Wind Energy

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Lecture 14: Mechanics of Wind (contd..)

Welcome back to this discussion on these things. So, we are talking about the mechanics of wind or the factors which are influencing or causing to have wind motion. So, we have already talked about so you have pressure differences, Coriolis force, centrifugal force, frictions. So, these are some of the influencing factor that causes the motion of the wind to happen.

So, we talked about how pressure gradients impact. So, from there, we kind of again talked about the Coriolis force and we got the correlation of the geostrophic wind velocity with that. Now we can move to look at some correlation. So, what we can look at this effect of pressure gradient and Coriolis force. Okay. So, geostropic wind is kind of a balance of pressure gradient and the Coriolis force. So, geostropic wind is a balance of pressure gradient and the, Now, what we can look at in a simple case of straight isobars, let us say we can plot like that. So, this is space, this is also space.

This is positive pressure and this is negative pressure. So, what we can have, these are the isobars. So. These are all. I show.

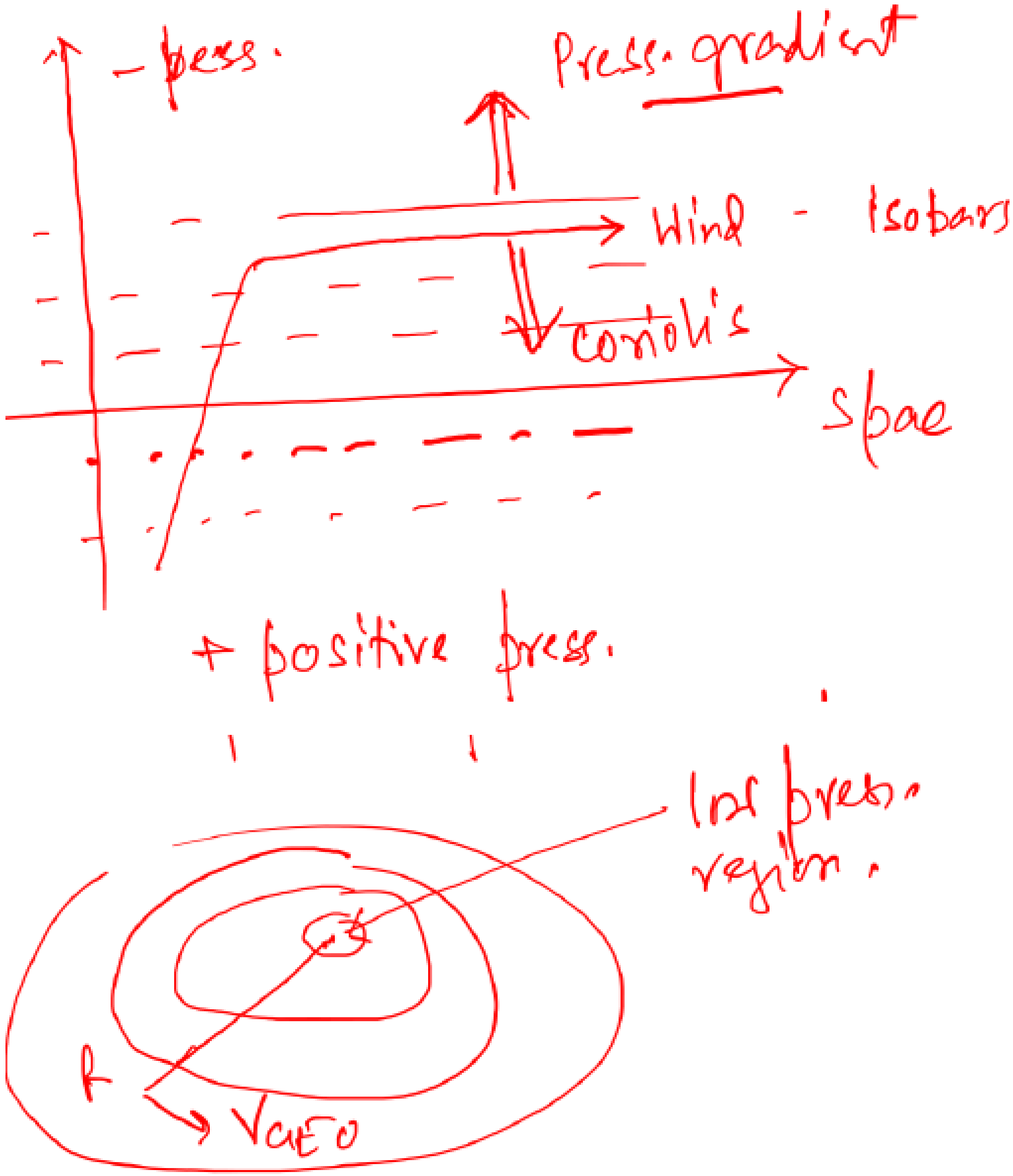
And. You can. So, this is where. You have wind. This is. Pressure.

