

Wind Energy

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Lecture 16: Atmospheric Boundary Layer (contd..)

We'll come back to our continued discussion on this atmospheric boundary layer and finding out the lapse rate. So, this is where actually we stopped in the last discussion where we wrote from assuming the ideal gas flow system of unit mass, which is going through and what is static change of state. So, using first law of thermodynamics, we wrote this where actually P is temperature, U is heat transferred, Here U is internal energy, H is enthalpy and this V is specific volume and C_p is constant pressure. specific heat okay these are the parameter that we have now if you consider the process is adiabatic that means there is no so which means dq would be zero okay If we do that.

Then from here. We get. $C_p dT$ minus one by rho dP . So that gives us One by rho.

dP . So what we can. Do that. We can. So replace this.

expression in this particular one so we had also let's say we had dp equals to minus rho dz so these two combining we can get dt by dz obviously adiabatic g by C_p . So, that's what we get. So, if I dT by dZ . Adiabatic is g by C_p . So let's say the g and C_p this variation.

Process is adiabatic (No heat transfer)
 $dq=0$

$0 = C_p dT - \frac{1}{\rho} dP$
 $\Rightarrow C_p dT = \frac{1}{\rho} dP$

had: $dP = -\rho g dz \rightarrow \left(\frac{dT}{dz}\right)_{ad.} = \frac{g}{C_p}$

With elevation. Is. Assumed to be. Negligible. Then the change in temperature.

Under adiabatic condition. Is constant. So, if you put a standard value of G . Which is 9.81 meter per.

Second square. And C_p 1.005. kilojoule per kg kelvin, then what we get dt by adiabatic is minus we miss the minus sign here so, this has to be minus so, this becomes nine eight by m degree c So, this typically gives the temperature decreases with increases. So, this tells me.

So, temperature decreases when elevation. So, with the increase in height for a system with no heat transfer, this is minus dt by dz It's a one degree C or 100 meter. So, that is what you get. And, this is what is known as lapse rate. So, this one has to also, I mean, be very specific.

It is a dry lapse rate because the dry adiabatic lapse rate is quite important in meteorological studies since a comparison of its value to the actual upset in lower atmosphere is a measure of stability of the atmosphere. Since, actual atmospheric condition will have most shares in other things, so comparing that would give you the stability of these things. Now, for comparative purposes, The international standard atmospheric lapse rate based on the metrological data has been adopted and defined specifically on the average in the middle latitude. The temperature decreases linearly with elevation up to 10,000 meters. So, the temperature decreases linearly up to about 10,000 meter.

So, which is 10.8 kilometer. Average temperature at sea level is 288 and decreases to 216. So, the standard lapse rate that is there, which is 216.7 minus 288 in 0 meter.

So, it is 0.0066 degree C by meter. So, that's the standard lab state based on intestinal condition is roughly 0.66 degree per 100 meters. That is as per the international standard.

Okay. We can see typically also the temperature profile changes from day to night due to heating of the Earth's surface. So, we can kind of have a representative graph. So, this is steam. This is so typically this is let's say did one or did I so did hi here is called the inversion right Okay. So, here, before sunrise, this solid line shows some kind of a temperature profile.

And this decreases with increasing height near the ground and reverse after sunrise, which is this kind of a dotted or dashed line. that changes with the sunlight so air is usually heated near the ground and the temperature gradient close to earth surface

increases with height up to height some z_i so the surface layer of air up to this height is called the so this is known as So up to this height it is known as. Above this particular height the temperature profile reverses. This is how the lapse rate is taken into consideration finding out the stability. dt by dz if it is greater than dt by dz adiabatic this situation is a stable situation okay obviously one has to note that the international upstate seldom occurs in nature so, this is why you need to have those meteorological data to monitor these things around your airports worldwide to determine the actual offset.

$$\left(\frac{dT}{dz}\right)_{ad.} = -\frac{g}{C_p} \quad ; \quad g, C_p \rightarrow \text{variations with elevation is assumed to be negligible}$$

$$g = 9.81 \text{ m/s}^2, \quad C_p = 1.005 \text{ kJ/kg}\cdot\text{K}$$

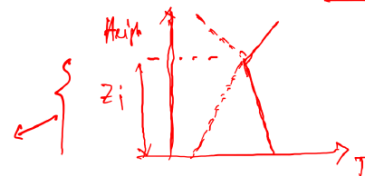
$$\left(\frac{dT}{dz}\right)_{ad.} = -\frac{0.0098 \text{ }^\circ\text{C}}{\text{m}} \rightarrow T \downarrow \text{ when } z \uparrow$$

$$\Gamma = -\left(\frac{dT}{dz}\right)_{ad.} \approx \frac{1^\circ\text{C}}{100 \text{ m}} \rightarrow \text{Lapse rate (Dry)}$$

