Introduction to Aircraft Control System

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Week – 12

Lecture – 60

Review of the Course

This is the last lecture for this course, Introduction to Aircraft Control System. Here I would like to revise some important topic what you have done in the last few lectures. First, let me rewrite in short form the linearized longitudinal motion.

$$\dot{X} = AX + BU$$

$$\begin{bmatrix} \Delta\dot{u} \\ \Delta\dot{w} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \end{bmatrix}_{4x1} = [A]_{4x4} \begin{bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta\theta \end{bmatrix}_{4x1} + [\square]_{4x2} \begin{bmatrix} \Delta\delta_a \\ \Delta\delta_t \end{bmatrix}$$

So, now if we want to find the transfer function for example, $\frac{\Delta u(s)}{\Delta\delta_a}$. So, for this case A matrix will be as it is, but B matrix we have to modify to find this transfer function. So, since B matrix has 2 columns where first column indicates the control $\Delta\delta_a$ and second column indicates $\Delta\delta_t$. So here we have to consider the first column of B matrix. And C matrix we have to modify accordingly. C matrix since we are considering the output, the output state should be u(s). So, C matrix we have $[1 \quad 0 \quad 0 \quad 0]$ multiplied by X vector. So, we will have Δu. So, based on this kind of manipulation in the matrices we can come up with the transfer function. And the formula we can use

$$\frac{\Delta u(s)}{\Delta\delta_a} = C(SI - A)^{-1}B$$

So, this is how we can find the transfer function. Now, similarly we can find $\frac{\Delta w(s)}{\Delta\delta_a}, \frac{\Delta q(s)}{\Delta\delta_a}, \frac{\Delta\theta(s)}{\Delta\delta_a}$. The C matrices would be $[0 \quad 1 \quad 0 \quad 0], [0 \quad 0 \quad 1 \quad 0]$ and $[0 \quad 0 \quad 0 \quad 1]$ respectively. A and B matrices would be same. Similarly if you want to find $\frac{\Delta u(s)}{\Delta\delta_t}, \frac{\Delta w(s)}{\Delta\delta_t}, \frac{\Delta q(s)}{\Delta\delta_t}, \frac{\Delta\theta(s)}{\Delta\delta_t}$. Then A matrix would be same and B matrix would be the second column of the control vector. Similalrly C matrices would be

$[1 \quad 0 \quad 0 \quad 0], [0 \quad 1 \quad 0 \quad 0], [0 \quad 0 \quad 1 \quad 0], [0 \quad 0 \quad 0 \quad 1]$ respectively. So, this is how we can find the transfer function of the individual states with respect to different control input. Now we can apply the classical control or we can design the autopilot.

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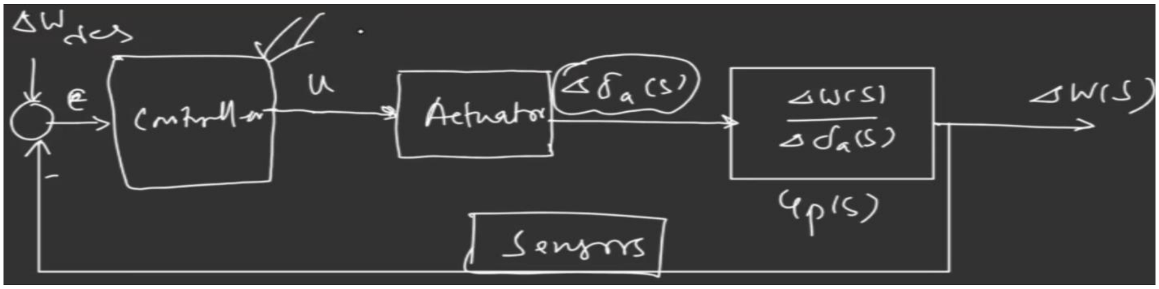

So, for example, if you want to design the autopilot for this system $\frac{\Delta w(s)}{\Delta \delta_a}$. So, this is the system and please remember this transfer function is basically plant transfer function. So, this is basically our $G_p(s)$ and we have the control input which is basically $\Delta\delta_a$ and this is given by the actuator. And into the actuator, we can apply some control u, it can be in terms of voltage because we are controlling the motor which can provide the required torque to the system. And this is the controller and this is error which is going to affect and this is the summing point and here we can set $\Delta w_{desired}$. This is the feedback loop here and $\Delta w(s)$ is the output and we have sensor here. So, this is how we can design the classical control system for the any transfer function in the above cases. . So, this is how we can apply the different control technique here and we can come up with our mission objective for the specific control system.

