Wind Energy

Prof. Ashoke De

Department of Aerospace Engineering, IIT Kanpur

Lecture 25: BEM theory

Okay, welcome back. So, we continue our discussion on the BM theory which is blade element theory and so if you recall that what we have been talking in the last session is that we got that velocity triangle where we had the effective wind magnitude based on the relative axial wind speed and which is also partly you can see this is your resultant effective wind and your axial wind would be in this direction which is your axial wind that would be U infinity A into 1 minus A. okay! and then, you have sorry you would be into one minus a and this direction is induced tangential wind which is nothing but r omega 1 plus a prime. So, and then you could find the effective win thing. Now, what you can hear, what we are trying to find out the lift and drag on the blade element. Okay.

So, that's what you are trying to find out the lift and drag on the blade element. Now, what we can write DL, DL is the lift component on this blade element which is shown again your flow angle free is alpha plus beta, one is the angle of attack and this is your chord line. what you get the dl is half rho w square d ab into cl and similarly so what you could do okay so probably this was 26 so dL is and then dD which is drag half of rho w square d AB into C d so these are two things now here what you have you have sine phi which is u infinity 1 minus a by w you have cos phi which is r omega 1 plus a prime by w. Then, what we have axial flow on all blade elements that is d of fa okay which is equals to b into dL cos phi plus dD sin phi.

Axial Wind $= U_\infty (1-a)$

$$\left.\begin{aligned} dL &= \frac{1}{2}\rho W^2 dA_B \cdot C_L \\ dD &= \frac{1}{2}\rho W^2 dA_B \cdot C_D \end{aligned}\right\}$$

Okay. And, we can have tangential force on all blade elements that would be dA p b into dL sin phi minus dd cos phi which is 28. So, this would be positive if the blade element produce power. Okay. So, now what we can do is that so these are what you have, I mean, in some text you can find this axial axial force the axial force is equivalent to term and normal force, so, some textbook can write this dFN but obviously tangential wall would remain as a tangential but then also one can find out using using dFT one can find the differential torque due to this Ft which is dQ B r dFT so this is also one can write down now what here we have axial and tangential force was induction a and a prime due to momentum balance as before.

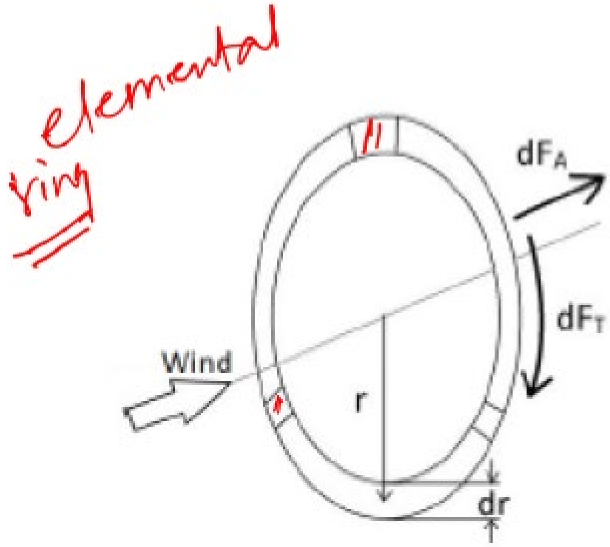
Axial Force on all blade elements:

$$dF_A = B \cdot (dL \cos\phi + dD \sin\phi) \quad \ldots \quad (27)$$

Tangential Force on all blade elements

$$dF_T = B \cdot (dL \sin\phi - dD \cos\phi) \quad \ldots \quad (28)$$

Okay. So that is there. So now, what we have Now we have axial force dFA, which is dM dot 2AU infinity, which is 29a, which is rho 2 pi r dr U infinity 1 minus A. So that gives dFA of rho u infinity square to pi r dr 4a 1 minus a, which is there.



