Wind Energy

Prof. Ashoke De

Department of Aerospace Engineering, IIT Kanpur

Lecture 15: Atmospheric Boundary Layer

welcome back to discussion of this different distribution of the wind speed, so, which are we talked about the normal distribution of the gaseous distribution as you can see here where you can use this probability density function and then you can estimate this so as I have emphasized that these distributions of the wind speed is very very important to estimate the power and all this. The second one that we can talk about is that Weibull distribution. This is another kind of distribution. So, there are different kinds of distribution which are possible. but wind energy calculation which we have also seen earlier, very commonly use this level distribution, okay! so, this distribution has one is the scale parameter, scale parameter of c and another is the ship parameter or factor which is so, this we have already seen that while talking about the Weibull distribution. So, here that f of u is represented as 1 minus exponential minus u by c to the power k.

So my p u would be f of u prime so, one can write k by c u by c k minus 1 into exponential minus u by c to the power k so it's taking the derivative of that so, you can get this. So, what you can show that the mean and when you compute from this c and k using the gamma function. So, we can use the gamma function. So, gamma of x is zero to infinity minus t to the power dt where gamma of n is defined and n minus 1 so gamma 1 is 1 gamma 2 is 1 like that so using this gamma function and this we can estimate the mean wind speed.

So, that can be you. C into gamma of one plus one by K. And my variance would be. C squared gamma one plus. Two by K minus C squared.

(b) Weibull distribution $\rightarrow$ scale parameter (c)

$\rightarrow$ shape " (K)

$$F(U) = 1 - \exp\left(-\left(\frac{U}{c}\right)^{K}\right)$$

$$P(U) = (F(U))' = \left(\frac{K}{c}\right)\left(\frac{U}{c}\right)^{K-1} \exp\left(-\left(\frac{U}{c}\right)^{K}\right)$$

$$\Gamma(x) = \int_{1}^{\alpha} e^{-t}\, t^{x-1}\, dt \quad \Big| \quad \Gamma(n) = (n-)! \quad \Gamma(1) = 1, \; \Gamma(2) = 1 \ldots$$

Gamma one plus one by K whole square. So, this you can write 0 to infinity u square p u du minus u bar square. So, one can write 0 to infinity u minus u bar square probability of u into du. So, that's how the variance is calculated. So, if you look at the variation of the gamma function with respect to k, this is how this varies so let's say 3 this is 1 this is gamma 1 plus 1 by k ok and this this is something 0.

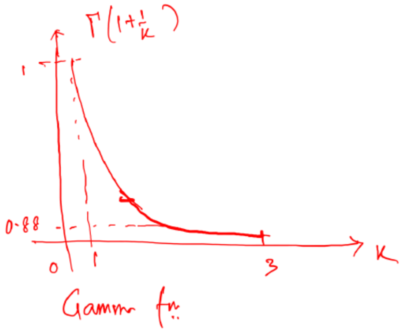
88 this would be 1 this is the variation of the function okay so if you have a this is my power curve and then go like this so this could be cut in speed cutting speed so this is cut out speed and this is design wind speed design wind speed this is for a given turbine which is fixed area for that. So, this is how the power curves would look like. Now, you can have another kind of distribution. So, these are different distributions which are possible. This is Rayleigh distribution.

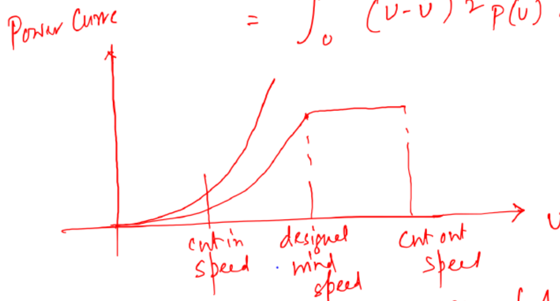
Okay. So, this is a special case of Weibull distribution with kappa equals to 2. Here gamma 1 plus 1 by 2 is root over pi by 4 and c is u bar root over pi by 4. So, here you have f of u one minus exponential u by c square and you have p of u two by c square u exponential minus u by c square. So, this So, Rayleigh distribution is a special case of Weibull distribution with kappa equals to 2. So, Rayleigh distribution corresponds to vector magnitude of two-dimensional Gaussian distribution.

$$\bar{U} = c\cdot\Gamma\left(1+\frac{1}{k}\right), \quad \sigma_U^2 = c^2\Gamma\left(1+\frac{2}{k}\right) - c^2\left(\Gamma\left(1+\frac{1}{k}\right)\right)^2$$

$$= \int_0^\infty U^2 p(U)\,dU - \bar{U}^2$$

$$= \int_0^\infty (U-\bar{U})^2 p(U)\,dU$$





Given function (fixed area)

(c) Rayleigh distribution

$k=2, \quad \Gamma\left(1+\frac{1}{2}\right) = \sqrt{\frac{\pi}{4}}, \quad c = \frac{\bar{U}}{\sqrt{\frac{\pi}{4}}}$

$F(U) = 1 - \exp\left(-\left(\frac{U}{c}\right)^2\right), \quad p(U) = \frac{2}{c^2}U\cdot\exp\left(-\left(\frac{U}{c}\right)^2\right)$

→ corresponds to vector magnitude of 2D Gaussian distribution.

