Wind Energy

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Lecture 23: Momentum Theory

We come back. So, we continue our discussion on now or rather move on discussion to so momentum theory calculation. So, what we have talked so far we have looked at the different, I mean, brief history of the horizontal axis wind turbine. And more importantly, we have discussed about like the different components of the horizontal axis wind turbine. And, then we looked at different, or rather discussed about those different components, their importance and things like that. We want to look at the aerodynamics of this horizontal axis wind turbine.

And in that context, the most important thing is that we have to see how we calculate this thrust force, I mean essentially the lift force, all the moments which are associated with that. Okay. So, to continue on that front, so what we are going to look at is the momentum theory. Okay, so we'll discuss about the momentum theory or somewhere you can think about it's called actuator disc model and from there we'll establish the wedge limit.

So, this we have, so this is essentially the CP max. That's the power coefficient, maximum power coefficient that a particular turbine can have. Earlier we have talked about it, but now we are going to see how that is going to arrive or achieve. So, we start with this, let's say the schematic. this see the turbine which is kept there and at different location.

So, what happens is if we extend this and try to draw the this is going in the direction of x and let's say if you have this velocity profile then you have this u naught here and then so one can think about this will go like this it will come like that And OK. So, let's say this could be u0. This is u1, u2, u3. So, that's all. And around that turbine, we have drawn this stream tube or stream lines.

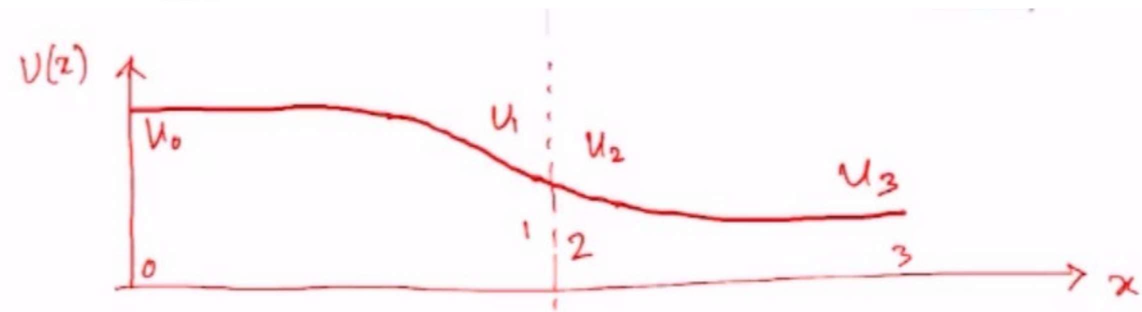
So, obviously, this turbine is modeled actuated disk. So, what you can see the wind is slower approaching at and after the wind turbine. So, this steam tube which is drawn here. This is defined as a tube whose boundaries are parallel to the fluid velocity of the wind passing through the turbine. This is what we have drawn here.

So, let's say if the P air which is power in the air that would flow through the actuated disk, if the actuator disk is not there, then this would be half rho A u0 cube, which is probably not achievable in reality. Because there could be losses and other things. So, at this point, reference zero position this is the velocity that you have and that much of wind energy power in the wind that is available but this cannot be achieved in practical scenario so this graph shows the this graph or plot which shows the axial wind velocity axial low end velocity which slows down as it approaches the turbine and is slowed down further as it passes through. So, what we have, I mean, we can write that u of minus infinity is u naught, u of 0 is u1 equals to u2 and u of plus infinity is u3. So, when you go further away in this side, then you have U naught.

When it reaches here, which is essentially the location of the disk. So, what we assume, we can assume there is no interaction of steam tube with outside. Okay. That's what is there. So, what we can do is that when we look at this, we can also look at the pressure situation.