Now, I would like to highlight what are the topics which were covered in this course. Since this is the last course I would like to go through the conclusion what are the different topics we have covered from different perspective, I would like to highlight. So, here first we started with the need for study of the control system. So, here we have discussed the importance of control system in aircraft for stability. So, these are the reasons why we have to study the control system. Then in the second part we have studied understanding of control system. So, here we have covered definitions of control system and their importance in aircraft. And also we have studied about the types of control system which are basically the open loop and closed loop control system. And also we have discussed what are the components in closed loop control system.

So, here we have discussed from different perspective like controller, then actuator, then plant, then we have talked about sensors. So, these are the main different blocks existing in the control system. So, these things actually come under control, navigation and guidance. And how these three different systems connected in closed loop such that the desired objective can be fulfilled. Then we have discussed about the linear LTI system, linear time invariant system.

So, here we have used the principle of superposition to fulfill the linearity of the system. And also how to identify if a system is LTI. Then next part we have discussed about the equilibrium point, which is one of the most important part in designing the control system. So, here we have discussed definition of equilibrium point and its significance in the control system. Also we have discussed how we can find the equilibrium point with examples. We have considered the pendulum system to find the equilibrium point. And then also we have used the small perturbation theory to find the linearized system of the non-linear system. So, here we have used the pendulum system and applied small perturbation theory. And then we have started with stability analysis. So, under this part we have discussed the concept of stability. Here we have discussed what it means for a system to be stable at an equilibrium point. Then we have discussed how we can study the stability of the linear system. So, here methods to determine system stability we have discussed. Then we started with the transfer function, how we can study the transfer function and how it is important for classical control synthesis.

So, here we have covered definition and importance of classical control synthesis. Also we have covered the derivation of transfer function from the linear system. Here, we have given the process and examples. So, here we have considered the aircraft attitude control problem. Thereafter the transfer function we have studied, so transfer function we can look at from different angles. So, we have studied open loop and closed loop transfer function in the presence of reference input and noise in the system. Then we have discussed how we can come up with the PID control. So, here we have discussed proportional control, derivative control and PID control. And what are the significance of the individual controls in the closed loop control system, we have discussed them in detail. Then we have discussed the time domain analysis. So, here we have covered two parts. So, here we came up with the different specifications for improving the transient response of the system. Then we have started with the steady state response. So, here we have discussed in the transient response about settling time, rise time, peak time, maximum overshoot. So, how this specification are very important for modifying the transient response.

So, in steady state response basically we have discussed the error analysis, how can we minimize the steady state error in the system. Then also we have discussed effect of disturbance in the control system. And then we have discussed, how can verify the closed loop system, whether it is stable or not. So, for that, we have come up with the Routh stability criteria. So, here we have come up with the concept of marginal stability. How many poles are on the right-hand side based on the polarity changes in the first column in the Routh table, we have discussed that as well. And then we have started with the root locus method. So, how we can graphically reproduce the locus, how the poles are changing in the S-plane, we have discussed them in detail. So, here we have discussed how the locus of the poles are moving in the S-plane, as we change the gain from zero to infinity. So, we have discussed in detail, how we can compute the root locus and how we can study and design root locus plot for the open loop transfer function and how we can study the closed loop system behavior. Then from the concept of root locus and Ziegler-Nichols method, we came up with the PID control design. And we have had many examples how we can come up with this PID control using the root locus and Ziegler-Nichols method. Then we have shifted our attention to the frequency domain. In frequency domain we have covered the time domain specification with the frequency domain specifications. So, basically the relation between frequency margin with time domain specification.

