Wind Energy

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Lecture 19: Wind Data Analysis (contd..)

we'll come back. So, now we'll continue the discussion on this statistical data analysis. Obviously, we have talked about this different direct method, method of beans and the statistical ways of, so they're the major uh aspect of it is this a different kind of probability or probabilistic distribution uh of the wind velocity so once you know this probability distribution function so what it allows that you can estimate the..

. So we have talked about..

. I mean, earlier we have talked about this Rayleigh distribution, which we have seen, which is represented like... which is a special case of.

.. u by square exponential minus pi by four u by u prime square and my cumulative distribution function will be one minus exponential minus pi by four u y u r square, so, this is what we have seen this is a kind of an special case of then we have this, Weibull distribution. That also. So, the probability distribution function of that. Based on two parameter. One is that.

And then the. A minus one. Exponential minus. U by C to the power K.

So my. this would be 1 minus exponential u by c to the power k so these are the distribution then we have also looked at normal or gaussian distribution okay so this is a different kind of distribution that we have seen which are there. So, what happens is that using this Weibull distribution function, so, we can find out the average velocity which is u bar c gamma of one plus one by k and gamma of x, which is gamma function is zero to infinity, d to the power minus dt. So, gamma function, one can estimate like root over two pi x x minus 1 e to the power minus x 1 by 12 x minus 139 by 840 x cube go on similarly for this weibull distribution sigma u where is u squared gamma 1 plus 2 by k plus 1 by gamma squared 1 plus 2 by k so minus 1 so this also you can find out So it's not quite straightforward to get C and K in terms of U by sigma U. However, there are some number of approximations that one can use because the factor that is going to play an important role in this estimation is C and K. There are some analytical or empirical let's say for k less than 10 greater than equals to 1.

Rayleigh distribution:

$$p(U) = \frac{\pi}{2}\left(\frac{U}{\bar{U}^2}\right)\exp\left[-\frac{\pi}{4}\left(\frac{U}{\bar{U}}\right)^2\right]$$

$$F(U) = 1-\exp\left[-\frac{\pi}{4}\left(\frac{U}{\bar{U}}\right)^2\right]$$

Weibull distribution:

$$p(U) = \left(\frac{k}{c}\right)\left(\frac{U}{c}\right)^{k-1}\exp\left[-\left(\frac{U}{c}\right)^k\right]$$

$$F(U) = 1-\exp\left[-\left(\frac{U}{c}\right)^k\right]$$

Normal/Gaussian

$$\bar{U} = c\,\Gamma\left(1+\frac{1}{k}\right), \quad \Gamma(x) = \int_0^\infty e^{-t}t^{x-1}\,dt$$

$$\Gamma(x) = \left(\sqrt{2\pi}\right)\left(x^{x-1}\right)\left(e^{-x}\right)\left(1+\frac{1}{12x}+\frac{1}{288x^2}-\frac{139}{51840x^3}+\cdots\right)$$

$$\sigma_U^2 = \bar{U}^2\left[\frac{\Gamma(1+2/k)}{\Gamma^2(1+2/k)}-1\right]$$

So, the approximation could be k equals to sigma u by u bar minus 1.086. Then what one can get u bar c equals to u bar by gamma 1 plus 1 by c. But obviously still you need to evaluate the gamma function here. Gamma function.

That one has to evaluate that. Now, some other empirical formula. So, this is the first one. This was by again Justice in 1978. Then again some empirical relation.

This was by Leisen. Here C by U bar. 0.568 plus 0.433 divided by k to the power minus 1 by k.

Then one can use some kind of which is log log plot. Okay. This was provided by Rodge and Nelson in 1994. So, this is a I mean, essentially, it's a graphical based approach to where you use this to get this estimation of directly that u cube mean which was u cube p of u du. So, this factor kappa is there then for example, energy pattern ke which is u cube bar.

by u bar cube which is gamma 1 plus c by k 1 by k this allows some calculation to estimate all these things to have these calculations now what you can have You can have extreme wind speed as well. So, the primary meteorological factor in evaluating a prospective wind turbine site is the mean wind speed. And another important consideration is the anticipated extreme wind speed. So, that means you have mean wind speed then you have anticipated extreme wind speed. This is the highest wind speed expected over some relatively longer period of time.