P_{air} = Power in the air = $\frac{1}{2} \rho A u_0^3$

(Not achievable in reality)



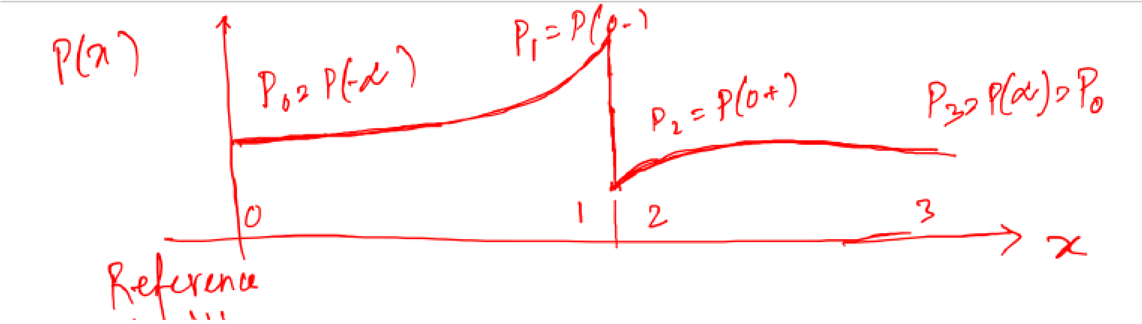
$u(-\infty) = u_0, \quad u(0) = u_1 = u_2$

$u(\infty) = u_3$

So, let's say we plot with x. So this is P of X. So, pressure goes on, shooting up, comes down, then stays somewhere. So, this is what happens. This is 1, 2, 3, again 0.

So, here P of 0 equals to P of minus infinity and P1 is P of 0 minus, P2 equals to P of 0 plus, P3 equals to P infinity equals to P naught. So, this is your reference position. And this is how the x varies. So, what it tells you is that the pressure is built up as the wind approaches the turbine and drops after passing the turbine. So, that's what you have.

what you now we can find out the mass flow rate through turbine which would be m dot rho a u1. So, here rho is I mean assumed to be kind of assumed to be constant as flow is incompressible. So, we are talking about that. So, then we have thrust of turbine that is force against wind.



Okay. So, that essentially difference in spaces in the OR area. So, we can write P2 minus P1. So, this one one This one we mark it 2, equation number 2. So, equivalently we can write say that momentum we can say dot U0 minus U3. So, this is our equation number 3.

Power extraction: $P = T \cdot u_1$ .... (4)

equivalently: change in kinetic energy: $P = \dot{m}\frac{1}{2}(u_0^2 - u_3^2)$ .. (5)

Given: $u_1 = u_2$, $P_3 = P_0$, & u_0, P_0 $\longrightarrow$ $u_1, u_2, P_1, P_2 \rightarrow ?$

thrust eqn: $T = A(P_2 - P_1) = \dot{m}(u_0 - u_3)$ ... (6)

Now, we get power extraction. So, that is P crossed into U1. Or, we can say change in let's say so this we can say equation 4. We can say this is equivalently change in kinetic energy. So, then we write power m dot half u naught minus u3 square.

That is equation number 5. So, let's say given u1 equals to u2, p3 equals to p0, and u0, p0, what we need to find out u1, u2, p1, p2. So, these we need to find out. So, the first equation that we have equals to A into P2 minus P1 equals to M dot U0 minus U0 minus U3.

Okay. So, that's what and now using bar null is equation without energy extraction what we can write we have p plus half rho u square equals to constant this is again along a streamline we can write that so therefore wind flowing through the disk, which is essentially along a steam line, but we can write u naught plus half rho u naught square equals to u1 plus half rho u1 square. that is 7. So, now what we write after passing the disk assuming energy loss at the disk we have E2 plus half rho U2 square p naught plus half rho u three square. It's a Bernoulli's equation that we write along different points. So, there's a reference point one, two and three.

