Wind Energy

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Lecture 26: BEM example

Welcome back. So, we are in the middle of the discussion on this momentum theory. So, what we have kind of done so far is look at that one dimension of momentum theory. So, essentially, looking at this one dimension, we have CP max, CT max, and all these limits. And essentially, this is where the power coefficients and the base limit, which were derived from here, came from the maximum or optimum power coefficient and things like that. Then, we looked at the momentum theory with weak rotation.

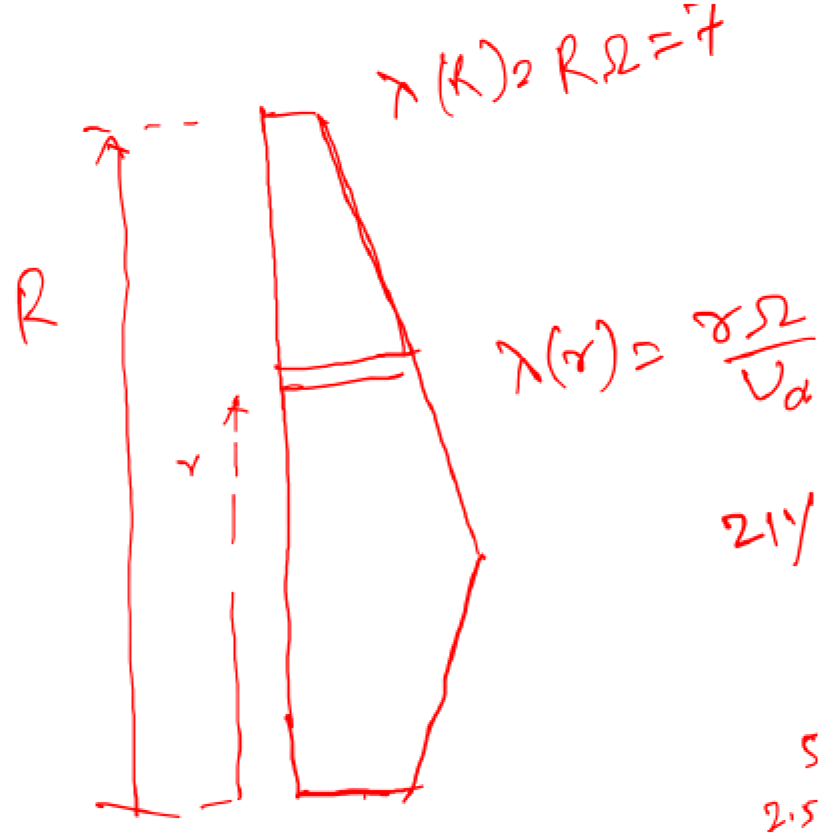
So, the first one, when we kind of talked about initially, doesn't consider the rotation of the weight, but the rotation of the weight can have a significant impact on the flow field or the parametric calculations, and then from there, we looked at the beam theory, which is. So, these are the things that we have derived so far. And I think we have stopped with this kind of condition in the last session. So, from here, we'll move on.

And we will now look at the BIM example. So, that would probably make things much clearer where we can. We'll look at some BIM. Example: okay, let us assume a few typical values. Okay, let's say lambda_r, which belongs to one to seven, then the number of blades is three.

Lambda_r is seven and A equals 1 by 3 for all R. Here, the consideration is that we are trying to extract maximum power due to the bridge limit. So, this is what we are trying to do. If I draw the system. So, let's say.

So, this is a street here where lambda of R is R omega divided by U infinity. That is R. And this distance is smaller. This is lambda of r, r omega, which are 7.

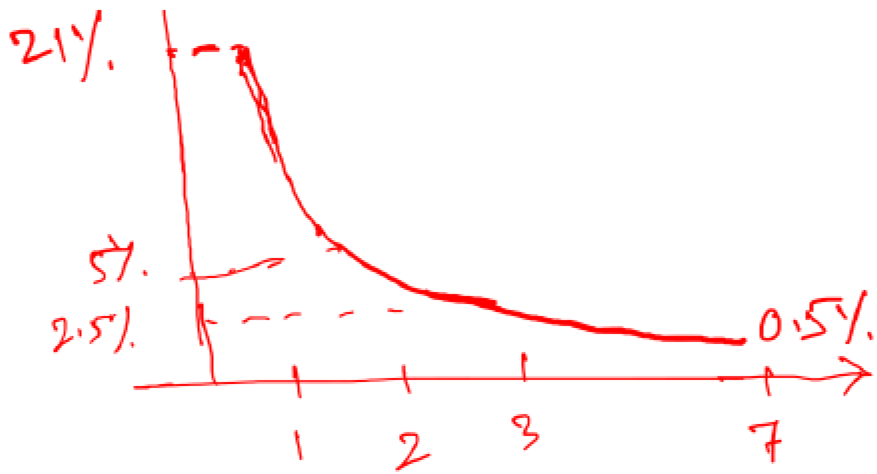
Okay. So, here, what we have is cl equal to 1, cd equal to 0.01. So, the idea is that the twist and pitch are optimally chosen so that the angle of attack alpha is constant at 5 degrees. So, that is something we So, now here A prime is something A 1 minus A by lambda R square 2 by 9 into 1 by lambda R square. So, what you kind of get to see is that one, two, three, let's say seven, so this goes like that and it kind of comes to a, so this is 21 percent, somewhere you get five percent, and this one is something 2.