(i) Analytical / Empirical (Justus, 1978) : for $1 \le k \le 10$ approx: $k = \left(\frac{\sigma_U}{\bar{U}}\right)^{-1.086}$

$$c = \frac{\bar{U}}{\Gamma(1+1/k)} \leftarrow \text{gamma fn.}$$

(ii) Empirical (Lysen, 1983) $\frac{c}{\bar{U}} = \left(0.568 + 0.433/k\right)^{-1/k}$

(iii) Graphical : Log-Log plot (Rohatgi & Nelson, 1994)

$$\overline{U^3} = \int_0^\infty U^3 p(U)\,dU = c^3\,\Gamma\left(1+\frac{3}{k}\right)$$

$$K_e = \frac{\overline{U^3}}{(\bar{U})^3} = \frac{\Gamma(1+3/k)}{\Gamma^3(1+1/k)}$$

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I mean, extreme wind speed are kind of some concern for design purposes is that it must be designed in order to withstand such extreme wind speed. Now, these are typically described in terms of recurrence. Specifically, an extreme wind is the value of highest wind speed, which is averaged over some approximated time interval with an annual probability of occurrence of 1 by n years. For example, the highest 10-minute average wind with 50-year recurrence period would have a probability of occurrence of.

.. Actually, determination of extreme wind speeds by actual measurement is quite difficult since it would require measurements over a long period of time. It is possible to estimate extreme wind speed power by using extremes over shorter period of time together with a statistical model. So, the most common statistical model for estimating extreme wind speed is the suitable statistical model is the Gumbel distribution. So, this is what is used to estimate extreme wind speed. So, the distribution of this P of ue 1 by beta exponential minus ue minus mu divided by beta exponential of minus exponential minus ue minus mu by beta here u is the extreme wind over some time period obviously it has to be satisfactory beta is sigma e root 6 by pi mu u is ue minus 0.

577 beta And ue bar is mean of a set of extreme values. And sigma e is standard deviation of the set. Standard deviation.

$$p(U_e) = \frac{1}{\beta} \exp\left(\frac{-(U_e-\mu)}{\beta}\right) \exp\left(-\exp\left(\frac{-(U_e-\mu)}{\beta}\right)\right)$$

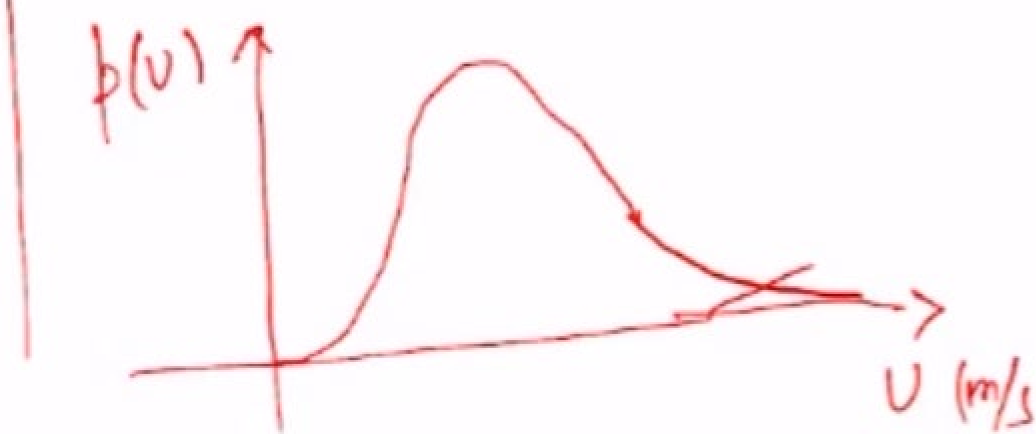
U_e = extreme wind over some time period

$\beta = (\sigma_e\sqrt{6})/\pi$, $\mu = \bar{U}_e - 0.577\beta$

$\bar{U}_e$ = mean of a set of extreme values.