Axial force: $dF_A = d\dot{m}\,(2aU_\infty)$ ... (29a)

$= \rho\, 2\pi r\, dr\; U_\infty(1-a)$ ... (29b)

$\Rightarrow dF_A = \frac{1}{2}\rho U_\infty^2\, 2\pi r\, dr\, 4a(1-a)$ ... (30)

Tangential momentum change:

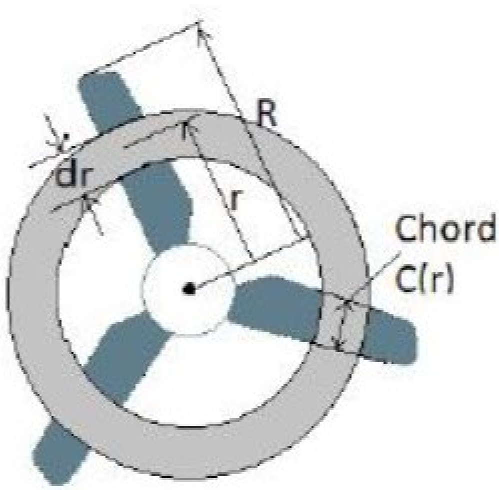
$dF_T = d\dot{m}\,(2a'.\,r\Omega) = \frac{1}{2}\rho U_\infty r\Omega\, 2\pi r\, dr\,(1-a)4a'$ ... (31)

So, that is DFT DM dot with a prime arc in a dot. which is half rho infinity r omega 2 pi r dr 1 minus a 4a prime. So, now from 27 equation equals to equation 30 and equation 28 equals to equation 31. So, we get two equations for two unknowns A and A', which need to be solved numerically.

But then you can find out then you can find out essentially from here a and a prime can be found out okay so that's how you get that you have these equations and then you get so let us now what let us first correct and simplify our equations to find out that thing. So, let us kind of try to do that. So, again we get back to this particular picture so that I try to make things more visible in a fashion that you can follow. So, now first we will write equation 27 equals to equation 30. So, what it is there, half rho W square B C into C L cos phi plus C D sin phi into dr equals to half rho 2 pi r dr u infinity square 4 a 1 minus a so this is 32a, so, from here what we get w square into b c into c l cos phi cd sin phi 2 pi r u infinity square 1 minus C.

Now, similarly, we have equation 28 equals to equation 30. Sorry, 31. Then you have half rho W squared. B C L sine phi minus C D cos phi D R. Half rho 2 pi R.

dr u infinity r for a prime 1 minus a which is 33a that gives w square b c cl sine phi minus cd cos phi 2 pi r square u infinity omega 4 a 1 minus a so this is what we have now, okay now, we can use the concept of solidity so we use um solidity sigma r is b r by 2 pi r and our local speed ratio lambda r is r omega by u infinity and w is u infinity square lambda r square 1 plus a prime square u infinity square 1 minus a square. So this you will have U infinity lambda R square 1 plus A prime square plus 1 minus A square. So let's say 34A, 34B. So what we have, this is our triangle that we have. So, this is w, this is p, so this is cos, this is sine.



q. 27 = q 30

$$\frac{1}{2}\rho W^2 B\, c\,(C_L\cos\phi + C_D\sin\phi)\,dr = \frac{1}{2}\rho\, 2\pi r\, dr\, U_\infty^2\, 4a(1-a) \quad \cdots \text{32a}$$

$$\Rightarrow W^2 \cdot B \cdot c\,(C_L\cos\phi + C_D\sin\phi) = 2\pi r\, U_\infty^2\, 4a(1-a) \quad \cdots \text{32b}$$

$$Eq. (28) = eq (30)$$

$$\frac{1}{2}\rho W^2 B c (C_L \sin\phi - C_D \cos\phi) dr = \frac{1}{2}\rho\, 2\pi r\, dr\, U_\alpha\, r\, \Omega\, 4a'(1-a) \quad \cdots (33a)$$

$$\Rightarrow W^2 \cdot B \cdot c (C_L \sin\phi - C_D \cos\phi) = 2\pi r^2 U_\alpha \Omega\, 4a'(1-a) \quad \cdots (33b)$$