$C_L = 1, \quad C_D = 0.01$

(Twist & pitch optimally chosen, so that the angle of attack α is cst. at 5°)

$$a' \approx \frac{a(1-a)}{\lambda_r^2} = \frac{2}{9} \cdot \frac{1}{\lambda_r^2}$$



5 percent, and this is 0.5. Okay, so this direction lambda with increasing r, so that's what you have. Okay, now. What we can get is an estimate of the local solidity using equation 35B, okay? So, what we get is sigma r over a one minus a CL lambda r one plus A prime CD one minus A dot square root of lambda r square one plus A prime square plus one minus A square.

So, what we get is eight by nine. 1 by lambda r 1 plus a prime plus 0.006, which is equivalent to 0. 1 by lambda r 1 plus a prime 1 plus 4 by 9 1 by a prime lambda r. So, this is also kind of equivalent to zero.

So, what we get is that 8 by 9, 1 plus a prime squared, which is 8 by 9, 1 by lambda r squared. So, that means my variation of the solidarity, this is lambda r with increasing r one two seven, so this is sigma r 89%. This is actually 22%. This would be 2%. So, that is how solidarity varies.

So, what we can find out essentially points here is that what does this mean for the quad length of C? What does it mean? So, we can say that sigma r equals b times c divided by 2 pi r; we have eight by nine one by lambda square. Now, we know that it equals two pi r divided by nine r squared by one over lambda r squared. Okay. So, that gets us to $\pi r b \lambda r^2 \times 8/9 \times 1/\mu$. So, mu is r by r, and lambda r is mu lambda.

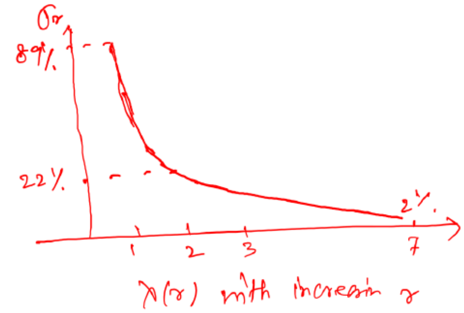
Estimate local solidity using eq. 35b

$$\sigma_r = \frac{4a(1-a)}{C_L \lambda_r (1+a') + C_D (1-a)} \cdot \frac{1}{\sqrt{\lambda_r^2 (1+a')^2 + (1-a)^2}}$$

$$= \frac{8}{9} \cdot \frac{1}{\lambda_r(1+a') + \underbrace{0.006}_{\approx 0}} \cdot \frac{1}{\lambda_r(1+a')\sqrt{1+\frac{4}{9}\underbrace{\frac{1}{(1+a')\lambda_r}}_{\approx 0}}}$$

$$\approx \frac{8}{9} \frac{1}{\lambda_r^2(1+a')^2} \approx \frac{8}{9} \cdot \frac{1}{\lambda_r^2}$$

What does this mean for Chord length of 'c'?



So, this can have μr λ by u infinity or ω by u infinity. So, this would become 2πr by b 2% of 1 by μ. So, which is 4% of r divided by mu. So, that means what happens is that let's say r equals 50 meters, this is 2 meters, and this is 10 meters. So, that tells us two important things here: one is that the wick rotates more if the turbine moves relatively slower, which is low lambda, and higher lambda leads to lower rotation.