So, what we do, you eliminate p one and p three from equation 6 7 and 8 what you get p1 equals to p naught plus half rho u naught square minus u1 square this is 9, p2 equals to p naught plus half rho u3 square minus u1 square n u1 equals to u1 minus p2 equals to half rho u0 square minus u3 square which is 11. you have this. Now, with equation six, what we write equal into first P2 minus P1. Okay. So, that's essentially 12 A.

So we can write A half rho U naught minus U3 U naught plus U3 rho A U1 u naught minus u three will be so that half of u naught plus u three equals to u one which is twelve c. So what happens is that basically if you look at this So, there is a drop. There is a drop.

eliminate, P_1 & P_3 from eq. (6), (7) & (8)

$$P_1 = P_0 + \frac{1}{2}\rho(u_0^2 - u_1^2) \quad \ldots \quad (9)$$

$$P_2 = P_0 + \frac{1}{2}\rho(u_3^2 - u_1^2) \quad \ldots \quad (10)$$

$$P_1 - P_2 = \frac{1}{2}\rho(u_0^2 - u_3^2) \quad \ldots \quad (11)$$

with eq.-(6), $T = A(P_2 - P_1) \quad \ldots \quad (12a)$

$$A \cdot \frac{1}{2}\rho(u_0 - u_3)(u_0 + u_3) = \rho A u_1 (u_0 - u_3) \quad \ldots \quad (12b)$$

$$\Rightarrow \frac{1}{2}(u_0 + u_3) = u_1 \quad \ldots \quad (12c)$$

Induction [illegible]

So, this is U3. This is U1. This is U naught. Okay. Now we introduce some kind of an induction factor. induction factor of A, which belongs to zero two. Now, what we got induction factor is defined as U naught minus U1 by U naught.

Now, using 12C, what we get u1 equals to 1 minus a u0 and then what we get u3 equals to 1 minus 2a u0 okay so this is what we get u1 and u3 using induction factor that's what you have now what we can compute power and thrust as function of A. So our P is rho U1 A half of U naught square minus U3 square, which is 13 A. So, this is half of rho u naught cube 1 minus a, 1 minus 1 minus 2a square. That is half rho a cube 4a into 1 minus a square.

So, that in C. So this guy is essentially, if I look at the power coefficient, which is CP, is the rotor power divided by power in wind. So, that means P by half rho A. So if I compare this, then what we have, this is essentially my CPA, the power coefficient. Similarly, we can write P that is m dot u naught minus u3 14a rho u1a u naught minus u3 14b you can have half rho a square 1 minus 1 minus 2a 14c up to 4a into 1 minus a 14d so again this is my city of a that is trust coefficient. If we compare this power coefficient and trust coefficient, so what you can have is that what you can have that CP of A 1 by A CT of A.

So this is what you get. Okay. So, one can have some power coefficient and the relationship between thrust coefficient. Okay. So, what we can do, we can maximize maximize our extraction. If you plot this guy, let's say 0, then 0.

Now, we can compute power & thrust on fn. a:

$$P = \rho u_1 A \frac{1}{2}(u_0^2 - u_3^2) \quad \ldots (13a)$$

$$= \frac{1}{2}\rho u_0^3 A (1-a)[1-(1-2a)^2] \quad \ldots (13b)$$

$$= \frac{1}{2}\rho A u_0^3 \underbrace{4a(1-a)^2}_{C_p(a) \rightarrow \text{Power coeff.}} \quad \ldots (13c)$$

Power coeff. (C_p)

$$= \frac{\text{Rotor Power}}{\text{Power in wind}} = \frac{P}{\frac{1}{2}\rho u_0^3 A}$$

$$T = \dot{m}(u_0 - u_3) \quad \ldots (14a)$$

$$= \rho u_1 A (u_0 - u_3) \quad \ldots (14b)$$

$$= \frac{1}{2}\rho A u_0^2 \cdot 2(1-a)[1-(1-2a)] \quad \ldots (14c)$$

$$= \frac{1}{2}\rho A u_0^2 \underbrace{4a(1-a)}_{C_T(a) \rightarrow \text{Thrust Coeff.}} \quad \ldots (14d)$$