σ_e = standard deviation of

$$F(U_e) = \exp\left(-\exp\left(\frac{-(U_e-\mu)}{\beta}\right)\right)$$



of the set. Okay. We can have the cumulative density function of that which is exponential of minus exponential of u e minus. Gumbel distribution also looks similar if this is my you this is p of u this also goes again so this has a similar tendency of um or similar kind of nature that one see in gaussian distribution, so, it has the, so, now what we get now we can get all the wind turbine energy production estimate so wind turbine energy production estimates. This is all let's say using statistical techniques or method. So, what you have is that for a given resume of probability density function P u. So, what we can get the average wind turbine power that is P bar w we can estimate P w u P of u d u.

So, this PW bar which is average wind turbine power can also be used to calculate a related performance parameter called the capacity factor. The capacity factor of a wind turbine at a given site is the ratio of the energy actually produced by the turbine. So, this is capacity factor which is actually produced by the turbine to the total energy that could have been produced if machine run at its rated power that means PR. So, PR is rated power that means this is how we estimate the capacity factor. So, obviously also one can determine the turbine power I mean curve using the turbine power curve, wind power coefficient Cp.

So, what happens is that once we have Cp as a power coefficient, so we have PwU would be top rho A Cp theta U cube. here eta is drive train efficiency which is essentially the generator power to rotor power okay and a density these are all road so then what we can get is that Cp is rotor power to power in the wind. So, P rotor up row Typically, CP is also a function of tip speed ratio. So, the tip speed ratio which is lambda is blade tip speed to wind speed which is omega r by u. So omega is the angular velocity which is in radian per second and r is the radius of the wind rotor.

So, if we assume a constant value for drive efficiency then what we can write assume constant value for What we write is that up rho pi r squared eta 0 to p of lambda u cube pu du. Now, using this one can estimate the power and all these things using statistical analysis. So, one can have some idealized machine productivity calculation. Here we use Rayleigh distribution to make our lightweight simpler. So, in idealized twin turbine, so since it is idealized, so that means no loss, the CP would be 16 by 27 as per base limit.

average wind turbine power ($\bar{P}_W$) = $\int_0^\infty P_W(U) \cdot p(U)\, dU$

CF = $\frac{\bar{P}_W}{P_R}$, P_R = Rated power ,

C_p = power coeff. , $P_W(U) = \frac{1}{2} \rho A C_p \eta U^3$ (η = drive train eff. $= \frac{\text{generator power}}{\text{rotor power}}$)

$$C_p = \frac{\text{Rotor Power}}{\text{Power in Wind}} = \frac{P_{rotor}}{\frac{1}{2}\rho A U^3}$$

tip-speed-ratio (λ) = $\frac{\text{blade-tip speed}}{\text{Wind speed}} = \frac{\Omega R}{U}$ (Ω = angular vel.

assume cst. value for η, $\bar{P}_W = \frac{1}{2} \rho \pi R^2 \eta \int_0^\infty C_p(\lambda) \cdot U^3 \cdot p(U)\, dU$

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This will again, I will talk in details when we will talk about the momentum theory and things like that. so, this is the maximum theoretical possible power coefficient then can have then what we can estimate so we know the roller distribution so we can estimate this pw bar is half rho pi r square eta zero to infinity cp lambda u cube 2 u by u c square exponential minus u by u c square uc is characteristics wind velocity which is 2u bar by root pi now for an ideal machine for an ideal machine eta equals to 1 And CP is equals to CP bridge limit, which is 16 by 27. So, what we have PW bar is up rho pi R square.

UC is Q. CP bridge. U by UC cube. u by u c square du by u c now one can now normalize the wind speed by defining dimensional wind speed some let's say dimensionless wind speed which is x u by u c then this integral becomes pw half rho pi r square uc cube 0 to infinity x cube 2x exponential minus x square dx. So, what you can see that wind machine constants have been removed. The integral now can be evaluated over all wind speed and this value would be, so if we integrate this, this would be c by 4 into root pi. So, what we have now pw is half rho pi r square uc cube 16 by 27 3 by 4 root pi now we can use the diameter b then we can little bit simplify it is rho by two-third b square into q. So, this is what you can see one can calculate the average productions and things like that using this kind of.

