Wind Energy

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Lecture 24: Momentum theory with wake rotation

Welcome back so, let us continue our discussion on this momentum theory and the extraction of the power coefficients and all this, so, what we discussed uh in the i mean in the last session that is that what is the maximum power coefficients which is essentially the theoretical base limit and then what is the thrust coefficient or maximum thrust coefficient that is possible so these are optimal so but what happens is that though we have got this cp max is 16 by 27 but theoretically possible rotor power coefficient but in practice three effects lead to decrease in the maximum achievable power coefficient so there are effects which does not allow to achieve this So, one is that rotation of the wake behind the rotor. Number two, finite number of blades and associated tip losses number three non-zero aerodynamic drag because drag is non-zero so this is what so what happens in that case though we have that maximum power coefficients theoretically supposed to be 16 by 27. So, what happens is that the overall turbine efficiency becomes a function of rotor power coefficient rotor power coefficient and mechanical, including electrical efficiency of the wind turbine. So, it becomes an overall efficiency now becomes an. So, what It does that eta overall becomes P out half rho A U naught cube. So, this you write eta mic into Cp.

So, the output power that you get it will have eta mic Cp up row u naught. So, that's what u active. In practice, there could be losses. And because of the loss, so you kind of And those are possible because there are rotation of the weight behind the rotor, there are finite number of blades which could happen.

So, the momentum theory analysis that we have carried out so far, it assumed no rotation which was imparted on the floor. Now what we can do, we can extend that analysis but the rotating rotor generates angular momentum which can be related to rotor torque. So, that means what we want to do now we want to include the weak rotation so that in our calculation that is something we can incorporate. So, what we do we use this weak rotation and rotor disk theory. So, what you have, so again, this is connecting with the picture where we have considered the turbine or positioning of the turbine with the reference position and the other position where you have.

$$\eta_{overall} = \frac{P_{out}}{\frac{1}{2}\rho A U_0^3} = \eta_{mech} C_p$$

$$P_{out} = \eta_{mech} \cdot C_p \cdot \frac{1}{2} \rho A U_0^3$$

So, what we have this similar picture where the turbine is there, but here we are looking at one particular aerofoil and considering this rotation. Obviously, here we can do can actually have tip speed of then this is r and let's say this is omega this would be r omega so if your speed at any r which is less than r would be r into omega so omega is 2 pi by t where t is the period of rotation. Okay. So, that's what you have. Now what you so here when it is going through this rotation here is deflected in the tangential direction by the blade.

So, here is deflected by the by the blade in the tangential direction so tangential now we define tangential induction factor or tangential induction which depends on r and this tangential velocity, V1 tangential. So, this particular picture now, along with the position where you can see this is the airfoil, that means the blade, and this is the airflow. And then blade is rotating like that. So, which kind of tells you a situation. I mean, like, if you look at that, that means this is the airfoil.

and this is the air coming in and blood goes in a rotation like that and that is why this r omega in this direction which is shown here okay, so, it's primarily the matter of velocity triangle how you kind of come across this thing tangential induction the tangential induction which depends on r.v1 and the tangential induced velocity is equals to V1 r omega A'. So, we define here A' as tangential induction factor. Earlier we have seen the axial induction factor. Now since we are considering the disc rotation, so we are taking into consideration the tangential induction factor as well.