Since, $\sigma_r = \frac{B \cdot C}{2\pi r} = \frac{8}{9} \cdot \frac{1}{\lambda_r^2}$

We know $C = \frac{2\pi r}{B} \cdot \frac{8}{9} \cdot \frac{R^2}{r^2} \cdot \frac{1}{\lambda_R^2}$

$$= \frac{2\pi R}{B \lambda_R^2} \cdot \frac{8}{9} \cdot \frac{1}{\mu}$$

$$\approx \frac{2\pi R}{B} \cdot 2\% \cdot \frac{1}{\mu} \approx 4\% \, \frac{R}{\mu}$$

$\mu = \frac{r}{R}$

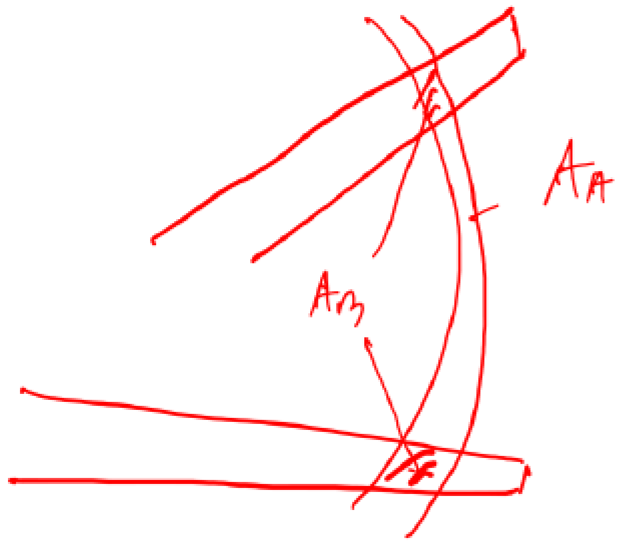
$\lambda_r = \mu \lambda = \frac{\mu R \lambda}{U_\infty} = \frac{r \Omega}{U_\infty}$.

For R = 50m get, C(R) = 2m & C(10m) = 10m

What you get from this is that the weight rotates more when the turbine moves relatively slower, and that is happening towards the hub, which is this portion that is close to the hub, while always towards the tip it rotates more; so, the weak rotation would be less. So, the majority of the impact from the weak rotation would be visible towards the hub of the blade. That is what is in one of the.

.. So, what it does tell us is that it's one of the design considerations that you will gain from this kind of analysis. And that tells you what you should consider and how you should consider these; these are the things one has to kind of consider during the design part. Now, there are other important equations, which are obviously important to remember: lambda r is r divided by r times lambda, which is always greater than 1. These are assumptions. Now, from the axial momentum balance, there is a force on the blade's area.

AB equals half rho AB lambda R, which is essentially W squared. So, we can think about this as a kind of annulus through which it cuts. So, this is A and this is AB. So, it equals thrust on the annulus if A equals U infinity 1 minus A infinity to a of rho a a infinity square four a one minus a; this is ct of a. We have local solidity, which is sigma R AB by AA, that is B into C by two pi R.



local solidity:

$$F_A = \rho A_A U_\infty (1-a)(U_\infty \cdot 2a)$$

$$= \frac{1}{2} \rho A_A U_\infty^2 \underbrace{4a(1-a)}_{C_T(a)}$$

$$\sigma_r = \frac{A_B}{A_A} = \frac{B \cdot C}{2\pi r}$$

Now we can find the optimal chord. Optimal chord. So, half rho U infinity squared AB into CL lambda R squared of rho U infinity squared for one minus a gives us sigma r L lambda R squared. One by lambda squared equals one by three, which is optimal.

Okay. What we do: 2πR into B, CLR, 8 by 9, 1 by λ squared, R squared by R squared. So, C by R, 1 by B, 2 pi by 9, 8 by lambda squared, r squared by r, which is proportional to 1 by r. What you get, I mean, one can see, I mean, so if this is r, your C by R is inversely proportional to r. So this tells us one thing. I mean, basically, for a fixed seal at an optimal angle of attack, the optimal chord is inversely proportional to the radius.

$$\frac{1}{2}\rho V_d^2 \cdot A_B \cdot C_L \lambda_r^2 = \frac{1}{2}\rho V_d^2 A_A 4a(1-a)$$

$$\Rightarrow \quad \sigma_r \cdot C_L = \frac{4a(1-a)}{\lambda_r^2} = \frac{8}{9}\cdot\frac{1}{\lambda_r^2}, \quad a = 1/3 \text{ (optimal)}$$

$$\Rightarrow \quad \frac{c(r)}{2\pi r}\cdot B \; C_L(r) = \frac{8}{9}\cdot\frac{1}{\lambda^2}\cdot\frac{R^2}{r^2}$$

$$\Rightarrow \quad c(r) = \frac{1}{B\cdot C_L(r)}\cdot\frac{2\pi}{9}\cdot\frac{8}{\lambda^2}\cdot\frac{R^2}{r} \propto \frac{1}{r}.$$