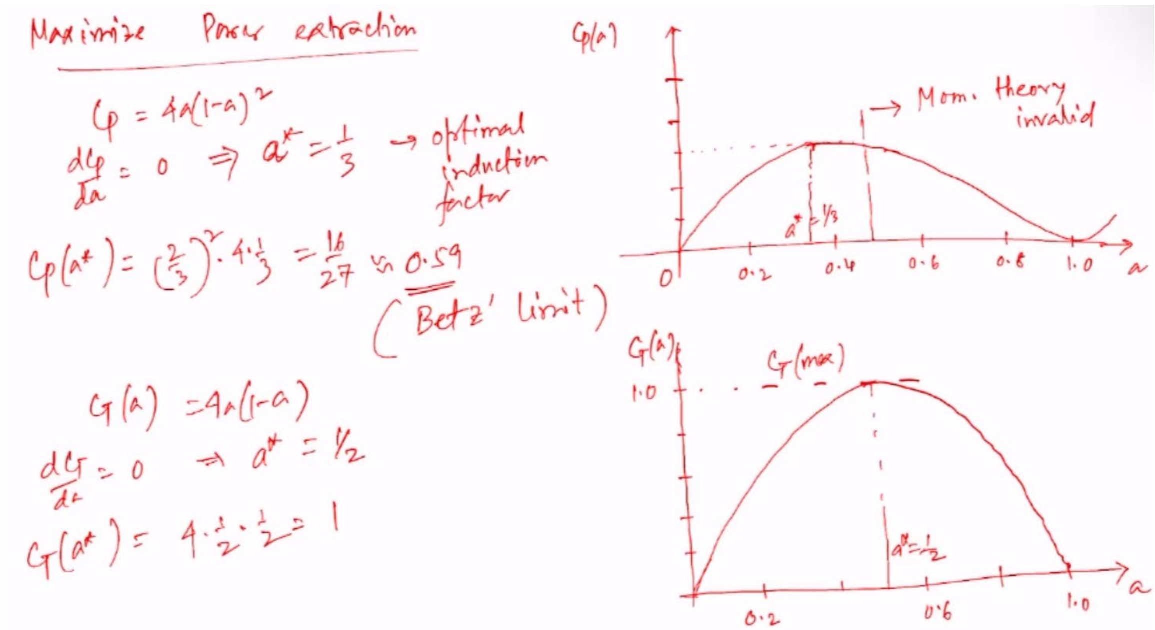
$$C_p(a) = (1-a)C_T(a) \quad \ldots (15)$$

2, 0.4, 0.6, 8, 1, and 1, 2, 3, So, this is CP of A. So, you can have a curve like this and a kind of a curve and then this goes like that. So, you can have some barrier here beyond which the momentum theory is invalid. And that is because my CP is for a one minus a square. So, BCP of BA is zero from which we get essentially a star which is 1 by 3.

So, the thing which is maximum at 1 by 3. This is a. This is at a star 1 by 3. And, so then obviously if a star is the optimal induction factor. So, at A star, Cp of A star is two-third square four into one-third, which is 16 by 27, that is five to nine.

That's what we'll known Bayes limit. maximum power coefficient can be like that. Similarly, you could plot the so you could plot So, this is at 0.6. So, you will have it at city of A.

Now, city of A is 4A into 1 minus A. DCT by DA equals to 0. gives us a star equals to half so this is at half so city at a star is four up into half which is one so this is one this is city max okay So that's what you get that these things. Obviously, this base limit that you have, I mean, so this is the maximum theoretical possible rotor efficient. But in practice, the effect lead to decrease in maximum achievable power coefficients and all this stuff. OK, so but this is what the theoretical calculation gives you.



So, we'll continue this discussion in the next session.