Gradient. And this side is. your who really is, okay! so, it just like I mean other way one can think about I mean to look at the same thing in kind of in some so, this is R and where you have VGEO, this is your low pressure region. So, this is illustration of the gradient with radius of curvature. So, in this simple picture here, you can see that the pressure gradient pushes the wind going upward and the core release force pushes wind downwards.

So, the result of that the wind travels parallel to the ice over. So, wind travels parallel to isobars. Because of these two forces which is cancelling each other. So, the result is that wind travels in parallel to isobar where the acceleration due to pressure gradient and

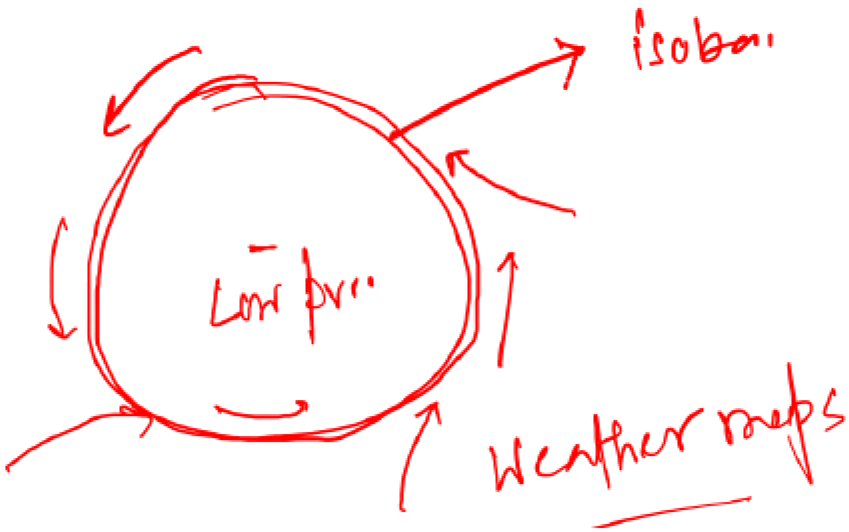
Coriolis effects are balanced. Acceleration which we have 1 by rho by dp dx, 2 sin p omega dot v gE o So, this is the acceleration due to.

Coriolis. Effect. And this is due to. Pressure. Gradient. They balances out.



So one thing you can note here. That. What you can note here is that.

$$\underbrace{-\frac{1}{\rho}\frac{\partial P}{\partial x}}_{\text{press. gradient}} = \underbrace{2\sin\phi\,\omega_0 \cdot V_{GEO}}_{\text{coriolis effect}}$$



VGEO. VGEO. to pressure gradient but parallel to isobars so this is what you get and for that you can expect to have some kind of an weather map obviously this needs to be complete circle so this is your low pressure and this is so what happens is that, so, this is how the in with the map you get to see the low pressure as we have seen here low pressure zone and high pressure zone which are getting created now we see the centrifugal acceleration. Geostrophic we will consider the pressure gradient and correlation but, however when isobars are curved which is almost always the case so, that if we draw a situation let's say, this is low pressure zone, this is radius and, then this is incoming wind stream and, this goes like that so, this goes like that This is VG equal to R omega. So, geostrophic wind consider this pressure gradient and the Coriolis force. But the isobars which are most of the times are cut. There is a third force which affects the wind is the centrifugal force.

So, obviously this arises. From. Traveling. In a circular path. So, anybody is traveling in a circular path.