T - decreases linearly up to about 10,000 m. ($\approx 10.8 \text{ km}$)

$$\left(\frac{dT}{dz}\right)_{\text{surface}} = \frac{(216.7 - 288)}{10800 \text{ m}} = -\frac{0.0066^\circ\text{C}}{\text{m}} \approx \underline{0.66^\circ\text{C}/100 \text{ m}}$$

z_i = inversion height
convective or mixing layer



Conclusion

In general, $\frac{dT}{dz} > \left(\frac{dT}{dz}\right)_{ad.} \rightarrow$ stable | Unstable
 To have stability: Atmosphere is more stable | Neutral

So, you have this daily balloon surrounding those airports. Also, to have stability, to have stability, it is not necessary for an inversion to exist. When one does exist, the atmosphere is even more stable. So this is how the atmospheric boundary layer stability can have some impact and obviously this can lead to different stables to unstable situation and or maybe staying at neutral stable condition so what it does that gives you an idea and all these things obviously when you talk about this atmospheric boundary layer the one of the important issue is the turbulence which we have seen some basics of the turbulence But now we will go a little bit more in details about those things, how this impacts the boundary layer and in turn the calculation of the wind energy and so on. So, the turbulence here in the wind is usually caused by the dissipation of wind's kinetic

energy into thermal energy via the creation and destruction of some smaller eddies or gusts.

obviously, any turbulent flow they can, i mean, it is more or less like an if you look at the speed so, this could be very much fluctuating on random in nature so, every flow will have some kind of an mean, okay obviously, the feature to characterize is that one has to talk about turbulence intensity, wind speed, probability density function, autocorrelation, integral time scale, integral time scale or length scale, PSD or spectral density so we have very briefly talked about these things and how they are interconnected to those things so so, when you talk about the turbulence so this field is essentially 3D in nature so the impact can be sensed both, I mean, in all longitudinal, lateral and vertical components. Obviously, longitudinal component in Pivotal Wind Detection. So, if I say that longitudinal in Pivotal Wind Detection is defined at $u(z,t)$, the lateral component which is perpendicular to $v(z,t)$ and the vertical component would be $w(z,t)$. So, essentially these are, if I have a vector, these are all these component u , v , w . That's the, so once you have as we have seen this particular, the instantaneous velocity, let's say U , which could be having two components.

One is the mean component and the fluctuating components. This is mean. And, then let's say, here, this is the fluctuating component. So, that means the flow will have some mean around that and fluctuating component. So if you have short term mean of the wind speed, then you can, for a, let's say, time period of Δt then you can find out this mean so here use the mean we can write $\frac{1}{\Delta t} \int_0^{\Delta t} u \, dt$ okay, also you can collect lot of samples and using the collected sample let's say sampling interval sampling interval is let's say Δt is Δt and you have collected number of sample then your Δt is $n \Delta t$ so, we can write u is $\frac{1}{n} \sum u$ So, this also we have seen that how this is one can think about short term average longitudinal wind speed.



instantaneous vel. $\Rightarrow u = U + \tilde{u}$ \tilde{u} = fluctuation component
time period: Δt , $U = \frac{1}{\Delta t} \int_0^{\Delta t} u dt$ U = mean

often used in time series observation and also we will use this in our discussion. So, next thing that you need to look at is the turbulence intensity. So, that is a basic measure of turbulence in the turbulent flow field. So, it's kind of a defined the ratio of the standard deviation of the wind speed to the mean wind speed. So, if my turbulence intensity is like that, then we can write sigma u by u.

Sampling interval: δt , N_s = no. of samples
 $\Delta t = N_s \delta t$, $U = \frac{1}{N_s} \sum_{i=1}^{N_s} u_i$ → short term average longitudinal wind speed.

So, this also you have seen. And this calculation, both the mean and standard deviations usually calculated over a time period longer than of the turbulent fluctuation, but shorter than the periods associated with one type of wind speed variation. This, Sigma u is essentially one by N_s minus one summation of one to N_s U_i minus U . That's the typical turbulence intensity varies from range of 0.