So, this one can say this corresponds to vector magnitude magnitude of 2d Gaussian distribution. Okay. So, these are different kinds of distribution that you can have. So, obviously, we had the simple one which is on the normal distribution of Gaussian distribution we had then we will distribution which we have talked enough because for different site and this is commonly used in the wind energy calculations then you can have the Rayleigh distribution which is again a special case of the distribution and things like that so so these are i mean i mean one can now talk about some kind of an thing let's say given a wind speed and power curve one can estimate what is the let's say given is the wind speed distribution and power of distribution. So, these are the things which are given.

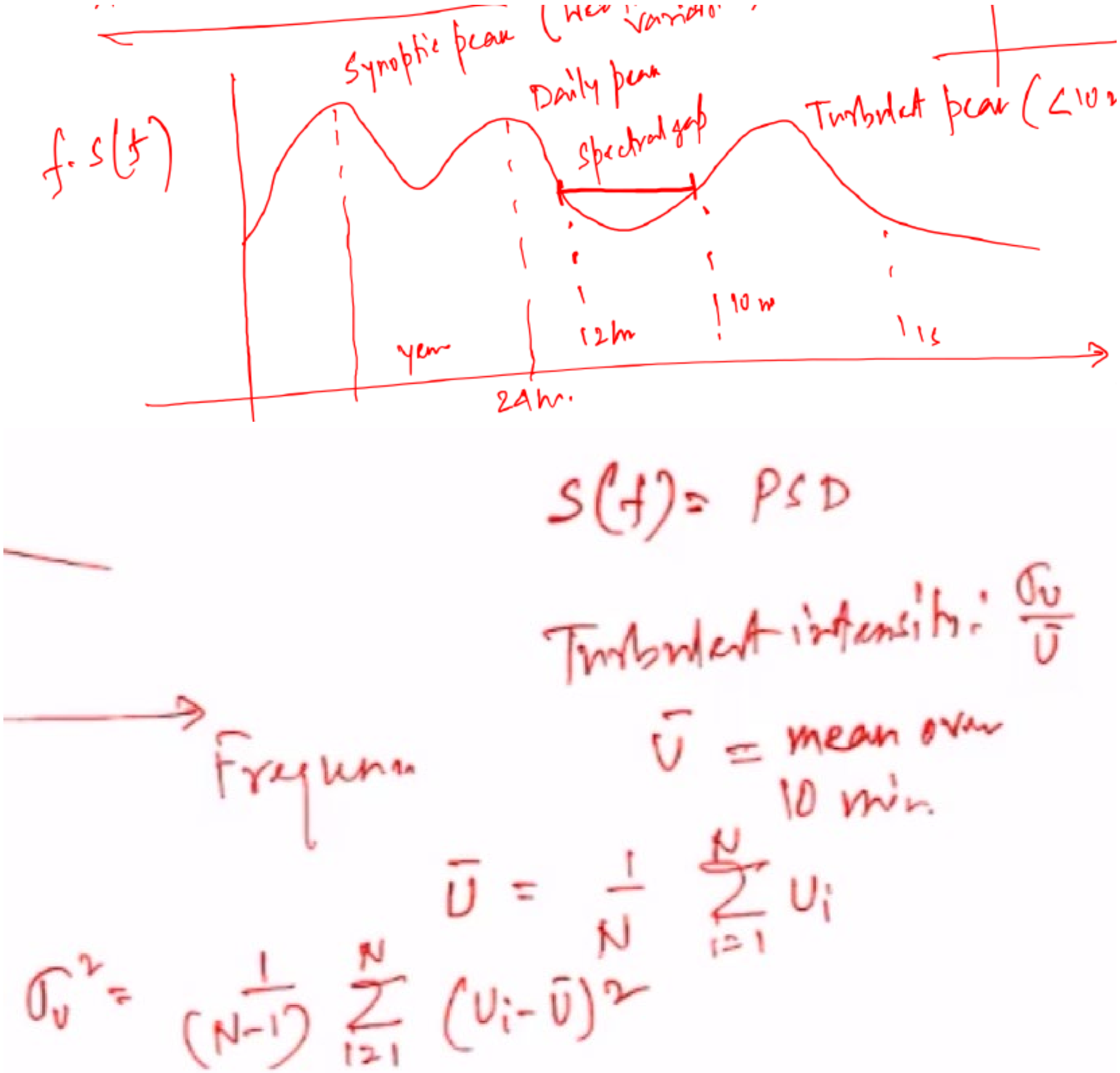
So, one can find out the average power. So, average power per year. So, let's say average power per year, which would be P average is So that's how, I mean, obviously some distribution which should have been given. And once you have the distribution, then you can easily estimate the average power carbon.