This is. The. So, this shows the. Situation where a circular. This is a circular. Isobar. And wind is travelling along the isobar.

Wind is travelling along the isobar. So, this is the figure is all about. So, the refinement of this VGO is the VG which is the gradient wind. and that would be at any radius if something is rotating with an angular velocity, then you can get that, so, my acceleration would be v g square by r which is omega square r equals to we can write omega square r square by r so r here is the radius of curvature of isobar. So, this is the gradient wind which is one can say the speed of wind along isobar.

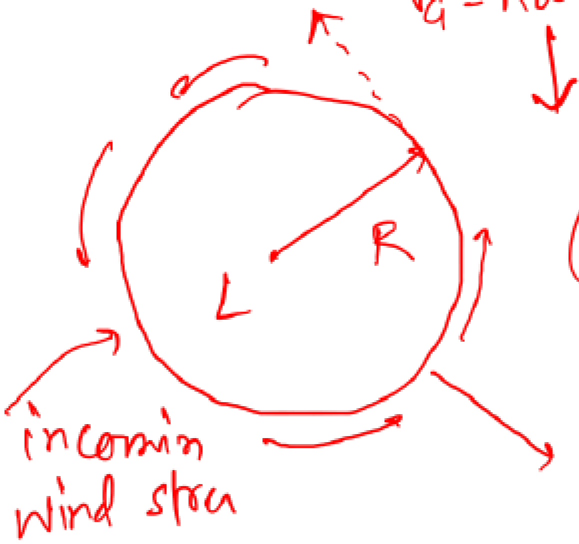
But one can note that VG is not equals to VGEO. Okay. But this is still parallel to the isobar. So, what we can do now so hence the calculating in acceleration which is minus 1 by rho del P by del X. So, one extra term is added due to this extra centrifugal force.

So, there would be 2 sine phi omega naught VG. So, plus VG square by R. So, this is due to this is due to centrifugal. So, what we can now write that vg square plus 2r omega naught sin phi vg plus minus del p by del x into r by rho zero so we get vg equals to minus r omega naught and phi plus minus r square omega naught square sine square phi minus del p del x by r by rho so what it here one has to note Vg is less than VGE0. To assess the relevance of centrifugal force compare core release the 2 sin 5 omega VGE0 with once we compare the core release that is core release is 2 sin phi omega naught vg and centrifugal which is vg square by r.

$V_G = R\omega$

Gradient wind
(speed of wind along isobar)

$V_G \neq V_{GEO}$



circular isobar & wind is travelling along the isobar

Centrifugal force — arises from travelling in a circular path

$$a = \frac{V_G^2}{R} = \omega^2 R = \frac{\omega^2 R^2}{R}$$

R = radius of curvature of isobar

$$a = -\frac{1}{\rho}\frac{\partial P}{\partial x} = \underbrace{2\sin\phi\, \omega_0 V_G}_{\text{Coriolis}} + \underbrace{\frac{V_G^2}{R}}_{\text{centrifugal}}$$

$$V_G^2 + (2R\,\omega_0 \sin\phi) \cdot V_G + \frac{-\frac{\partial P}{\partial x} \cdot R}{\rho} = 0$$

$$V_G = -R\omega_0 \sin\phi \pm \sqrt{R^2 \omega_0^2 \sin^2\phi - \frac{\frac{\partial P}{\partial x} \cdot R}{\rho}}$$

Note! $V_G < V_{GEO}$

coriolis $(2\sin\phi\, \omega_0 V_G)$

Centrifugal $\left(\frac{V_G^2}{R}\right)$

Ratio: $\frac{2\omega_0 \sin\phi \cdot V_G}{V_G^2/R} = \frac{2\omega_0 \sin\phi\, R}{V_G}$.