So the tangential induction factor would come into the picture now, so, here in this particular picture as we see with an initial tangential velocity which is here where the air is coming towards the rotor blade that is zero the tangential velocity downward which is going to be v3, so, we can estimate that so tangential velocity downward is v3 which is r omega 2a prime, so, the change in tangential momentum would be m dot v3 minus zero, so, that is what because the initial tangential velocity is zero and at the downward side it is v3, so, if you see this particular plot which clarifies a lot of things actually the what happens is the like air pushes the blade so essentially these are the rotor blades airfoil which is shown here so there would be blades and the air is coming through these blades are rotating like that, so, here pushes the blade upward because of this is happening because here you get this resultant lift force which pushes this to rotate and the air is deflected downward okay! so, that's what this rotational aspect of it or the rotation of this disc has been brought in to encounter the losses due to this disc rotation and all these things. So, what you see that now with this we can now move to find what we can find is that We can find or we can now compute V3 of R for given A omega R U infinity, obviously all these things. Okay, so based on so here we consider an infinite seminal annulus of area DA. So, infinite seminal annulus of area DA. and we see this from the center at r that will be small dr and the complete radius of the rotor is r so here ba is 2 pi r here okay! so, that's what you have then what you get now if we so now we'll continue with the same numbering system so this is 16 then what we can write is that 0 to r 2 pi r dr is pi r square which is 17.

$$dA = 2\pi . r . dr \quad \ldots\ldots (16)$$

$$\int_0^R 2\pi r \, r \, dr = \pi R^2 \quad \ldots (17)$$

then you can have the infinitesimal power extracted that is dp half rho u infinity cube de into cp into now to harvest this power via rotary motion with the angular velocity of omega, we need a tangential force of Bf. Thus, what we can have dp equals to df r into momentum. Now, since f equals to m dot delta v which is due to change in momentum and where my M dot is rho A U infinity one minus A. Okay. So, what we can have or can write D F equals to rho D A U infinity 1 by A, V3 minus 0.

$$dP = \frac{1}{2} \rho u_\alpha^3 . dA . C_p(n) \quad \ldots (18)$$

$$dP = dF . r . \Omega \quad \ldots (19)$$

$$[\dot{m} = \rho A U_\alpha (1-a)]$$

$$dF = \rho\, dA\, U_\infty (1-a)(V_3 - 0) \quad \cdots \quad (20)$$

Now, using equation 18, 19 and 20, we get that up rho U infinity cube dA CpA R omega rho dA Q infinity 1 minus A V3. This is 21A. Then we have Q infinity square Cp of A R omega 1 minus A V3. So, from here we get V3 equals to 2 U infinity squared A into 1 minus A by R omega. So, that's what at the downstream location that is here at 3 we get a V3.

$$\frac{1}{2}\rho U_\infty^3\, dA\, C_p(a) = r\Omega \rho\, dA\, U_\infty (1-a) V_3 \quad \cdots \quad (21a)$$

$$\frac{1}{2} U_\infty^2 C_p(a) = r\Omega (1-a) V_3 \quad \cdots \quad (21b)$$

$$\Rightarrow V_3 = \frac{2U_\infty^2 a(1-a)}{r\Omega} \quad \cdots \quad (22)$$

Now, we can now write since we have A prime of R is V3 of R by 2R omega which is the radial induction factor sorry tangential induction factor so this is tangential induction factor so what we get a prime of r u infinity square a into 1 minus a r squared omega squared which means d3 is proportional to 1 by r obviously for a equals to constant and with the local speed ratio lambda r equals to mu lambda mu r omega by u infinity r omega by u infinity where mu is r by r we also have A prime R equals to A into one minus C by seminar square. So, we can conclude. So, what we can conclude is that weak rotates more if the turbine moves Relatively. Lower.

That means. Low. Lambda. And. Higher lambda.

Leads to. Less. Rotation. So. Lambda is the. Speed. ratio okay so that can be connected with this weak rotation, so, what you get is that you if your lambda is low the turbine moves relatively slower.

Since $a'(r) = \dfrac{V_3(r)}{2r\Omega} \longrightarrow$ tangential induction factor.

$$a'(r) = \frac{U_\alpha^2 a(1-a)}{r^2\Omega^2} \quad \ldots \quad (23)$$

Which means, $V_3 \propto \frac{1}{r}$ (for $a = $ Cst.). And with the local speed ratio:

$\lambda_r = \mu\lambda = \mu \dfrac{R\Omega}{U_\alpha} = \dfrac{r\Omega}{U_\alpha}$, Where, $\mu = \dfrac{r}{R}$, We also have:

$$a'(r) = \frac{a(1-a)}{\lambda_r^2} \quad \ldots \quad (24)$$

So, which is the case towards the hub. But when it goes towards the tip which is high lambda then leads to less weight rotation. So, that means close to the hub where the lambda is low you have more weight or weight rotates more. That means this rotation of the weight Is going to impact.