That thing. So, now what we can look at the some pixel properties of wind or statistical properties of wind. So, let's say some pixel properties of wind. Okay. So, here what we can have, let's say if your wind speed variation is there, which is some kind of an, then you can find out autocorrelation and PSD. So, which is PSD is nothing but power spectral density.

If a Fourier series is taken, then the spectral density is obtained. It often kind of an, let us say, if you take a Fourier series and then this is the frequency, then it will look like, like that so this is f into s of f here s of f is the phd power spectral density this is an example of courier series and so this is your you can say this is my synoptic peak weather system transition. This say that daily peak. This is called the spectral gap.

Okay. It could be 2 hour to 10 minutes. This is 1 second. This could be less than 10 minutes. This is let's say 24 hour. This is kind of a year.

So, this is a density is a Fourier transform. So, this is how. What happens is that the turbulence happened at time scale which is very small or sampling time scale could be 10 minutes or so. Turbulence intensity one can turbulent intensity can be defined as variation by u bar where u bar is mean over 10 minute. Sigma is a standard deviation.



Then what we can say that u bar is 1 by n, summation of i equals to 1 to n ui and you can write sigma square 1 by n minus 1, summation of i 1 to n ui minus ui square. This is how you can estimate these things. So, another interesting quality is the autocorrelation. So, this autocorrelation is autocorrelation. So, this is a method of finding repeating pattern such as periodic wind patterns.

In case of wind it is looking for dependence of wind on conditions of the previous instant of time. So. the autocorrelation function, which is let's say r, rt delta, one can estimate, let's say r delta, 1 by sigma square, n minus r, 1 to n minus r, ui minus ur square, ui plus r minus u bar. Here the delta t is sampling time, r is the lag number and r delta t is the lag time. So, if you can plot that, let's say t photo correlation function so, this is t this is let's say integral time scale of t this is my rt photo correlation and this is one so, This graph shows that wind is strongly autocorrelated at a very short lag time, not so strongly at longer lag time.

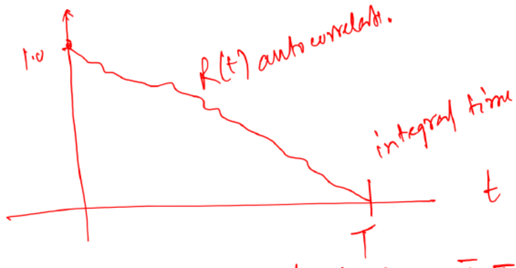
Autocorrelation fn. ($R(r\Delta t)$)

$$R(r\Delta t) = \frac{1}{\sigma^2 (N-r)} \sum_{i=1}^{N-r} (U_i - \bar{U})^2 (U_{i+r} - \bar{U})$$

So, this shows that strong correlation at very short time. but not so strongly correlated in longer lag time. This is kind of expected because we expect wind one second ago to have a big influence on the current wind, while not so much for the wind from a day ago. So, that means in a smaller scale, you would expect more radiation and more influence on the wind speed variation, whereas you do not expect that the impact would be too high from previous day or day before that, or maybe few days before. So, short times variations would be strongly correlated, which clearly you can see from here, but for a longer duration, effects would be as less as possible.

that's how the wind characteristics is all about okay now from here one can estimate integral length scale so you can estimate integral l which is l it u bar into t so this is kind of approximation size of turbulent interruption. This is what you can find out from these different correlations and all these things. What now we can see, so that gives you an idea of about how these things are correlated and not so what we want to look at it again go back to this so these are kind of looking into the distributions and then from there getting the correlations and the integral scale calculations So, what we would like to now talk about and look at the effect of the surface temperature and how the atmospheric labs actually play a role. Obviously, we need to see how temperature variation takes place on the surface. So, we can quickly look at atmospheric density and pressure okay these are important aspect because this going to be because wind power is a function of air density because we know this density so one hand we talk about wind power which is also variation with the density and then density is variation density is also kind of a function of pressure and temperature and temperature again is function of height so there is an interconnectivity essentially Wind power.