So, this also tells us from this momentum theory how the quad is varying because if you see this location, you have different quads here; the different quad here is different. So, in a horizontal axis wind turbine, which is very typical, when you move the area from the rotor hub towards the tip, there are different kinds of airfoils with different kinds of chords that are used. So, that is another important design consideration that one has to take into account while doing all this design. I mean, in totality, you have to see that what we can look at is that linear taper is a practically great design. Okay, so in practice, linear taper is also used, so this is what it kind of shows.

So, this is what it is. So, this is the linear taper part. Okay. It is also used because the chord close to the hub is taken into account. To compensate for the lower solidity, one can increase the angle of attack. In order to increase the CL of R accordingly.

Here, the drag loss in the inner part of the blade is less important to us. Now, we have already looked at the flow angle and all these aspects. So, just to recap that part. So, this will be the beta.

This would be alpha. This is phi, which is alpha plus beta. And this is the direction of the W, and that is a quad line. So, here beta is twist. Alpha is the angle of attack.

So, alpha becomes phi minus beta. So, fix alpha for the best CLYCD. So, what you can do is optimize the CL by the CD, which is going to be the best. For that, you can adjust the alpha. So, what you have then is sine phi, which is 1 minus a lambda r, 1 plus a prime, 1 plus 1 minus a square, 1 plus a prime square, which is equivalent to 0. So that gives you 1 minus A over lambda r, 1 plus A prime, which is 2 thirds, and 1 over lambda r.

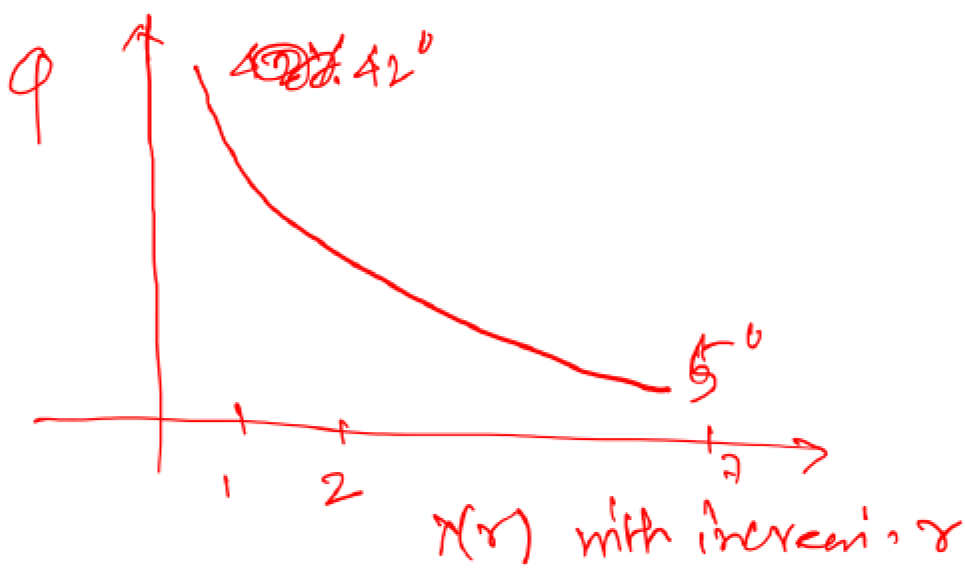
A equals 2 thirds, and A prime equals 0. So, this is what you get. So, now if you plot, let's say if I plot phi, this might be lambda r with increasing r, one, two, let's say seven, so that goes like this; this could be around 42 percent, this can be 42 degrees, and this would be around 5 degrees, like that, so p would be the sine inverse. Two-thirds of r by r lambda, two-thirds of r lambda into 1 by r, which are inversely proportional to r. Okay, so another plot we can look at is the similar way one two seven, which is beta.

$$\sin\varphi = \frac{(1-a)}{\lambda r(1+a')\sqrt{1+\underbrace{\frac{(1-a)^2}{\lambda^2(1+a')^2}}_{\simeq 0}}} \simeq \frac{1-a}{\lambda r(1+a')} \simeq \frac{2}{3}\cdot\frac{1}{\lambda r}$$