Use solidity $\sigma_r = \frac{Bc}{2\pi r}$, local speed ratio $\lambda_r = \frac{r\Omega}{U_\alpha}$

$$\& \quad W = \sqrt{U_\alpha^2 \lambda_r^2 (1+a')^2 + U_\alpha^2 (1-a)^2} \quad \cdots (34a)$$

$$= U_\alpha \sqrt{\lambda_r^2 (1+a')^2 + (1-a)^2} \quad \cdots (34b)$$

So, what we have, sin phi, 1 minus a, root over lambda r squared, 1 plus 1 minus a squared, and cos phi, lambda r one plus a prime lambda r square one plus a prime square plus one minus a square so that's what is happening okay so, what we therefore we get the equivalent formula from equation 32 B, what we get that lambda r 1 plus a prime squared plus 1 minus a into sigma r cl so what so cl into basically what we have lambda r 1 plus a prime root over of lambda r square 1 plus square plus 1 minus a square cd into 1 minus a lambda r square 1 plus a prime square plus 1 minus a square equals to 4a into 1 minus a. So, if we, so that we can write lambda r 1 plus e prime square. So, what we get plus 1 minus a square sigma r del lambda r 1 plus a prime pd 1 minus a equals to 4 into 1 minus a so 32 b so this is what we have used so w square so this is supposed to be lambda square so this is supposed to be okay lambda square plus so 32 we get where b okay okay so this is essentially w square so this has to be squared perfectly fine now singularly from equation 33b we get lambda r squared plus 1 prime squared 1 minus squared by lambda r sigma r cl1 minus a minus cd1 plus a prime equals to 4a prime minus 1 minus a. So this is essentially from that. Now what happens if we kind of now dividing both 35B and 35B by equation 36 for each side, we have lambda R T L lambda R one plus A prime dd one minus e l one minus a dd lambda r one plus a prime a by a prime so you we get this so what you have is that uh so now If you recall from rotor disc theory, A prime equals to A into 1 minus A by lambda R square.

$$\sin\varphi = \frac{(1-a)}{\sqrt{\lambda_r^2(1+a')^2+(1-a)^2}}, \quad \cos\varphi = \frac{\lambda_r(1+a')}{\sqrt{\lambda_r^2(1+a')^2+(1-a)^2}}$$

Therefore, we get the equivalent formula:

From eq. (32b),

$$\left[\lambda_r^2(1+a')^2+(1-a)^2\right]\cdot\sigma_r\cdot\left[C_L\frac{\lambda_r(1+a')}{\sqrt{\lambda_r^2(1+a')^2+(1-a)^2}} + C_D\frac{(1-a)}{\sqrt{\lambda_r^2(1+a')^2+(1-a)^2}}\right] = 4a(1-a) \quad \ldots (35a)$$

$$\Rightarrow \sqrt{\lambda_r^2(1+a')^2+(1-a)^2}\cdot\sigma_r\cdot\left(C_L\lambda_r(1+a') + C_D(1-a)\right) = 4a(1-a) \quad \ldots (35b)$$

From eq. (33b)

$$\frac{\sqrt{\lambda_r^2(1+a')^2+(1-a)^2}}{\lambda_r}\cdot\sigma_r\left(C_L(1-a) - C_D\lambda_r(1+a')\right) = 4a'(1-a) \quad \ldots (36)$$

Dividing both (35b) ~~by 36~~ by eq (36) for each side, we have:

$$\lambda_r\frac{C_L\lambda_r(1+a')+C_D(1-a)}{C_L(1-a)-C_D\lambda_r(1+a')} = \frac{a}{a'} \quad \ldots (37)$$

So what we have A prime equals to A by lambda R, 1 by A minus CD by CL lambda R 1 plus A prime. divided by lambda r 1 plus a prime cd by cl 1 minus a so, it's more like a little bit of algebra of doing that so, we can write a 1 minus a by lambda r square 1 minus cd by cl lambda r 1 plus a prime by 1 minus a 1 plus cd by C L 1 minus A 1 by lambda r. So, 38P. And, if we get the quadratic equation in A prime, so, we get A prime square 1 plus C D by C L into 1 by lambda r A prime minus A into 1 minus A by lambda r square minus A by lambda r. So, here only one only positive solution is meaningful.