Then we have started the frequency response how we can analyze the Bode plot. So, here we have discussed how we can come up with the magnitude and phase plot for varying frequency. So, here we have discussed also the phase crossover frequency and gain crossover frequency and which gives us the gain margin and phase margin. And we can also come up with the relative stability. Based on relative stability we can come up with whether the system is stable or not. Then we have concluded with an example on aircraft attitude control problem using PID control. So, here you have used the frequency analysis concept. So, these are the contents we have discussed in the first half of the classical control part. Then, we started how we can come up with the aircraft motion in 6 DOF. So, then we have started the 6 degrees of freedom equation of motion of the aircraft. So, here we came up with the two sets of equation. One set is represented by the longitudinal motion and another one is the lateral directional motion. So, for both the cases we came up with the linearized model using the concept of small perturbation theory. After the linearization is to this equation, then we came up with the system in form of $\dot{X} = AX + BU$ which is basically the state space model. So, here then we came up with different approximations. For the longitudinal motion, we had two approximations.

One is the short period and another is the phugoid or long period. So, these motions are characterized by the roots of the characteristic equation or the roots of the system matrix. So, here we have also discussed the dynamic analysis based on the period, cycle and time to half and full magnitude. So, these are the topics we have covered in the longitudinal approximation. Then we also discussed about the approximations in lateral directional motion. So, here we came up with the three different approximations. One was the spiral, second roll mode and then the Dutch roll. So, these are the approximations we came up with the roots of the characteristic equation. And based on the characteristic equation, we came up with eigen values based on the damping ratio and natural frequency of the roots.

Then also we have discussed the dynamic analysis for these three motions using the concept, what we have done for the longitudinal approximations. Then we discussed how we can come up with the transfer function for the state space model. So, here $\dot{X} = AX + BU$. This is the state space model and how we can come up with the transfer function for the different states with the different controls. So, here we have used two methods. One method was a brute force method. Here we have used the Cramer's rule. And another method was $C(SI - A)^{-1}B$. And using these two methods, we can come up with the transfer function for all the states with respect to different inputs. And once we have the transfer function, we can apply the linear controls, what we have done many times in this course.

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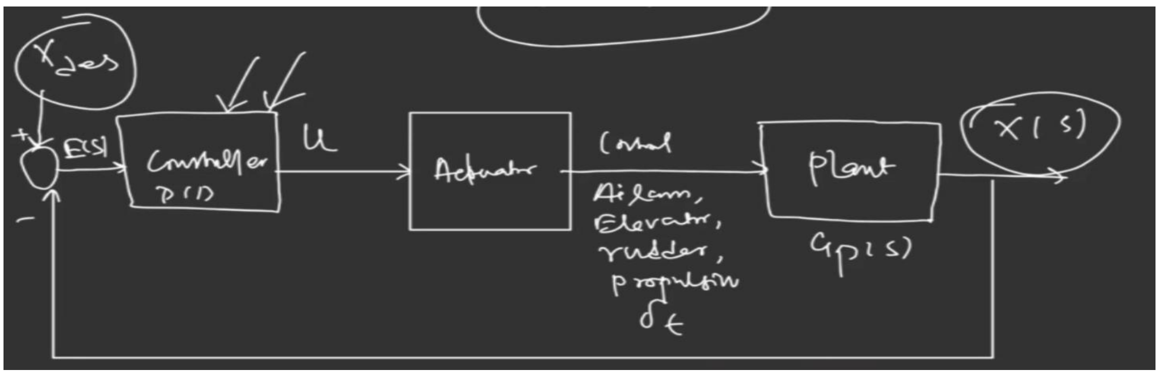

Suppose this is the plant, which is written in terms of transfer function. This is basically $G_p(s)$. Then we can provide the control, which can be your aileron or elevator, propulsive force, which is maybe δ_t. And these are the control inputs, which is going to the plant. And this actuator is affected by the control input, which can be given by the controller. In the first part in this course also, we have come up with different concepts how we can design PID control. So, in this course, mostly we have designed the PID control. And here if you want desired value X_{des} to be tracked. So, error is $E(s)$ and $X(s)$ is the output. So, how we can design this control effectively with the concept we have discussed in this course, which can help us to track X_{des} and X. So, this is how we can study the autopilot for the aircraft system and the concept we have discussed in this course can be applied to any other dynamical system. Because only the system will change, but the concept of designing control will be remain same. These are very introductory course, how we can come up with the basic controls for the different systems, of course, including the aircraft systems. Thank you very much and wish you all the best and I hope you are going to design your own autopilot for the systems, for the aircraft or the UAV you are working on. Thank you very much.