$\left(a = \frac{2}{3}, a' = 0\right)$

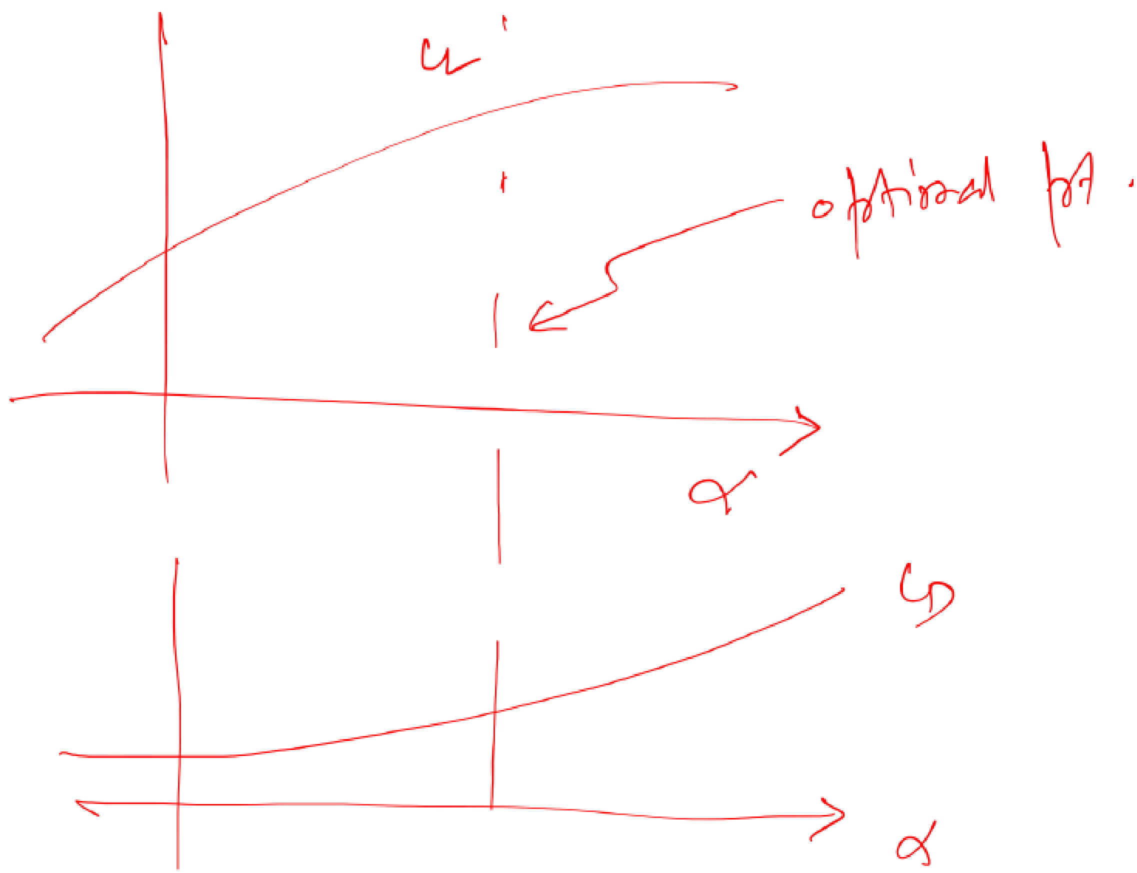
$$\varphi = \sin^{-1}\left(\frac{2}{3}\frac{R}{r\lambda}\right) \simeq \frac{2}{3}\frac{R}{\lambda}\cdot\frac{1}{r}$$

inversely proportional to r



Beta can vary in a similar fashion as well, so here it could be around 37 degrees. This can go almost to zero degrees; this is where it varies. So, for a given profile, with CL, CD, and alpha fixed at alpha naught equal to five degrees, you can see that what happens is that this is the variation of CL with alpha. And this is alpha; this is the CD variation. This could be somewhere near the optimal point, so what you need to consider is that you try to optimize CL by CD.

Basically, we optimize or base CL on CD; we try to have it. And for that, if it's the alpha, then the alpha and the other parameters can be determined. So, all of these go into a kind of design consideration, and all these things, how this momentum theory can be applied to the design; obviously, one can turn these into the initial design one needs to have; one can have complete numerical-based data and carry out this design. Okay. So, now what we would like to see, because we have seen that these airfoils and all these things, is what is important to discuss now: what kind of airfoils or how the airfoil behaves.



So, we are going to have a discussion about the airfoil. And we're also going to talk about weaknesses and all this before we move to the other part of the discussion, okay? That will continue in the second session. Thank you.