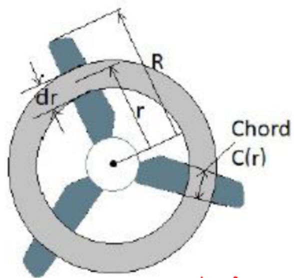
More predominantly. Close to the hub. Whereas the impact would be. Less predominant. To the tip.

Because of this. Connectivity. So this is what. We can get a relationship. Between.

This weak rotation. And all this. Now, what we can look. we can look at the another theory which is called blade element or blade momentum which is BEM theory. So, this is blade element theory or BEM theory, which is very, very well known in the context of these calculations of the turbine. So, we get the annually independent from each other like rotor disk.

And, assume the aerodynamic lift and drive accounting lift and drag accounting to 2D airfoil theory so what we say that the solidity at radius r is defined as sigma r which is b into c of r by 2 pi r where b is the number of blades. So, the overall solidity is total blade area divided by this area. We get sigma is zero to r er dr by pi r square. So, that's what we have. So, what we have is function of r A prime is also function of R.

So these are already we have. So, what we can now, we can see that this is, now allow me to have this thing that, Now we can see how in the context of, now we can see if we see the, so this is blade element drop view at i. So, but if you see, this is how the wind is coming. These are the turbine which is rotating. So, we have taken a small area here, which is da, cr into dr. And at this particular small infinite similar area, the blade speed would be omega into r because it's at the r.



$a = a(r)$, $a' = a'(r)$

Blade Element / momentum (BEM) theory

'solidity' at radius $r \Rightarrow \sigma_r = \frac{B \cdot c(r)}{2\pi r}$

B = # of blades, so the overall solidity is total blade area divided by disc area:

$$\sigma := \frac{B \cdot \int_0^R c(r)\,dr}{\pi R^2} \quad .. \quad (25)$$

So the tangential velocity is a r omega. And axial wind, which is from free stream to reach to this disk is u infinity 1 minus a. Now, this is the situation where you can see that this turbine which is exposed to the incoming wind and then if you look at the top view at r you get this essentially the velocity triangle and you can see you have the blade velocity here okay you have blade velocity here you have induced tangential velocity which is coming from here this is blade speed and this is induced tangential wind direction here alpha is the alpha is angle of attack that means if my turbines are there so every turbine airfoil this is my wind direction so the airfoil angle so that's what it is shown and here the effective wind direction is shown like that alpha is the angle of attack beta is the set pitch angle at R and P is essentially alpha plus beta, which is nothing but row angle. So, the axial wind which is coming, that is this one. So, that's coming perpendicular to that.

So, my blade lies, then the axial wind comes perpendicular to that. And this is my relative wind. and this is my tangential vector so once you get an concept of this particular velocity triangle then if everything else would follow simply the calculation around this. So here, what you can write my effective wind magnitude is w, which would be one can write that which would be u infinity square, one minus a square. If, I look at that velocity triangle, this is my velocity triangle that I am trying to find out.

So, this is my W. So, this would be using your typical Pythagoras theorem plus R square omega square 1 plus A prime square. Now, 2D lift coefficient. That would be CL of alpha.

Effective wind magnitude : $W = \sqrt{U_\infty^2(1-a)^2 + r^2\Omega^2(1+a')^2}$

2D drag coefficient. CD of alpha. And the area of blade element that is DAB which is C dot DR. So what we can find out lift and drag of blade element. So, that now one can find out. So, essentially once you have this velocity triangle understood, then you can find out the lift and drag of blade element.

Okay. So, we'll continue this discussion. We'll stop it here. Thank you.