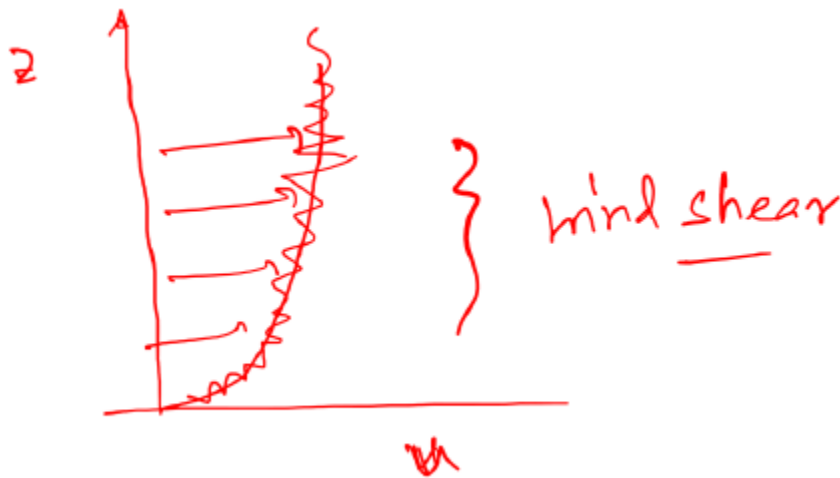
1 to 0.45 and dependency. I mean, in general the highest turbulence intensities one can observe at the lowest wind speed but the lower limiting value given depends on the location specific terrain features surface conditions etc so, now the other thing is the wind speed probability density function This also we have seen that because wind speed has a variation over time and those variations to capture those variations we have looked at at least the Gaussian distribution which is normal distribution, Weibull distribution and the Rayleigh distribution. We have seen the quality density function of those distributions.

because these distributions are very important to have in and also we have looked at the property of autocorrelation so how they are correlated the data and the between two spatial locations and how closely and we talked about that as well now what and then we have found from autocorrelations we can find out the integral length scale time scale all this So, that's how from the autopollution function you can find those out. And then power spectral density that is another thing that we have seen that is PSD or spectral density function. So, this is used for kind of a dynamic analysis of the system so often this number of power spectral density function are used as model in wind energy engineering and when the representative turbulence or spectral densities are unavailable so there are theoretical models which are available for different flow fields and things like that so those are kind of used.

So, now another thing is that wind speed variation. Wind speed variation with height. So, wind speed also varies with height. Then you will have I mean the We have seen that the variation profile could be like that. So, there is a variation of the speed with height.

So, this is essentially going to lead to wind shear. And wind shear influences both assessment of wind resources and the design of the wind turbine. because assessment of wind resources over a wide geographical area might require that anemometer data for a large number of sources again from design expect aspect a rotor belt failure life will be influenced by the cycle loads resulting typically from rotation through a wind field that is varying with the so this variation with height or indirectly which we term as wind shear, this has impact in design, this has impact in assessment. So, this is an important parameter. Typically there are two mathematical models which exist or laws one can say generally what it has been used that the this vertical profile of wind speed over a region of homogeneous flat terrain, that is field deserts and the first approach is kind of having a log law okay, and then second is the power law profile so kind of if you see this two-dimensional feature.

This is the mean variation of the profile. Actually, this can vary like this. Log law or logarithmic profile, which is often known as log law. So, I mean there are different ways also to arrive at the prediction of logarithmic wind profile. So, what we can say that near surface of the earth, the momentum equation reduce it to $\frac{\partial p}{\partial x} = \frac{\partial \tau_{xz}}{\partial z}$ so here the direction wise we can say that x and z these are the direction so, p is the pressure τ_x is the shear stress in the direction x was normal coincide with z pressure is independent of z and integration yields that $\tau_{xz} = \tau_0 + z \frac{\partial p}{\partial x}$ where τ_0 is surface value of τ_{xz} .



$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial z} (\tau_{xz})$$

since near the surface the pressure gradient is small so what we can do we can neglect this term so we can neglect this term because near surface assuming that pressure gradient near surface is small. So, with that, you can also use Prandtl mixing length theory, which gives us τ_{xz} is $\rho L^2 \frac{\partial u}{\partial z}$. So, ρ is the density of the air, u is horizontal component of the velocity. So, now if we combine this and this, what we get $\frac{\partial u}{\partial z}$ equals to $\frac{1}{L} \tau_{xz}$ by which is u^* by L .

Use Prandtl mixing length theory:

$$\tau_{xz} = \rho L^2 \left(\frac{\partial u}{\partial z} \right)^2$$

$$\frac{\partial u}{\partial z} = \frac{1}{L} \sqrt{\frac{\tau_{xz}}{\rho}} = \frac{u^*}{L}$$

Where you start. Is. So this is my. Hello. So. If we assume.

Let's say. You assume. Mood surface. Then L equals to κz . where κ equals to 0.4, which is known as one common constant. So then we can integrate this from Z_0 to Z , where Z_0 is Surface.

Roughness. Length. We get u of $z = u_{star}$ by κ .

ln. Red by z . So, this is. Known as. Logarithmic. So, this is what you get, which is also known as the log law. So, obviously this further we can develop and come to a conclusion. final expression for that so, this logarithm profile is one of the profile which has been used to calculate the wind shear and things like that okay so, we'll stop here and we'll continue this discussion and finish it off in the follow-up session

$$\boxed{u(z) = \frac{u^*}{\kappa} \ln\left(\frac{z}{z_0}\right)} \rightarrow \text{logarithmic wind profile.}$$