Idealized, No loss, $C_p = 16/27$ (as per Betz limit)

$$\bar{P}_W = \frac{1}{2}\rho\pi R^2 \eta \int_0^\infty C_p(\lambda) \cdot U^3 \left\{ \frac{2U}{U_c^2} \exp\left[-\left(\frac{U}{U_c}\right)^2\right]\right\} dU$$

U_c = characteristic wind vel: $= 2\bar{U}/\sqrt{\pi}$

For an ideal m/c, $\eta = 1$, $C_p = C_{p_{betz}} = \frac{16}{27}$

So, $$\bar{P}_W = \frac{1}{2}\rho\pi R^2 U_c^3 \cdot C_{p,betz} \int_0^\infty \left(\frac{U}{U_c}\right)^3 \left\{\frac{2U}{U_c}\exp\left[-\left(\frac{U}{U_c}\right)^2\right]\right\} \frac{dU}{U_c}$$

dimensionless wind speed $(x) = \frac{U}{U_c}$

$$\Downarrow$$

$$\bar{P}_W = \frac{1}{2}\rho\pi R^2 U_c^3 \, C_{p,betz} \underbrace{\int_0^\infty x^3\left\{2x\exp\left[-x^2\right]\right\}dx}_{= \frac{3}{4}\sqrt{\pi}}$$

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So, once the probability distribution is known then you can estimate this and then using the annual or productions for this thing also you can do some productivity calculation for real wind turbine. So, this is used. We will distribution. So here, what we have the PW bar would be due to infinity.

PW you you do. So, what we get using you distribution function so we can write PWU d of fu and for Weibull distribution f of u is 1 minus exponential u by c to the power k so once we replace this thing so what we can get is j equals to 1 to nb exponential u g minus one c to the power k minus exponential u j by c to the power k p w u j minus one plus u j by two so this is some kind i mean the statistical methods you can see that has the similar expression that we have. So, these are the kind of ways that one can estimate this. So, the final thing that we would like to touch is that regional wind resource assessment. So, what has been observed is that the major barrier for development in many regions of the world is a lack of reliable and detailed wind resource data. So, one of the first steps required for regional wind is an estimate of the available wind So, there are many methods which one can use to estimate that.

we have, $\bar{P}_W = \frac{1}{2}\rho\pi R^2 U_c^3 \left(\frac{16}{27}\right)\left(3/4\right)\sqrt{\pi}$ - | Diameter = D

$$\bar{P}_W = \rho\left(\frac{2}{3}D\right)^2 \bar{U}^3$$

Productivity Calculation for Real Wind Turbine (Use Weibull distribution)

$$\bar{P}_W = \int_0^\infty P_W(U)\, p(U)\, dU = \int_0^\infty P_W(U)\, dF(U)$$

$$F(U) = 1 - \exp\left[-\left(\frac{U}{c}\right)^k\right]$$

$$\bar{P}_W = \sum_{j=1}^{N_B}\left\{\exp\left[-\left(\frac{U_{j-1}}{c}\right)^k\right] - \exp\left[-\left(\frac{U_j}{c}\right)^k\right]\right\} P_W\left(\frac{U_{j-1}+U_j}{2}\right)$$

So, some methods we will just, one is the Polk-Rohr method, measurements only. Then you can measure, correlate, then predict. You can use global database. You can use wind atlas methodology. You can have site database modeling, then mesoscale modeling.

combined meso or microscale modeling. So, these are different way one can do the resource assessment for regional resource assessment because these are again to since there is a lack of reliable and detailed data, so regional database assessment is very, very important. One can estimate using different ways to get some idea about all these things so what we have done is that we looked at the atmospheric boundary layer how the lapse rate is happening and then we have also at this different statistical methods and distribution function how one can use those to estimate the wind power and all these things i mean the major key idea here is that all these different things are they are available in nature because nature's surface is not smooth there would be terrain there would be some roughness and all this is going to have some impact on the wind velocity profile or one can say that there is the effect of that atmospheric boundary layer and then that's going to affect your wind shear and wind shear is going to be key design parameter for turbine. So, that will see how wind shear, wind bay effect and all this when you go detailed discussion on this turbine parameters and all these things. So, with that you have a fairly good idea how what are the resource of the wind motions how that is happening, how that is impacting these things. Now, we can move to discussion about power calculations and all this. So, that we will do in the next session.