$r\Delta t$ = lag time.



integral time scale T } ⇒ Strong correlation at very short time.

Integral length scale $(L) = \bar{U} \cdot T \approx$ size of turbulent interruption.

This is. Wind power. Which becomes. Inherently.

Function of. Temperature. Now. If you assume. That.

Ideal. Gas law. Ideal gas law. then the density and pressure variation can be estimated like that or I can say density is P by RT which is kind of a 3.4837 P by T. So you can get that thing where the units are P is in Pascal, Kelvin, so that's how you can get that. So the international standard atmosphere, assume that the sea level temperature and pressure, if you look at sea level, the pressure would be 101.

325 kPa. And temperature is 288.15 Kelvin. And obviously from here you can get sea level density is 1.225 kg per meter cube. So air pressure decreases with elevation above sea level. Up to 5000 meter elevation the pressure so let's say up to 5000 meter elevation, the pressure variation is like that 101.

29 minus 0.011837 into height plus 4.793 into 10 to the power minus 7 z square. So here z is the elevation. in meter and pressure that we will get here in kp of course the actual pressure may vary about to the standard pressure as weather pattern changes usually in practice at any given location the daily and seasonal temperature fluctuations have a much greater effect on air density than do daily or seasonal change in pressure in air moisture. So, that importance issue is that so one can with the height you can actually get this pressure temperature and all these things. Now, there is a stability of Atmospheric boundary layer.

Atmospheric Density & Pressure (Wind Pow) $P \propto \rho$

$\rho \neq f(p, T)$

$T = f(\text{height})$

Ideal gas law :

$p = \rho R T$; $\rho = \frac{p}{RT} = 3.4837 \frac{p}{T}$

Sea level: $p = 101.325$ kPa

$T = 288.15$ K.

$\rho = 1.225$ kg/m3.

upto 5000m elevation:

$$p = 101.29 - (0.011837) z + (4.793 \times 10^{-7}) z^2 \ (\text{kPa})$$

(z = elevation in m)

This is very very important. We, just talked about that some conditions the boundary layer could be stable at some point of time it could be unstable and the stability is an important factor. The particular important characteristics of the atmosphere is their stability. So here it tries to resist the vertical motion or to support existing turbulence. so the stability of ABL is a determining factor for the wind speed gradient so this is an determining, factors for wind speed radius that is essentially one can think about winds here so which are experienced in the first few hundred meters above the ground so because you have a particular layer boundary layer which is there atmospheric stability is usually classified as stable neutral stable or unstable that we have already seen and the stability of earth's atmosphere is governed by this vertical temperature distribution which will result from the radiative heating or cooling of its surface, so, this could be stable, neutrally stable or unstable and this is connected with the obviously the temperature distributions of the surface which is again directly connected with the radiativity or cooling and subsequent convective mixing of the air adjacent to the surface. So, now we can see how these things changes, so, the thing that we can look at is the lapse rate so the lapse rate of the atmosphere is as the rate of change of temperature i mean this is rate of change of temperature with height so that is what the lapras is So, one can easily determine the lapse rate by considering the change in pressure with height and obviously using conventional thermodynamic relationship.

Now, if the atmosphere is approximated as a dry, no water vapor in the mixture. So you consider, assume the atmosphere to be dry. then ideal gas relation would, I mean, be applicable and what we can write is that the dp is a change in pressure is rho g dz wherem p is atmospheric pressure rho is atmospheric density z is elevation above ground. G is local gravitational acceleration.

So you can assume constant here. You can constant here. This negative sign here this is coming because of the convention that height is measured positively upward. So, since we are going above the height that's what P decreases with the P decreases with the height. So, what we can write from the first law of thermodynamics for an ideal gas closed system of unit mass undergoing a quasi static change of state that we can write that d of q equals to d of u plus p of dv which is one can write dh minus v dp equals to cp dt minus 1 by rho dp okay so this is what you can write and we can use this and connect with the pressure variation with the height we'll try to find out the lapse rate, which is kind of going to give you an idea about the condition of the atmospheric boundary layer, which is stable, neutrally stable, or unstable. That, on the other hand, allows to consider the wind shear.

$$dp = -\rho g dz.$$

$p \rightarrow$ atmospheric pres.
$\rho \rightarrow$ " density.
$z \rightarrow$ elevation above groun.
$g \rightarrow$ local gravitational acc. (assume cst. here)

1st law of TD for an ideal gas closed system of unit mass undergoing a quasi-static change of state:

$$dq = du + p dv = dh - v dp = C_p dT - \frac{1}{\rho} dp$$

T = Temp.
q = heat transferred

That, wind shear is an important factor to be considered during design calculations. Okay, we'll stop the discussion here.