Wind Energy

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Lecture 20: Turbine Calculation

Welcome back to the discussion of this wind energy. So, now to continue our discussion, what we have discussed so far is the basic of all the turbine, turbine technology, then most importantly the wind conditions, wind characteristics and how the atmospheric boundary layer affects the calculation of the power and then specifically the distribution function that we have and so now at least we have some of the fundamentals of the fluid mechanics We have understanding about the wind characteristics, wind speed, the effect of terrain, surface roughness, and all these things. Now, we'll actually look at the basic turbine calculation. And from there, we'll try to estimate the power and all these things. OK. So, what we do, so we'll try to estimate power density and blade area.

So, what we do is that let us try to estimate how much power can be captured by a given blade area AB. So, here AB is the blade area. Okay. And for this given blade area, how much power can be captured? So, here what we are considering, we are considering only outer part of a rotor blade.

So, essentially if you see this is kind of a turbine rotor here and these are three blades and looking from the, I mean, kind of a side come front view depends on the orientation. So, outer part of a rotor blade Which is essentially close to the close to wing tips. And this one, this outer part moves with a speed which is VB. which has been shown here in the picture. So, you see this is the area.

This is the speed of the outer part. So, this moves with the speed VB in cross wind direction. So, one can Note here the inner part of the blade which is this portion. This is inner part. Inner part of the blade moves lower.

So, at least They are not of the focus at this moment. So, what we are looking at is the outer part which moves with a velocity Vb in cross weight direction. Now, we simplify further by assuming that the blade tips move straight, not a circular part. So here, in this picture, we try to, in this area, if you provide the direction like this, this is my wind direction, and the blades are having an airfoil shape, so having a representative airfoil, and this is the VB, and this is effective velocity. So here, we simply assume that assume the blade tip moves straight so not a circular, So, now the motion of the blade tip can be compared to a sailing boat moving half wind or crossing.

So, what we can do the motion of blade tip can now be compared to a sailing boat moving upwind or crosswind. Okay. So, if you look at that, so this one if you try to look at this picture, the top view, so the picture would look like that. So, essentially the top view of this one, so what we can write, the effective wind V e bar is V minus it's an simply using the velocity triangle yet one can write that okay. So, now what we can write from here, V e bar is V0 minus 0 Vb which is V minus Vb.

So the Effective wind magnitude is VB squared plus B squared which is VE. This is what you get. What we have to find out, we have to now determine the forces on wing. So, this is for the time being we talk about the blade tip area of AB. So, what we need one basic fact from the aerodynamics which is forced on a body is going to be proportional body in a moving fluid proportional to the dynamic pressure and area.

$$\text{Effective wind } (\bar{V}_E) = \bar{V} - \bar{V}_B \quad , \quad \bar{V}_E = \begin{bmatrix} V \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ V_B \end{bmatrix} = \begin{bmatrix} V \\ -V_B \end{bmatrix}$$

$$|\bar{V}_E| = \sqrt{V_B^2 + V^2} := V_E$$

Now this force can be decomposed into two components. One is lift, one is drag. The lift is perpendicular to the wind velocity So this would be perpendicular to wind velocity vector and this is aligned with the velocity. Now, let's say you have CL is the lift coefficient and CD is the drag coefficient. Then, we have the lift force FL up CL through V square drag force FD up CD.

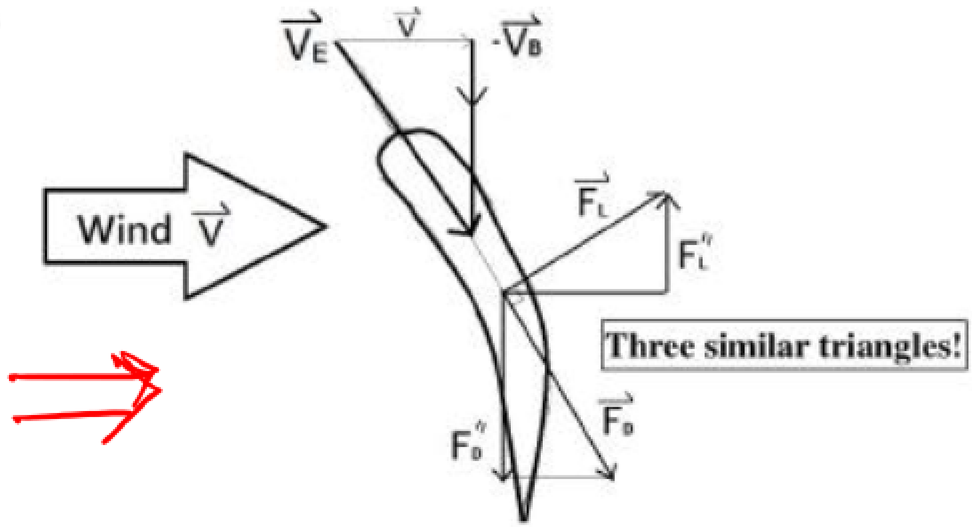
So, our CL and CD So they depends on angle of attack. They depends on Reynolds number. So, which is essentially the inertia to viscous force effect. So, typically the any good wing you will expect to have let's drag high lift similarly for turbine blades also same logic applies where you expect more lift because the lift force is the force which allows this turbine to rotate and you would expect to have as less drag as possible okay so Typically, in context of aerodynamic performance, we try to see lift to drag ratio, which is essentially CL by CD. This is a good interpretation in terms of if you talk about cell planes, if you talk about aircraft.

