one minus a rho u pi r cube omega dr and from blade element theory we had dfn which is b half rho u rel square C l cos phi plus C d sin phi C dr. And d q b half rho u rel square C l sin phi minus C d cos phi C r dr. So, these are what you have from the momentum theory and element theory. So, our starting point would be that to move forward with this thing so that we kind of uses those.

Mon:

$$dT = \rho U^2 4a(1-a)\pi r\, dr \qquad dF_N = B\tfrac{1}{2}\rho U_{rel}^2 (C_l \cos\varphi + C_d \sin\varphi)\, c\, dr$$

$$dQ = 4a'(1-a)\rho U \pi r^3 \Omega\, dr \qquad dQ = B\tfrac{1}{2}\rho U_{rel}^2 (C_l \sin\varphi - C_d \cos\varphi)\, c r\, dr$$

So, now what we do using equation 11 and 13 of BEM theory, we write dFn which is sigma prime pi rho u square one minus a square by sine square phi C L cos phi cd sine phi r dr and we have dq sigma pi rho q square 1 minus a square sine square phi cl sine phi minus cd cos phi r square dr where sigma prime is local solidity, which is defined as sigma prime PC by here. So, that is what we have. So, what happens is that in general, the calculation of the induction factor a and a so this is the practice uh to find a a prime we set cd equals to zero For airfoils with low drag coefficient, this simplification introduces some negligible errors. So, when the torque equation from momentum and blade element theory are equated, that is equation 2 and 23 with Cd0. So, what we do using equation 2 and 23 along with cd equals to zero what we get a prime by one minus a sigma prime cl by four lambda r sine phi so this is 25.

so this is what happens is that this is we are equating torque equation of momentum theory to blade element theory. These two are equated. Similarly, the normal force equation, so using 1 and 22, So that means you equate the normal force equation of momentum theory and the blade element theory. So, what we get A by 1 minus A sigma prime C L cos P 4 sin square phi which is 26. Now, you can do some algebraic manipulation using equation 5 which refers to your a a prime p lambda, so, what we am saying that using some algebraic manipulation and using equation 5 25 and 26 what you can get you get CL equals to four sine phi cos phi minus lambda r sine phi sigma prime sine phi lambda r cos phi.

Using eq. (1) & (13),

$$dF_N = \sigma' \pi \rho \frac{U^2(1-a)^2}{\sin^2\varphi}\left(C_l \cos\varphi + C_d \sin\varphi\right) r\,dr \quad \ldots (22)$$

$$dQ = \sigma' \pi \rho \frac{U^2(1-a)^2}{\sin^2\varphi}\left(C_l \sin\varphi - C_d \cos\varphi\right) r^2\,dr \quad \ldots (23)$$

σ' = local solidity

=

$$\sigma' = \frac{Bc}{2\pi r} \quad \ldots (24)$$

Practice, to find $a, a' \longrightarrow C_d = 0$

Using eq (2) & (23) along with $C_d = 0$

$$\frac{a'}{1-a} = \frac{\sigma' C_l}{4\lambda_r \sin\varphi} \quad \ldots (25)$$

(equating torque eq. of mom. theory = blade element theory)

Using (1) & (22),

$$\frac{a}{1-a} = \frac{\sigma' C_l \cos\varphi}{4 \sin^2\varphi} \quad \ldots (26)$$

Using some algebraic manipulation, using eq-(5), (25), (26)

And, what you get A prime by one plus A prime sigma prime CL four cos phi So other useful relationship can be derived other useful relationships, which are a by a prime equals to lambda and by 10 phi equals to so let's say 29 equals to one by one plus four sine squared phi by sigma prime CL cos P then A prime equals to 1 by 4 cos P plus 4 cos P divided by sigma prime CL minus one so this is what you get that these are the set of equation that you have that you often for this so now one can look at the solution methods of all this equation that you essentially obtain by doing this calculations and then we can see so effectively what one has to do is that finding out this a a prime lambda c and all these things so these are your design parameters because you have a fixed wind speed and for that what are these induction factors and Cl Cd variations and all these. We will discuss about the solution method in the next section.

$$C_l = 4\sin\phi \frac{\cos\phi - \lambda_r \sin\phi}{\sigma'(\sin\phi + \lambda_r \cos\phi)} \quad \ldots (27)$$

$$\frac{a'}{1+a'} = \sigma' C_l / (4\cos\phi) \quad \ldots (28)$$

other useful relationships:

$$\frac{a}{a'} = \lambda_r / \tan\phi \quad \ldots (29)$$

$$a = \frac{1}{1 + 4\sin^2\phi / \sigma' C_l \cos\phi} \quad \ldots (30)$$

$$a' = \frac{1}{[(4\cos\phi / \sigma' C_l) - 1]} \quad \ldots (31)$$