Recall, from rotor disc theory $a' = \frac{a(1-a)}{\lambda_r^2}$

$$a' = \frac{a}{\lambda_r}\,\frac{(1-a) - \frac{C_D}{C_L}\lambda_r(1+a')}{\lambda_r(1+a') + \frac{C_D}{C_L}(1-a)} \quad \ldots (38a)$$

$$= \frac{a(1-a)}{\lambda_r^2}\left(\frac{1-\frac{C_D}{C_L}\lambda_r\frac{1+a'}{1-a}}{1+\frac{C_D}{C_L}(1-a)\cdot\frac{1}{\lambda_r}}\right) \quad \ldots (38b)$$

And if we get the quadratic eq. in a'

$$a'^2 + \left(1+\frac{C_D}{C_L}\cdot\frac{1}{\lambda_r}\right)a' - \left(\frac{a(1-a)}{\lambda_r^2} - \frac{C_D}{C_L}\cdot\frac{a}{\lambda_r}\right) \quad \ldots (39)$$

Here, only ~~one~~ positive solution is meaningful

$$a' = -\frac{1+\frac{C_D}{C_L}\cdot\frac{1}{\lambda_r}}{2} + \sqrt{\frac{\left(1+\frac{C_D}{C_L}\cdot\frac{1}{\lambda_r}\right)^2}{4} + \frac{a(1-a)}{\lambda_r^2} - \frac{C_D}{C_L}\frac{a}{\lambda_r}} \quad \ldots (40)$$

So, what we get A prime equals to minus one plus CD by CL one by lambda R by two plus minus. So, it would be only plus one plus CD by CL into one by lambda R square by four A into one minus A by lambda R square. minus C D by L A by lambda R. So this is what. So, that's what we get.

And for a special case let's say for C D equals to 0 we set A prime equals to half plus minus half plus 1 by 4 plus a into 1 minus a by lambda r square which is 41a. So, we can write minus half plus half 1 plus 4a 1 minus a by lambda r square 41b. which is minus half plus half plus one by four, four A into one minus A by lambda R square. So, this is we can write A into one minus A by lambda R square plus four, which is 41 D. So, this line that we are writing by using Taylor series expansion root over 1 plus x which is 1 plus x by 2 plus order of x square and so on.

So, this is the higher order term that we have left it back. So, this is what you get. okay, that if the drag is zero then you get a prime which is the radial induction factor uh kind of so essentially no drag condition your a prime is a into 1 minus a by lambda r squared so that's the relationship but no drag condition is more like and very much theoretical condition because whenever these turbine blades are exposed to this rotation and wind speed and such things then it is very very difficult to have zero drag condition, so, what we have rather when these are exposed to that rotation then this lift flows on the rotor blade it causes the blade to rotate and there would be some drag and there could be delay i mean i mean the loss in the drag and then this drag can affect the power output. So, that is what your overall power output having those efficiencies which are considered to be because whatever available wind power is there, you may not be able to extract that completely through that. So, this is what we have talked about is that we have actuated this theory that is a momentum theory.

For $C_D = 0$, we set, $a' = -\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{a(1-a)}{\lambda_r^2}}$ .. (41a)

$= -\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{4a(1-a)}{\lambda_r^2}}$ .. (41b)

by using taylor series expansion: $\sqrt{1+x} = 1 + \frac{x}{2} + O(x^2)$..

$= -\frac{1}{2} + \frac{1}{2} + \frac{1}{4}\frac{4a(1-a)}{\lambda_r^2}$ .. (41c)

$= \frac{a(1-a)}{\lambda_r^2} + \underline{O(\lambda_r^{-4})}$ .. (41d)

higher order term.

So, we talked about actuated this theory and then we have talked about blade in event theory. So, these are both of them are kind of momentum theory. But when you talked about actuated disc theory, we have not considered the rotation of the weight, which is there. And because of that, you may start getting some performance effect. And in the weight rotation, we consider the rotation of the weight into the system.

And that's what we talked about the BEM theory, where you consider all these things. So we'll look at some of these things. details further and continue the discussion in the next session thank you