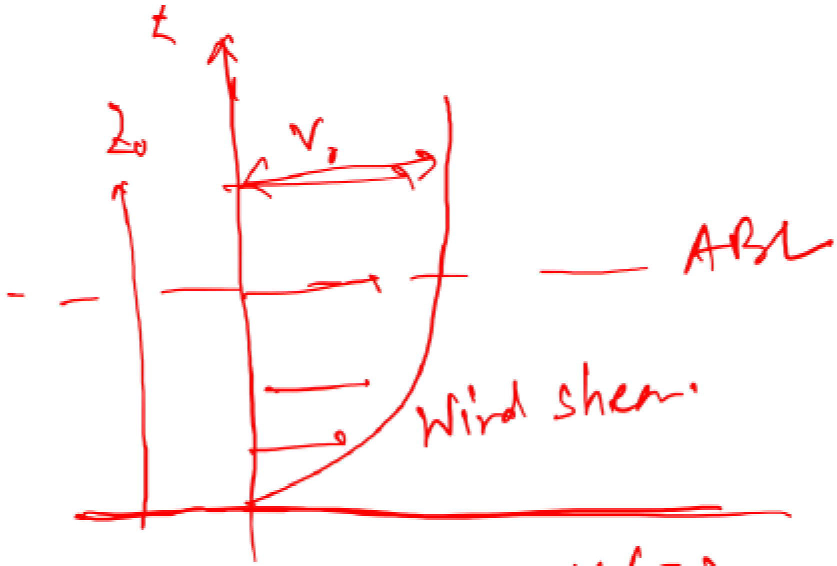
We can compare the ratio between these two. So, the ratio is 2 omega naught sin phi vg vg square by r. So, this gets omega naught and phi r by h. So, one can get an estimate between correlates and centrifugers which is going to dominate and try to find out what is going to be important and not.

Okay. So, now we are going to look at the effect of, so, friction essentially this depends on surface properties, but generally it slows down the air only in the atmospheric boundary layer. So, friction decreases pore release. and centrifugal forces. So, therefore what happens at a very low altitude winds tend to move towards the direction of the negative pressure gradient. So, that's why at very low altitude wind in more towards the direction of negative pressure gradient.

But at Earth's surface, if you see, so this is why it goes like that. So, this is atmospheric boundary layer. This is because of winds here. So, at Earth's surface, here the wind speed is zero. So, wind shear is often destined by a logarithmic profile.

Wind shear is always defined by a logarithmic profile. So one can say that if this is, let's say V(E) and at a height Z0, this is let's say V naught and this is direction. Then Vnaught is speed at altitude z equals to z0. Zr is roughness length. So, this is typically order of few millimeter or flat grounds, then you will have this V(Z), V naught log of Z by Zr log of Z0 by ZR.

So, this is the typical logarithmic profile that can be used to calculate the wind shear. I mean, this is something one can think about this friction that's going to. So, we would talk, I mean, look at this wind shear and effect of the wind shear when we'll be talking about the design aspect of it. So, this another parameter which is very important. So, we try to see this how this weather maps actually shows the low pressure high pressure zones and how the wind actually forms.



$V(E)$

$$V(z) = \frac{V_0 \cdot \log(z/z_r)}{\log\left(\frac{z_0}{z_r}\right)}$$

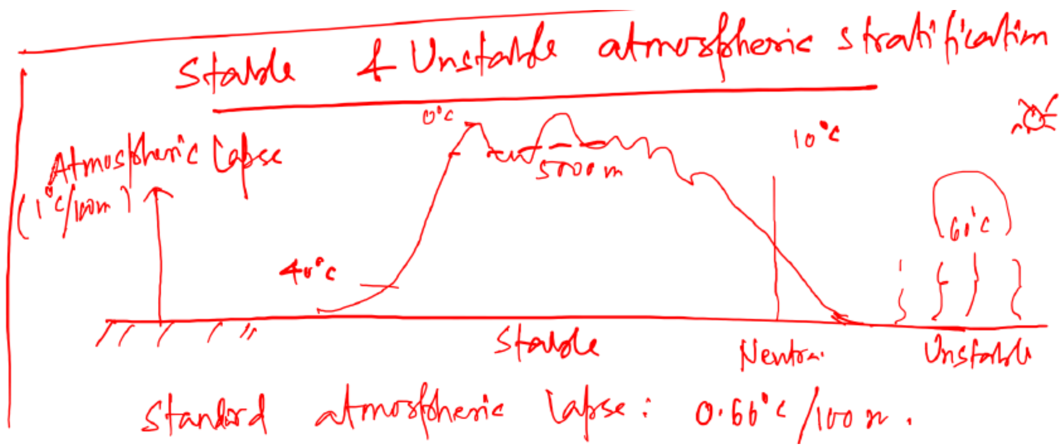
Wind shear

V_0 = speed at altitude $z = z_0$

Z_r = Roughness length. (~ mm. for flat grounds)

