Introduction to Aircraft Control System

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Week – 12

Lecture – 59

Transfer Function for Longitudinal Approximations (Cont.)

In this lecture, we will see how can we find the transfer function of a coupled system and also how can we design that closed loop control system, we will discuss in detail. So first let me discuss the problem. So, we have considered the linearized pitch dynamics of an aircraft at a steady state or steady level flight

$$\Delta\dot{\alpha} = -0.313\Delta\alpha + 56.7q + 0.232\Delta\delta_e$$

$$\dot{q} = -0.0139\Delta\alpha - 0.426q + 0.0203\Delta\delta_e \dots\dots Eq(1)$$

$$\Delta\dot{\theta} = 56.7q$$

Where q is pitch rate, $\Delta\alpha$ is deviation of angle of attack from stream angle of attack α_0, $\Delta\theta$ is deviation of pitch angle from trim pitch angle θ_0, $\Delta\delta_e$ is change in elevator deflection. The elevator servo dynamics is given by

$$\Delta\dot{\delta}_e = -10\Delta\delta_e + 10v$$

Here v is the input voltage or control voltage to the elevator servo. And if you see the closed loop block diagram for this particular system,

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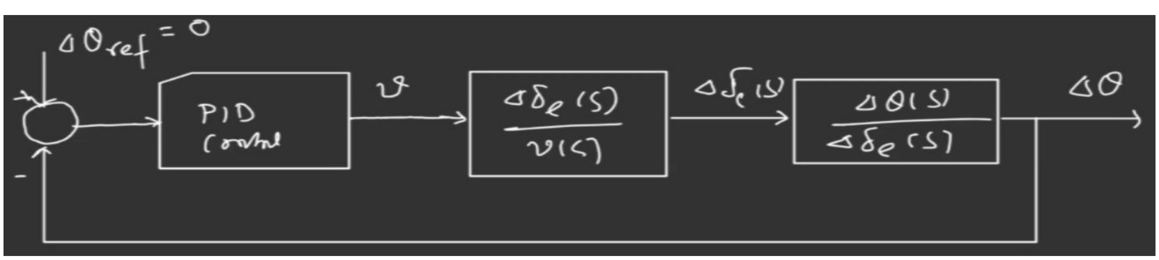

We have to design PID controller. We will go step by step how we can design the PID controller. The output of the PID controller is generated from a data stream which is going to the servo system here which is $\frac{\Delta\delta_e(s)}{v(s)}$ and which gives the elevated control to the plant which is denoted by $\frac{\Delta\theta(s)}{\Delta\delta_e(s)}$ and the output is $\Delta\theta$ and we are having summing point here where we have desired value of $\Delta\theta_{ref}$ which is assumed to be zero for this particular problem. So this is the closed loop block diagram for this particular problem and the first question is find out the transfer functions $G(s) = \frac{\Delta\theta(s)}{\Delta\delta_e(s)}$ which is basically the plant transfer function and also we have to find $H(s) = \frac{\Delta\delta_e(s)}{v(s)}$.

Second question is to sketch the root locus plot for the system with OLTF=G(s)H(s). So we will design the PID control for this problem using the root locus and Ziegler-Nichols method. Third question is to design a PID controller for the above system using Ziegler-Nichols method. So this is the problem for us now we will go step by step how we can solve this problem. So here if you notice we have to find the transfer function $\frac{\Delta\theta(s)}{\Delta\delta_e(s)}$. But if you look at our dynamics model, this is coupled actually. So we have to use the brute force method or the crammers rule, what we have discussed in the previous lecture we can apply those method and we can find this transfer function. Here I am not writing the dynamics part again, I am directly applying the Laplace transform of those equations. Applying Laplace transform to Eq. (1) by assuming zero initial conditions for each state variable and writing in matrix form

$$\begin{bmatrix} s+0.313 & -56.7 & 0 \\ 0.0139 & s+0.426 & 0 \\ 0 & -56.7 & s \end{bmatrix} \begin{bmatrix} \Delta\alpha(s) \\ q(s) \\ \Delta\theta(s) \end{bmatrix} = \begin{bmatrix} 0.232 \\ 0.0203 \\ 0 \end{bmatrix} \Delta\delta_e(s)$$

$$\frac{\Delta\theta(s)}{\Delta\delta_e(s)} = \frac{1.151s + 0.1774}{s^3 + 0.739s^2 + 0.9215s}$$