So, it gives you this ratio. can allows you to understand how far that plane can go and so this number in context of aircraft aerodynamics it is known as gliding number okay so for sail plane for example distance travel would be CL by CD into altitude. So, that's a good marker how you estimate the slip to drag ratio. To some extent, this factor is also quite helpful in terms of turbine calculations and all this thing so now for our rotor blade what we get the this is the for our rotor blade we get this velocity triangle so on top of the airfoil and this is wind velocity so the effective velocity and then you have the lift force you have drag force We have other components. We have three similar triangle.

So, what you have here is for rotation of the wind turbine, We are interested in the force component. We are interested in the force component in the direction of motion of the wing which is F parallel as its product with Vb is the mechanical power production. So, what we get is P of B is F dot Vb. Now what we have if FL plus double prime. So, this is along the direction of the motion of the blade or here the We or you said blade.

So, what we can write here? You can write a fill into V by B. Minus a B into. VB by VE. So.

For our rotor blade:



$$F'' = F_L'' + F_D'' = F_L \cdot \frac{V}{V_E} - F_D \cdot \frac{V_B}{V_E}$$

$$P_B = \frac{1}{2}\rho A_B V_E^2 V_B \cdot \frac{1}{V_E}\left(C_L \cdot V - C_D \cdot V_B\right)$$

$$P_B = \frac{1}{2}\rho A_B V^3 \lambda^2 \underbrace{\sqrt{1+\frac{1}{\lambda^2}}\left(C_L - C_D \cdot \lambda\right)}$$

A FL double prime. as the components of lift parallel to the blade movement direction and if D double prime is components of drag parallel to the blade movement detection. So, these are all we can write essentially this we can write from all this velocity triangle. Okay. Now, once we bring everything together, what I mean, essentially this, if you put it back in the power, so what we get of row AB, V square, AB, 1 by V, PL into V. So, once I put this back and also write the expression for FL and FD, what we have.

These are the expression for FL and FD. So, by putting everything together, essentially that's what we get. Now, let's say we use ship speed ratio, which is lambda, that is dv by

v. So, that means we have VB equals to lambda V. So what we have PE equals to 1 plus lambda into VB.

Or one can simplify 1 plus 1 by lambda square into lambda V. So, which is again one can write 1 plus 1 by lambda square into VB. So, that's the relationship that you get between VE and VB. So, that now if we bring this thing here.

What do you have? We be. Up go. Any. Requeue. Lambda squared. 1 plus 1 by lambda square L td into lambda.

This is the power harnessing factor. Power Furnishing.

So. This. Or. Come. They'll see there. I'm restriction. If. lambda equals to CL by CD then PV equals to zero. So that means no power is generated.

And similarly CL by CD is the maximum possible speed of wing tips if the generator is fixed up. That means there is no torque. What you get here is that you have this picture where this turbine, let's say the turbine is facing like that. So these are kind of rotating. so, if you look at so this is rotating now, if you look at from this view this is how it this particular picture it is the blade sections are careful section so for that rotor blade if you look at from the top and this direction this is how you get the velocity triangle so in terms of wind speed in terms of effective wind velocity in terms of blades tip speed and then you can estimate this mechanical power production and that is also dependent on your CL and CD and CL and CD can maximize this or minimize the situation so now what you can let's say if we take some typical value let's say a typical value for lambda equals to 7 and let's say if cl is 1.5 cd equals to 0.05 then we can calculate the power harnessing factor which is lambda square by lambda square L by CD into lambda. So, essentially that becomes 57, something approximate. For lambda equals to 20, we can even get equals to roughly 200 which is quite high. This number is a remarkably high number.

a typical value for $\lambda = 7$, if $C_L = 1.5$, $C_D = 0.05$

Power Harnessing factor $(\xi) = \lambda^2 \sqrt{1+\frac{1}{\lambda^2}}\,(C_L - C_D \cdot \lambda) \simeq \underline{\underline{57}}$

Okay. So, this is remarkably high number. So, now Xi shows how many times more power a blade area can harvest compared to the energy in the wind which would pass through the window of the same size as the blade area. Now, so this power harnessing factor gives you an idea how many times more the power a blade area can harvest compared to the energy in the wind. What you can see that as the inner part of the blade moves slower, their lambda is smaller. and therefore also their harnessing factor.

So, that's what if you look at the blade, the kind of having a shape like that, which goes like this. Okay. So, what happens is that since the inner part of the blade moves slower, lambda is also smaller for that because lambda is the ratio of the R omega. I mean, lambda A at any location is omega or omega into R by V. So, since the inner part and R is calculated from here.

So, the inner part of the blade if there is a rotation like that So, since that is moving slower, the lambda is smaller. So, also there power harnessing factor is smaller. And this is primarily the reason why the blades become thicker toward the center. So, that's why the blade becomes thicker toward the center. As you can see in this picture that the blades become thicker toward the center.

So, this is how you can see how this power harnessing factor, tip speed ratio, and end of the day, the production of the power from a particular blade. So, slowly you can see from the perspective of the design, you can immediately get to see that why the blade shape in wind turbine looks like that. They have a thicker configuration close to the root or rotor hub and when you go towards the tip it becomes thinner this is because I mean later on also again we can see when we bring in the structural issue into that okay so now with this you have an idea these blades are having aerofoil shape and how aerofoil is going to play a key role. So, we'll discuss those issues now. Okay? Thank you.