It's all because of this different altitude and rotation of the earth. Okay. So, now what we look at in there are also issues due to issues like, we can say that, stable and unstable, atmospheric stratification. So, what happens is that, When the sunlight falls on earth's surface, the surface becomes hot, so also rising piece of air becomes relatively hotter, so it rises. And rising air actually expands and therefore gets cooler.

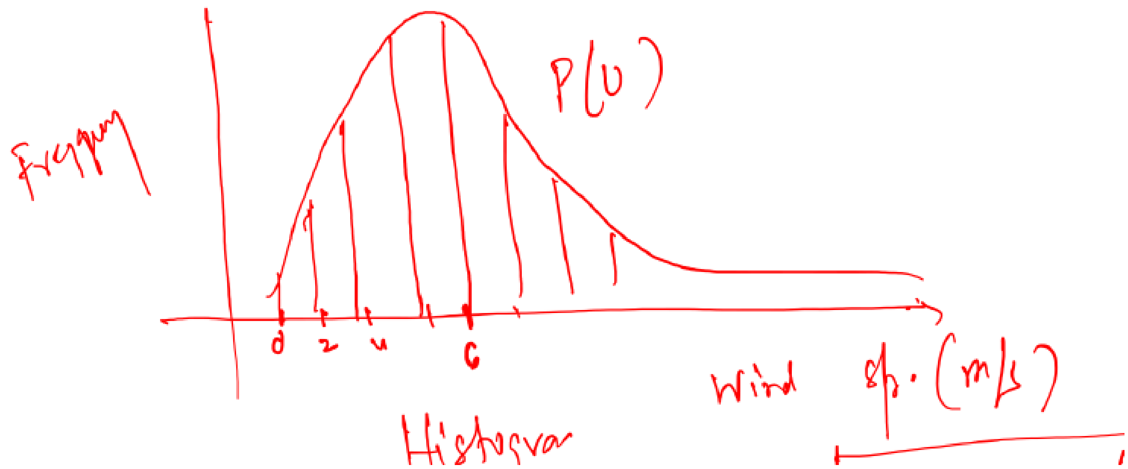
So, there is a dry adiabatic lapse rate which is about 1 degree per 100 meters. If I have to kind of plot it, so let's say then, so this is stable zone. So, that is, let's say some kind of a 5,000 meter.



This is something neutral. Okay. this is unstable because let's say like this there is a sun at this location this could be 10 degree c this is what happens let's say This is my atmospheric lapse, which is typically 1 degree C per 100 meter. So, if the ambient air gets cooler, slow down one degree per meter, it means that the atmosphere is stable. If it gets cooler faster, it is unstable. Atmospheric lapse plays an important role. So, the typical standard atmospheric lapse, which is 0.

66 degree C per 100 meter. So, this is correspond to a stable stratification even more stable is an inversion. So, generally wind shear is stronger for stable condition, less mixing between layer occurs. So, thus you have less momentum is transferred at strong winds, mixing less to neutral condition. So, obviously the atmospheric lapse rate equals atmospheric these things. what we can look at is that statistics of wind so at a given site wind speed and direction, so, that means the speed and direction is a function of time so, if only speed is regarded one can plot time series data let's say we can plot like that this is time and this is let's say one location any more meter So, this can vary like that.

So, one can, this is essentially one can think about that, let's say, hourly average over one year. It's just some sample data. So, once you have, you can find out the mean and also the variance with the hourly average with wind speed over the year okay, so, what you can have let's say varying wind speed and you can have let's say zero two four six eight and so on so i mean, you can have some kind of a curve like that with all this data so this is frequency this is probability of u this is one speed meter per second this is called the some kind of a histogram so different distribution can be used to describe this p(u) which is probability density probability density function of wind speeds that is the pdfs and we can compute It's Q, which is the cumulative distribution function, which is CDA. So, these are some kind of a statistical characterizations of the distribution, which comes from, again, statistical mechanics, but this is used in turbulence as well. So, the property that it satisfied, Q of Q use 1.