So this is the plant transfer function. Now we have to find the H(s), so let's take another equation which is the servo system and if you apply the Laplace transform with zero initial condition we can find

$$H(s) = \frac{\Delta\delta_e(s)}{v(s)} = \frac{10}{s+10}$$

Now to the second part of question.

$$OLTF = G(s)H(s)k$$

$$= \frac{k\ 10\ (1.151s + 0.1774)}{s(s+10)(s^2 + 0.739s + 0.921s)}$$

Zeros: $z_1 = \frac{-0.1774}{1.151} = -0.1541$

Poles: $p_1 = 0, p_2 = -10, p_{3,4} = -0.3695 \pm (0.886)i$

Here, $n = 4, m = 1, n - m = 3$

Angle of asymptotes: $60°, 180°, -60°$ ...

Centroid: $\frac{-10.5849}{3} = -3.5283$

p_1, p_2 and z_1 lie on the real axis. As one angle of asymptote is 180 degree so one branch will go from $p_2 = -10$ to $-\infty$ and another branch will go from $p_1 = 0$ to $z_1 = -0.1541$ Now, angle of asymptote from p_3:

$$= 180° + \angle(p_3 - z_1) - \angle(p_3 - p_2) - \angle(p_3 - p_1) = 75.76°$$

As the root locus is symmetric w.r.t the real axis, angle of departure from p_4 will be $-75.76°$

Intersection with imaginary axis:

$$s(s + 10)(s^2 + 0.739s + 0.9215) + 10k(1.151s + 0.1774) = 0$$

For intersection with $j\omega$ axis, $s = j\omega$, Solving , we get $\omega = 0$ and

$$\omega = \pm\sqrt{\frac{9.215 + 11.5k}{10.74}}$$

Here we can find $\omega = 0$ and $k = 0$ is satisfied or you can say is a solution. Again we can write

$$\frac{8.311 \pm \sqrt{8.311^2 - 4 * 1.774 * k}}{2} = \frac{9.215 + 11.51k}{10.74}$$

We find $k = 5.6031$ and $k = -0.9937$. As $k > 0$, we have $k = 5.6031$. Also $\omega = \pm 2.6197$. So the root locus intersects imaginary axis at $\pm 2.6197i$. Hence the ultimate value is $k_u = 5.6031$ and $\omega = 2.6197$. This k value we can find from the another approach what we have done in the first half of the course which is basically the Routh-Herwitz criteria, through that Routh table we can also find this ultimate k. From the characteristic equation we can find the Routh table and we can find the value and now if we are having this frequency we can also find the time period

$$T_u = \frac{2\pi}{\omega} = 2.39$$

So using these two parameters we can find the PID gains using the Ziegler-Nichlos as following

$$k_p = 0.6k_u = 3.3619$$

$$k_i = \frac{0.6k_u}{0.5T_u} = 2.8034$$

$$k_d = (0.6k_u)(0.125T_u) = 1.0079$$

So these are the gains for the PID controls, now you can structure the PID controller as

$$3.3619 + \frac{2.8034}{s} + 1.0079s$$

This is the PID control structure which can be appropriate in our control block diagram and from this we can come up with a system which is going to be stable because this is the gain we found through the Ziegler-Nichols rule. So let's stop it here, we have another lecture in this course where I will give the brief of the course, how it was conducted, what were the contents and I will give brief conclusion of this course. Then we'll wind up this course. Thank you.