P(U) → probability density fn. of wind speeds (PDFs)

F(U) → cumulative distribution fn. (CDF)

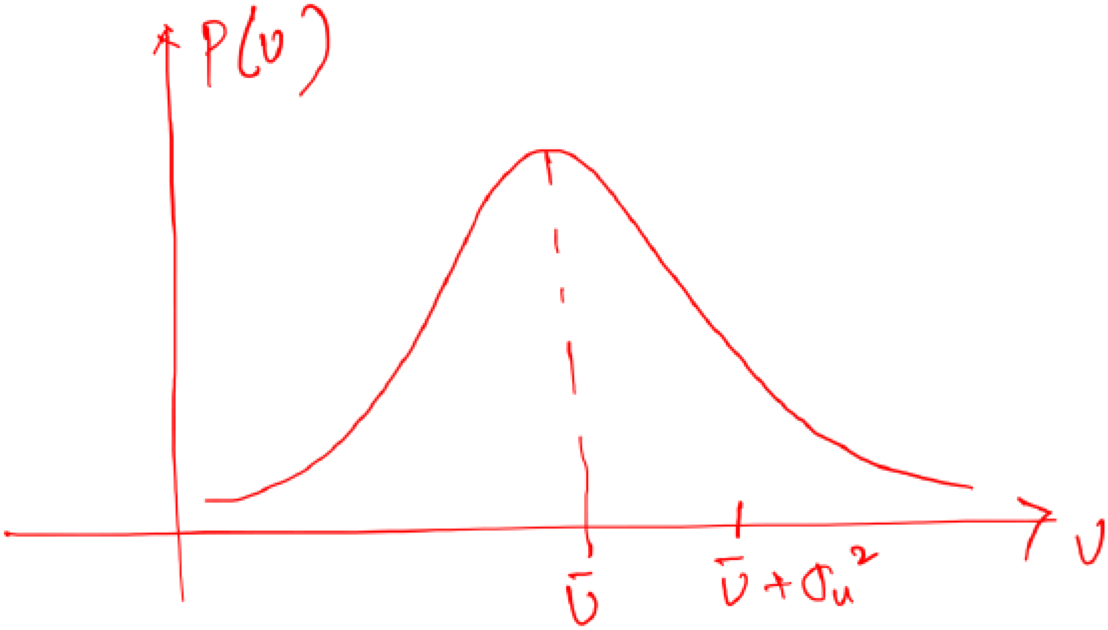
$$\int_0^{\infty} P(U) \cdot dU = 1$$

$$F(U) = \int_0^{U} P(U)\, dU$$

okay, and one can write a few is zero to u, u u d u so that means i can have e of u is f of u prime so that is the relationship that one can have between one can have between this probability distribution functions with the cumulative distribution function. So, we can have some example of different distribution function. So, the simple one could be Gaussian or normal distribution. So, which can be like this.

I can have U which goes like that. This is U bar, then this is U bar plus sigma U square. This is probability of U. So this is a simple Gaussian distribution or normal distribution and the distribution function is well known which one can write 2 pi sigma u square into exponential minus u minus u bar square by 2 sigma u square. So, that's the distribution function for Gaussian distribution that you have. Similarly, Another distribution which we have already talked about, that is, so this distribution, I mean, we talked about is that we will distributions, that is another kind of distribution function, which we'll look at it, how the, so this distributions are primarily coming because as we have seen that the wind speed is varying with time.

As, I have been emphasizing that the variation is over the month, hour, week, they are varying. Then you try to make a distribution of that particular. But again, this is very, very specific to a particular location where you would like to estimate that. And, then the distribution function, which is very critical, in getting the wind power calculated because we have already seen these distribution functions are definitely advantageous in getting power calculations compared to the power calculation based on average wind speed. Okay, we will stop here and we will continue this distribution discussion for next session. Thank you.



Examples

(a) Gaussin (Normal) distributi

$$P(U) = \frac{1}{\sqrt{2\pi\sigma_u^2}} \cdot \exp\left(-\frac{(U-\bar{U})^2}{2\sigma_U^2}\right)$$